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Riemann Zeta Function & Deeper Developments in Mathematics

Introduction:

The aim of this thesis is to provide systematic information on very complicated open problem of mathematics such as the truth of Riemann Hypothesis and the analysis of the Riemann Zeta Function. We know that this problem seems beyond human comprehension and represents rigorous statistical finger print therefore we have to analyze further sources of information for this problem.

The accurate direction of development stages for handling this problem is important as well therefore we will analyze the direction of the many problems in light of the renowned mathematician Einstein's thinking while modeling the ideas for solution.

Thesis starts with the earliest writing along with some unknown writing prints of the one of the oldest civilization known as Indus valley civilization and on some development history of mathematics while keeping in touch with the Riemann Zeta function.

Thesis consists of five chapters

The first chapter reflects the earliest stage of human mind and its ways of modeling the ideas at the earliest stages of the human civilization in Indus Valley Civilization.

The second chapter is on certain facts about prime numbers which helps to understand the structures of equations and hence to

Random Matrix Theory while the third chapter is on Eigenvalues the very important and ground making branch of mathematics.

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(ii)

Forth chapter develops the link of field of values, Eigen values & Random Matrix Theory.

Fifth chapters suggest collective deeper developments in mathematics to handle open problems of mathematics waiting for solution for more than 150 years.

Submitted by **Shagufta Nazir Ahmad**

CONTENTS

Chapter 1

Unknown Writing System Of Indus Valley Civilization

"We can't skip the history" *Abraham Lincoln*

The earliest civilization of the world

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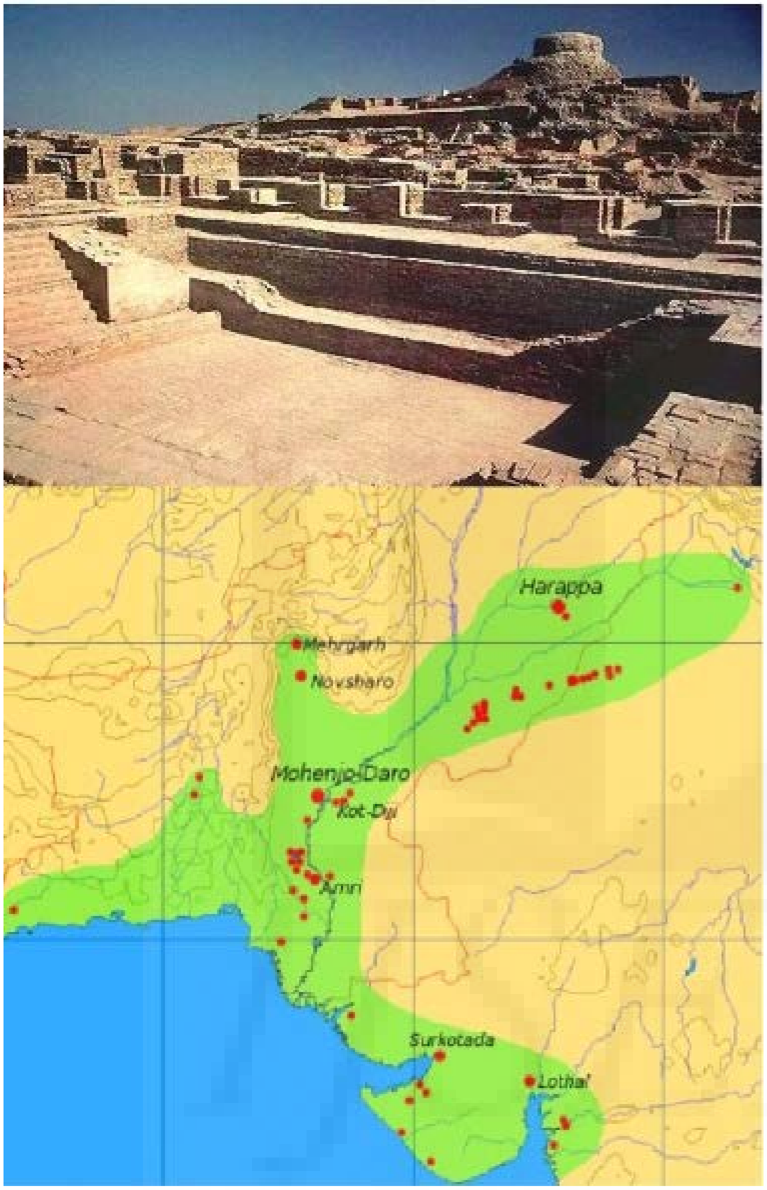

