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A Fuzzy Data Envelopment Analysis model to evaluate the Tunisian banks efficiency

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Abstract— Data Envelopment Analysis (DEA) is widely applied in evaluating the efficiency of banks with crisp inputs and outputs. However, in real-world problems inputs and outputs are often imprecise and vague. In this paper, we develop first a method based on arithmetic operations to solve fuzzy data envelopment analysis models (FDEA). The method transforms the FDEA model into linear programming problem which gives crisp efficiency scores. Then we propose a methodology to evaluate the performance of commercial Tunisian banks in terms of several financial and non-financial data. FDEA is used to calculate the efficiency score of each bank. The results show that, in a competitive environment, non-financial inputs and outputs should be taken into account in order to obtain credible and realistic efficiency scores.

Index Terms— Arithmetic operations, Efficiency, Fuzzy Data Envelopment Analysis, Tunisian banking sector.

1. INTRODUCTION

Data Envelopment Analysis (DEA), developed by Charnes et al. [1] has emerged as an important tool to evaluate the efficiency of a set of "Decision Making Units" (DMUs) using multiple inputs to product multiple outputs. It has been extensively applied in performance evaluation and benchmarking in a wide variety of contexts including educational departments in public schools and universities, health care units, agricultural production and essentially banks. While traditional DEA requires precise data for its analysis, the evaluation environment often involves vagueness and uncertainty. As system complexity increases, obtaining precise data becomes a difficult task. Furthermore, decision-makers often think and operate based on vague linguistic data (e.g., quality is "good", on time performance is "poor"). In these cases, fuzzy set theory can be a powerful tool to quantify imprecise and vague data in DEA models. FDEA models (DEA models with fuzzy inputs and fuzzy outputs) take the form of fuzzy linear programming models. Several methods were developed to solve FDEA models. These methods are usually categorised into four approaches. The tolerance approach was one of the first fuzzy DEA models that was developed by [2] and further improved by Kahraman and Tolga [3]. In this approach the main idea is to incorporate uncertainty into the DEA models by defining tolerance levels on constraint violations. This approach fuzzifies the inequality or equality signs but it does not treat fuzzy coefficients directly. Although in most production processes fuzziness is present both in terms of not meeting specific objectives and in terms of the imprecision of the data, the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp.

The α-level approach is the most popular FDEA model. The main idea is to convert the fuzzy DEA model into a pair of parametric programs in order to find the lower and upper bounds at an α-level of the membership functions of the efficiency scores. Kao and Liu [4] used this approach to transform the fuzzy DEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Their method found approximately the membership functions of the fuzzy efficiency measures by applying the α-level approach and Zadeh's extension principle [5]. Saati et al. [6] suggested a fuzzy CCR model as a possibilistic programming problem and transformed it into an interval programming problem using α-level based approach. The resulting interval programming problem could be solved as a crisp LP model for a given α with some variable substitutions. Saati and Memariani [7] suggested a procedure for determining a common set of weights in fuzzy DEA based on the α-level method with triangular fuzzy data. Liu [8] developed a fuzzy DEA method to find the efficiency measures embedded with assurance region (AR) concept when some observations were fuzzy numbers. He applied an α-level approach and Zadeh's extension principle to transform the fuzzy DEA/AR model into a pair of parametric mathematical programs and worked out the lower and upper bounds of the efficiency scores of the DMUs. Wang et al. [9] proposed a fuzzy DEA–Neural approach with a self-organizing map for classification in their neural network. Kao and ling [10] formulated developed a pair of two-level mathematical programs to deal with qualitative data.

The fuzzy ranking approach is also another popular technique that has attracted a great deal of attention in the fuzzy DEA literature. In this approach the main idea is to find the fuzzy efficiency scores of the DMUs using fuzzy linear programs which require ranking fuzzy sets.

The fuzzy ranking approach of efficiency measurement was initially developed by Guo and Tanaka [11]. Tlig and Rebai [12] proposed an approach based on the ordering relations between LR-fuzzy numbers to solve the primal and the dual of FCCR. They suggested a procedure based on the resolution of a goal programming problem to transform the fuzzy normalisation equality in the primal of FCCR. Also, Guo et al. [11] initially built fuzzy DEA models based on possibility and necessity measures and then Lertworasirikul et al. [13] have proposed two approaches for solving the ranking problem in fuzzy DEA models called the ''possibility approach'' and the ''credibility approach.'' They introduced the possibility approach

from both optimistic and pessimistic view points by considering the uncertainty in fuzzy objectives and fuzzy constraints with possibility measures. In their credibility approach, fuzzy DEA model was transformed into a credibility programming-DEA model and fuzzy variables were replaced by ''expected credits,'' which were obtained by using credibility measures. Tavana et al. [14] proposed three fuzzy DEA models with respect to probability–possibility, probability–necessity and probability–credibility constraints.
FDEA was applied in some studies to evaluate the efficiency of banks. BO [15] proposed a fuzzy super-efficiency slack-based measure DEA to analyze the operational performance of 24 commercial banks facing problems on loan and investment parameters with vague characteristics. Kao et Liu [16] used FCCR model to predict the performance of 24 commercial banks in Taiwan based on their financial forecasts. Wu et al. [17] used FBCC model to deal with environmental variables in order to assess the efficiency of bank branches from different regions in Canada. Yalcin et al. [18] proposed a multi-criteria decision model to evaluate the performances of Turkish banks. Pramodh et al. [19] develops a novel measurement technique Data Envelopment Analysis (DEA)-Fuzzy Multi Attribute Decision Making Hybrid to measure the productivity levels of Indian banks and rank them.
In this paper a method based on arithmetic operations between triangular fuzzy numbers is developed as a new way to treat FDEA models. Following this method, FDEA models are transformed into crisp linear programming problems. In addition, a methodology is proposed
to deal with non financial data, the innovation level as input and the customer's satisfaction as output, in assessing the performance of commercial banks in Tunisia.
The rest of the paper is organised as follow: section 2 describes the DEA and FDEA. Section 3 presents some basic definitions and arithmetic operations between triangular fuzzy numbers. Section 4 provides the method used in this paper. Section 5 gives the proposed methodology for measuring the efficiency of Tunisian banks. Finally, section 6 concludes the paper, and discusses some future research directions.

2 DEA AND FDEA

2.1 DEA model

The model of Charnes et al. [1] called CCR model, and the BCC model named after Banker, Banker et al. [20] are the frequently used models. The primary difference between the two models is the treatment of returns to scale. The CCR model assumes constant return to scale. The BCC model is more flexible and allows variable returns to scale. Other DEA models exist and all are extensions of the CCR model. In our paper. Consider N decision making units (DMU_s), each consumes varying amounts of m different inputs $(x_1,....,x_m)$ to produce s different outputs $(y_1,....,y_s)$. The programming statement for the CCR model (input oriented) is:

$$\begin{aligned} \max h_0 &= \sum_{r=1}^{s} v_r y_{r0}, \quad r=1,....,s, \\ s.t \quad & \sum_{i=1}^{m} u_i x_{i0} = 1, \; i=1,....,m, \\ & \sum_{i=1}^{m} u_i x_{ij} \geq \sum_{r=1}^{s} v_r y_{rj}, \; j=1,.....,N, \\ & u_i, v_r \geq 0. \end{aligned} \qquad (1)$$

Where u_i is the weight associate to the ith input and v_r is the weight associate to the rth output. The target DMU (DMU_0) is technically efficient if and only if $h_0 = 1$. It can be seen from (1) that the essence of CCR model is that the DMU_0 evaluated tries to find out its weight vector to maximizing its weighted output with the constraints that its weighted input is fixed as unity and the weighted output is not larger than the weighted input for all DMUs. In addition to the CCR model, other well-known DEA models include the "BCC" model, , the "additive" model, the "free disposal hull" (FDH) model, and the "slacks-based measure of efficiency" (SBM) model. More details on other DEA models and their applications can be found in [21].
In this paper, the focus will be on the CCR model because the CCR model was the originalDEA model. All other models are extensions of the CCR model obtained by either modifyingthe production possibility set of the CCR model or adding slack variables in the objective function. Hence, an approach developed for solving the CCR model can be adapted for other DEA models.

2.2 FDEA model

Fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming models. The CCR model with fuzzy coefficients is given as

$$\begin{aligned} \max \; & \sum_{r=1}^{s} v_r \tilde{y}_{r0}, \quad r=1,.....,s, \\ s.t \; & \sum_{i=1}^{m} u_i \tilde{x}_{i0} = 1, \quad i=1,....,m, \\ & \sum_{i=1}^{m} u_i \tilde{x}_{ij} \geq \sum_{r=1}^{s} v_r \tilde{y}_{rj}, \quad j=1,....,n, \\ & u_i \geq 0, \quad v_i \geq 0, \end{aligned} \qquad (2)$$

Where $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are respectively the ith fuzzy input used and the rth fuzzy output produced by DMU_j.
The interpretation of constraints of FCCR model is similar to the crisp CCR model. The difference between the two models resides on the manner of resolution. The crisp CCR model can be simply solved by a standard LP solver. For the FCCR model, the resolution is more difficult and requires some ranking methods for ranking fuzzy sets.

3 PRELIMINARIES

3.1 Basic definitions

Definition 3.1 [22]. A fuzzy number $\tilde{M} = (m_1, m_2, m_3)$ is said to be a triangular fuzzy number if its membership function is given by :

$$\mu_{\tilde{M}}(x) = \begin{cases} \left(\dfrac{x-m_1}{m_2-m_1}\right), & m_1 \le x \le m_2 \\ \left(\dfrac{x-m_3}{m_2-m_3}\right), & m_2 \le x \le m_3 \\ 0, & otherwise \end{cases}$$

Definition 3.2 [22]. A triangular fuzzy number $\tilde{M} = (m_1, m_2, m_3)$ is said to be non-negative fuzzy number if $m_1 \ge 0$.

Definition 3.3 [22]. Two triangular fuzzy numbers $\tilde{M} = (m_1, m_2, m_3)$ and $\tilde{N} = (n_1, n_2, n_3)$ are said to equal if and only if $m_1 = n_1, m_2 = n_2, m_3 = n_3$.

Definition3.4 [23]. A ranking function is a function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{M} = (m_1, m_2, m_3)$ be a triangular fuzzy number, then.

(i) $\Re(\tilde{M}) = \dfrac{m_1 + 2m_2 + m_3}{4}$

(ii) $\tilde{N} \le \tilde{M}$ if $\Re(\tilde{N}) \le \Re(\tilde{M})$

(iii) $\tilde{N} \ge \tilde{M}$ if $\Re(\tilde{N}) \ge \Re(\tilde{M})$

3. 2 Arithmetic operations

In this subsection, arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers R, are reviewed [22].

Let $\tilde{M} = (m_1, m_2, m_3)$ and $\tilde{M} = (n_1, n_2, n_3)$ and $\tilde{N} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers then

(i) $\tilde{M} \oplus \tilde{N} = (m_1 + n_1, m_2 + n_2, m_3 + n_3)$

(ii) $-\tilde{M} = -(m_1, m_2, m_3) = (-m_3, -m_2, -m_1)$

(iii) $\tilde{M} - \tilde{N} = (m_1 - n_1, m_2 - n_2, m_3 - n_3)$

(iv) Let $\tilde{M} = (m_1, m_2, m_3)$ be any triangular fuzzy number and $\tilde{X} = (x_1, x_2, x_3)$ be a non-negative triangular fuzzy number then

$$\tilde{M} \otimes \tilde{X} = \begin{cases} (m_1x_1, m_2x_2, m_3x_3), & m_1 \ge 0 \\ (m_1x_3, m_2x_2, m_3x_3), & m_1 < 0,\ m_3 \ge 0 \\ (m_1x_3, m_2x_2, m_3x_1), & m_1 < 0 \end{cases}$$

4 SOLVING DEA MODEL

In this section, a new method based on arithmetic operation between triangular fuzzy numbers is proposed to solve the FCCR model. In order to describe our method, we firstly introduce the following notations:

$\tilde{x}_{i0} = \left((x_{i0})^l, (x_{i0})^c, (x_{i0})^u\right)$ is the ith fuzzy input of target $DMU\,(DMU_0)$.

$\tilde{x}_{ij} = \left((x_{ij})^l, (x_{ij})^c, (x_{ij})^u\right)$ is the ith fuzzy input DMU_j.

$\tilde{y}_{r0} = \left((y_{r0})^l, (y_{r0})^c, (y_{r0})^u\right)$ is the rth fuzzy output of target $DMU\,(DMU_0)$.

$\tilde{y}_{rj} = \left((y_{rj})^l, (y_{rj})^c, (y_{rj})^u\right)$ is the rth fuzzy output of DMU_j.

$v_r\ (r = 1, \dots, s)$ is the weight of output r.

$u_i, (i = 1, \dots, m)$ is the weight of input i.

Using the above notation, The FCCR model can be written as:

$$\begin{aligned} &\max h_0 = \sum_{r=1}^{s} \left(v_r (y_{r0})^l, v_r (y_{r0})^c, v_r (y_{r0})^u\right), \quad r = 1, \dots, s \\ &s.t \quad \sum_{i=1}^{m} \left(u_i (x_{i0})^l, u_i (x_{i0})^c, u_i (x_{i0})^u\right) = (1,1,1) \quad i = 1, \dots, m \qquad (3) \\ &\sum_{r=1}^{s} \left(v_r (y_{rj})^l, v_r (y_{rj})^c, v_r (y_{rj})^u\right) \le \sum_{i=1}^{m} \left(u_i (x_{ij})^l, u_i (x_{ij})^c, u_i (x_{ij})^u\right) \\ &u_i > 0, v_i > 0, \ j = 1, \dots, n \end{aligned}$$

Using arithmetic operations, defined in Section 3.2 and Definition 3.4 the fuzzy linear programming problem, obtained in Step 3, is converted into the following CLP problem:

$$\begin{aligned} &\max h_0 = \Re\left[\sum_{r=1}^{s} \left(v_r (y_{r0})^l, v_r (y_{r0})^c, v_r (y_{r0})^u\right)\right], \quad r = 1, \dots, s \\ &s.t \quad \sum_{i=1}^{m} u_i (x_{i0})^l = 1 \quad i = 1, \dots, m \\ &\sum_{i=1}^{m} u_i (x_{i0})^c = 1 \quad i = 1, \dots, m \qquad (4) \\ &\sum_{i=1}^{m} u_i (x_{i0})^u = 1 \quad i = 1, \dots, m \\ &\Re\left[\sum_{r=1}^{s} \left(v_r (y_{rj})^l, v_r (y_{rj})^c, v_r (y_{rj})^u\right)\right] \le \\ &\Re\left[\sum_{i=1}^{m} \left(u_i (x_{ij})^l, u_i (x_{ij})^c, u_i (x_{ij})^u\right)\right], \quad j = 1, \dots, n \\ &u_i > 0, v_i > 0 \end{aligned}$$

Using the expression of the fuzzy ranking function, we obtain the following linear programming problem:

$$\max h_0 = \frac{1}{4}\sum_{r=1}^{s} v_r\left(\left(y_{r0}\right)^l + 2\left(y_{r0}\right)^c + \left(y_{r0}\right)^u\right), \quad r = 1,.....,s$$

$$\begin{aligned} s.t \quad & \sum_{i=1}^{m} u_i\left(x_{i0}\right)^l = 1 \quad i = 1,.....,m \\ & \sum_{i=1}^{m} u_i\left(x_{i0}\right)^c = 1 \quad i = 1,.....,m \qquad (5) \\ & \sum_{i=1}^{m} u_i\left(x_{i0}\right)^u = 1 \quad i = 1,.....,m \\ & \frac{1}{4}\sum_{r=1}^{s} v_r\left(\left(y_{rj}\right)^l + 2\left(y_{rj}\right)^c + \left(y_{rj}\right)^u\right) \le \frac{1}{4}\sum_{i=1}^{m} u_i\left(\left(x_{ij}\right)^l + 2\left(x_{ij}\right)^c + \left(x_{ij}\right)^u\right) \\ & u_i > 0, v_i > 0, \; j = 1,...,n \end{aligned}$$

The resolution of this model gives a crisp efficiency score of the target DMU (DMU_0).The three first constraints serve to normalize the objective function and the last mean that the efficiency score function should not be greater than 1.

Definition 4.1. DMU_0 is efficient if $h_0 = 1$. Otherwise, it is inefficient.

We note that our method is very useful for decision makers witch prefer crisp efficiency scores rather than imprecise or interval values. In addition it is easy to implement and avoid the need to fix some specific values as an alpha cut as in the alpha cut approach, a possibility level as in the possibility approach and a preference level as in the tolerance approach.

5 Methodology for Measuring the Efficiency of Tunisian Banks

Applications of DEA in banking industry are numerous “[24]-[25]”. In these studies, only financial data are used. However, it is not always sufficient to evaluate bank efficiency by taking only financial inputs and outputs as a basis. Nowadays, we see that non-financial performance criteria show up as an emerging asset especially in performance measurement of banks. In general terms, non-financial criteria are defined as the criteria which cannot physically be measured and always given in the form of linguistic or imprecise variables.

In This paper, our study aims to evaluate the efficiency scores of 14 commercial Tunisian banks during the period 2010-2012. The used data are categorised into two kinds, financial and no financial data. we focus on the intermediary approach. Three inputs (deposits, labour and fixed assets) and two outputs (loans and portfolio investment) have been used. The source of these data is the PATB (Professional association of Tunisian Banks). The fixed assets, deposits, loans and portfolio investment are measured in TND and labour is measured in terms of number of staff.

Non financial-data are obtained by two questionnaires, the first is addressed to the technical person of each bank and aims to obtain information of the level of innovation and the second is addressed to the customers of each bank in order to obtain information in customers’ satisfaction.

In the next subsection, we propose a three-step method for converting non financial data to fuzzy numbers.

5.1 Conversion of non financial data

Step 1 To measure customer satisfaction, we used five linguistic variables: "Not at all satisfied, (NS)", "unsatisfied, (UNS)", "moderately satisfied, (MS)", "satisfied, (S)" and "very satisfied, (VS)". These linguistic terms are distributed on a scale [0, 20] as follows:

TABLE I
linguistic terms for customer’s satisfaction

terms	NS	UNS	MS	S	VS
classes	[0,4]	[3, 8]	[7,12]	[11,16]	[15,20]

Step 2 First, for each bank we calculate the frequency of respondents for each linguistic variable in order to construct satisfaction intervals. Then, we calculate a satisfaction interval for each bank with this formulate

$$\left[a_i, b_i\right] = f_{i1}\times\left[0, 4\right] + f_{i2}\times\left[3, 8\right] + f_{i3}\times\left[7, 12\right] + f_{i4}\times\left[11, 16\right] + f_{i5}\times\left[15, 20\right]$$

Where $f_{ij}\left(i = 1,....,14;\; j = NS,.....,VS\right)$ is the frequency of respondents corresponding to the ith bank and the jth variable.

Step 3 Once the satisfaction intervals are determined, a triangular fuzzy number is assigned to each bank with the following characteristics:

The lower value $m^l = a_i$

The middle value $m^c = \dfrac{a_i + b_i}{2}$

The upper value $m^u = b_i$.

With the same manner, we use the above three steps to obtain fuzzy numbers for the innovation level input.

Step 1 To measure the innovation level, we used five linguistic variables: "Very Low, (VL)", "Low, (L)", "moderate, (M)", "High, (H)" and "very high, (VH)". These linguistic terms are distributed on a scale [0, 20] as follows:

TABLE II
linguistic terms for the innovation level

variable	VL	L	M	H	VH
classes	[0, 5]	[4, 9]	[8, 13]	[12, 16]	[15, 20]

First, we calculate the frequency of respondents for each linguistic variable in order to construct innovation level intervals. Then, we calculate innovation level interval for each bank with this formulate

$$\left[a_i, b_i\right] = f_{i1}\times\left[0, 5\right] + f_{i2}\times\left[4, 9\right] + f_{i3}\times\left[8, 13\right] + f_{i4}\times\left[12, 16\right] + f_{i5}\times\left[15, 20\right]$$ W

where $f_{ij}\left(i = 1,....,14;\; j = VL,.....,VH\right)$ is the frequency of respondents corresponding to the ith bank and the jth linguistic variable. From these intervals, a triangular fuzzy number has Constructed for each bank.

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5.2 Results

Tables gives results of efficiency scores estimated according to the DEA method under the assumption of FCCR model. The average efficiency score over all the period is 0.867 with FCCR. This score is 0.845 when non financial data are not taken into account. The increase of the efficiency scores with FCCR model can be explain by the use of non financial data, customer satisfaction as output and innovation level as input. With the FCCR, the average efficiencies vary between 84.6% and 89,1% . The highest scores are obtained in 2010 and the lowest are registered in 2011 which is the year of Tunisian revolution. The results indicate that large banks (STB, BIAT and BNA) are less efficient than small banks (BFT, BC,…) and medium-sized banks(BT,BH, AB). This classification can be explained by several factors including especially the poor quality of services offered to their customers. However a big part of these customers are dissatisfied.

TABLE III
Efficiency Scores of Tunisian Banks

	2010	2011	2012
BNA	0.786	0.699	0.703
STB	0.774	0.702	0.719
BIAT	0.802	0.700	0.725
UIB	0.890	0.801	0.801
BH	0.825	0.799	0.815
ETTIJARI	0.877	0.805	0.8100
BT	0.888	0.804	0.939
UBCI	0.911	0.900	0.905
ATB	0.912	0.899	0.900
AB	0.840	0.798	0.821
BFT	0.976	0.950	1
CB	1	1	0.979
BTS	1	1	1
ABCT	0.999	0.987	0.998

In addition, the low level of efficiency of these banks is essentially comes back to the specialization of these banks, in spite of the enactment of the law 2001-65 of 10 July 2001 which consists of the universality of banking. The BNA is the least efficient bank, and it was specialized in loans to the agricultural sector which is the main source of increased bad debts with the tourism sector. Again, the Bank of Housing (BH) remains the dominant bank of habitat credits, its share remains elevated in this market and that is around 58% in 2011. Also, these banks have the highest volumes of nonperforming loans (NPLs), 39% for STB and 35% for the BNA in 2010, and they are submitted to a public control, which can generally lead to decrease the efficiency level. This policy of financing of real estate, agriculture and tourism sectors (40% of loans to the tourism sector are considered non-performing), through the pursuing a policy of easy credit, contributed to the heaviness of non-performing loans (NPL). Also, the big-sized banks have more expenses on average (personnel expenses, Interests incurred and similar charges) than those of small and medium sizes.

Foreign banks and mixed banks are significantly more efficient than domestic banks. This superiority efficiency score of foreign banks and mixed banks can be explained by better resource management and better organization. In addition, these banks make big part of their capital in new technologies and diversified their marketing strategies in order to satisfy their customers.

Conclusion

To date DEA is widely applied to measure the performance of the banking system since it is capable of evaluating the efficiency of DMUs with multi-output and multi-input. Most previous studies used the conventional DEA with crisp data. However we can sometimes take into account no financial data (imprecise data) to evaluate the efficiency of banks more accurately and realistically. In this study, first, we propose a method based on arithmetic operations between fuzzy numbers to solve FDEA model. The obtained crisp linear programming problem provides an exact efficiency score for each bank. Second, we develop a methodology to deal with non financial data, the innovation level as input and the customer's satisfaction as output, in assessing the performance of commercial banks in Tunisia. This methodology provides a procedure for converting linguistic variables into fuzzy numbers. Theses fuzzy numbers represent the parameters of the FCCR used to obtain the efficiency score of each bank. The empirical results show that the small and mid-sized banks are the least efficient because they spend much of their total budget for investment in new technologies. A further study may compare the results obtained in this paper with the ones from the other methodologies.

References

[1] A. Charnes, W.W. Cooper, E. Rhodes, "Measuring the efficiency of decision-making units", European Journal of operational research, vol. 2, pp. 429–444, 1978.

[2] Sengupta, J.K., "A fuzzy systems approach in data envelopment analysis". Computers and Mathematics with Applications, Vol.24, pp.259–266, 1972.

[3] Kahraman, C. and Tolga, E., "Data envelopment analysis using fuzzy concept". 28th International Symposium on Multiple-Valued Logic, pp.338–34, 1998.

[4] Kao, C. and Liu, S.T., "Fuzzy efficiency measures in data envelopment analysis", Fuzzy Sets and Systems, Vol. 113, No.3, pp.427–437, 2000.

[5] Zadeh, L.A., "Fuzzy sets as a basis for a theory of possibility", Fuzzy Sets and Systems, Vol. 1, pp.3–28, 1978.

[6] Saati, S. and Memariani, A., "Reducing weight flexibility in fuzzy DEA", Applied Mathematics and Computation, Vol. 161, No. 2, pp.611–622, 2005.

[7] Saati, S. and Memariani, A., " A note on measure of efficiency in DEA with fuzzy input–output levels: a methodology for assessing, ranking and imposing of weights restrictions by Jahanshahloo et al.", Journal of Science, Islamic Azad University, Vol. 16, No. (58/2), pp.15–18, 2006.

[8] Liu, S.T., "A fuzzy DEA/AR approach to the selection of flexible manufacturing system", Computer and Industrial Engineering' Vol. 54, pp.66–76, 2008.

[9] Wang, C.H., Chuang, C.C., Tsai, C.C., "A fuzzy DEA–neural approach to measuring design service performance in PCM projects", Automation in Construction, Vol. 18, pp.702–713, 2009.

[10] Kao.C, Ling, P. H, "Qualitative factors in data envelopment analysis: A fuzzy number approach", European journal of operational research, 211, pp. 586-598, 2011.

[11] Guo, P., Tanaka, H., "Fuzzy DEA: a perceptual evaluation method", Fuzzy Sets and Systems, Vol. 119, No.2 , pp.149–160, 2001.

[12] Tlig, H.and Rebai, A., " A mathematical approach to solve data envelopment analysis models when data are LR fuzzy numbers", Applied Mathematical Sciences, Vol. 3, No. 48, pp.2383–2396, 2009.

[13] Lertworasirikul, S., Fang, S.C., Joines, J.A. and Nuttle, H.L.W., "Fuzzy data envelopment analysis (DEA): a possibility approach", Fuzzy Sets and Systems, Vol.139, No 2, pp.379–394, 2003.

[14] Tavana.M, Damghani K.K, Nezhad S.S, "A fuzzy group data envelopment analysis model for high- technology project selection: A case study at NASA", Computers and Industrial Engineering, 66, pp.10-23, 2012.

[15] BO, H., Ching, C.C., Yung, H.C. and Ching, R.C., "Using fuzzy super-efficiency slack-based measure data envelopment analysis to evaluate Taiwan's commercial bank efficiency", Expert Systems with Applications, Vol.38, pp.9147–9156, 2011.

[16] Kao, C. and Liu, S.T., "Predicting bank performance with financial forecasts: A case of Taiwan commercial banks", Journal of Banking & Finance, Vol. 28, pp.2353–2368, 2004.

[17] Wu, D.D., Yang, Z. and Liang, L., "Efficiency analysis of cross-region bank branches using fuzzy data envelopment analysis", Applied Mathematics and Computation, Vol. 181, pp.271–281, 2006.

[18] Yalcin, N.S., Ali, B. and Kahraman, C., "Fuzzy performance evaluation in Turkish Banking Sector using Analytic Hierarchy Process and TOPSIS", Expert Systems with Applications, Vol. 36, pp.11699–11709, 2009.

[19] Pramodh, C., Ravi, V. and hushanam, N , " Indian banks' productivity ranking via Data Envelopment Analysis and Fuzzy Multi-Attribute Decision-Making hybrid", International Journal of Information and Decision Sciences, Vol. 1, pp.44 - 65, 2008.

[20] Banker, R., A. Charnes and W.W. Cooper, "Some models for estimating technical and scale inefficiencies in data envelopment analysis", Management Science 30, pp.1078-1092, 1984.

[21] Bardhan, I., W.F. Bowlin, W.W. Cooper and T. Sueyoshi, "Models and Measures for Efficiency Dominance in DEA, Part I: Additive Models and MED Measures," Journal of the Operational Research Society of Japan 39, pp.322-332, 1996.

[22] A. Kaufmann, M.M. Gupta, "Introduction to Fuzzy Arithmetic Theory and Applications", Van Nostrand Reinhold, New York, 1985.

[23] T.S. Liou, M.J. Wang, "Ranking fuzzy numbers with integral value", Fuzzy Set. Syst. 50, pp.247–255, 1992.

[24] Aly, H.Y., Grabowski, R., Pasurka, C. and Rangan, N., "Technical, scale and allocative efficiencies in U.S. banking: An empirical investigation", The Review of Economics and Statistics Vol. No.72, pp.211-218, 1990.

[25] Bauer, P.W., Berger, A.N., Ferrier, G.D., Humphrey, D.B., "Consistency conditions for regulatory analysis of financial institutions: A comparison of frontier efficiency methods", Journal of Economic and Business, Vol. 50, pp. 85–114, 1998.

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