The earliest civilization of the world

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Map of the early stages of the world

Map of the early stages of the world

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Introduction

Indus Mathematics

The earliest evidence of the use of mathematics in South Asia is wondrously seen in the artifacts of the Indus Valley Civilization. The people of Indus valley were among the first to develop a system of uniform weight and measures. Their measurements were extremely precise; e.g. their smallest division marked on an ivory scale found in Lothal was approximately 1.704 millimeters, hence being the smallest division ever to be recorded in the Bronze Age.

Archaeologists have uncovered evidence of practical use of mathematics in the civilization's history.

The ancient civilization's earliest pioneers manufactured bricks whose dimensions were in the proportion 4:2:1, considered to be one of the most favorable for the stability of a brick-based structure. They also used standardized system of weights based on the ratios of, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500 with the unit weight equal to approximately twenty-eight grams (Approximately equal to an English ounce or the Greek uncial). Accordingly, they produced weights in regular geometric shapes, which included hexahedra, barrels, cones and even cylinders

demonstrating basic knowledge of geometry.

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Archaeologists also managed to find the monument of a Mohenjo-Daro ruler whose unit of length 3.4 cm was *divided in to ten equal parts amazingly. Bricks manufactured often had dimensions of the same integral multiples of this unit of length.Unique Harappan inventions include an instrument which was used to measure whole sections of the horizon and the tidal lock (for raising and lowering boats between stretches of water of different levels on river and canal waterways).The engineering skills of the Harrapans were remarkable, especially specializing in the building of docks after a careful study of tides, waves and currents. There was standardization in the size of bricks all sites, the predominant size being 28x14x7 cm* or a ratio of *4:2:1 as mentioned previously. Some specialized brick shapes such as the wedge-shaped ones were used in the constructions of wells have been discovered as well. There was a significant variation in the size of residential houses ranging from single-room tenements to units with*

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courtyards and up to a dozen rooms of various sizes, to much larger houses with several courtyards and tenements.

Archaeologists have also uncovered evidence of the presence of an astronomical system in relation to the ancient religious practices. The lunar calendar Nakshatra is very similar to the Chinese calendar as both can be dated back to 2400 B.C. Nakshatra is based upon principles quite different than those of ancient Greek and Egyptian calendars, but is likely enough to have originated from the ancient Indus civilization itself. The date of its composition coincides with that era of urbanization in that

area that dates back further than the urbanization period in China. Egyptian and Greek Calendars were based on the observations of the "helical" rising and setting of stars at

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dawn whereas the calendars of the Indus Valley and the Chinese were based upon the observations of the stars that lay opposite the Sun.

The plans of cities in the I.V.C. were built on a grid pattern and carefully oriented according to the cardinal directions, which must have been obtained by some astronomical observations. These factors further indicate that the Indus Valley Civilization was indefinitely the oldest urban civilization in the region of Asia.

The Indus Valley civilization is one of the world's three earliest urban civilizations along with Mosopotamia and Egypt .This civilization existed from about 2600 BCE to about 1700 BCE, which means it existed at about the same time as Egyptian and Sumerian civilization.
In 1922, archaeologists found the remains of an ancient city called Harappa. They found another city, located 400 miles southwest of

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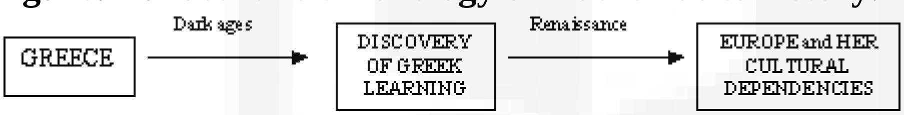
Harappa, called Mohenjo-Daro. Harappa is a city in Punjab northeast of Pakistan and about 35 km southwest of Sahiwal.

Mohenjo-daro was a city of the Indus Valley Civilization built around 2600 BC and is located in the Sindh Province of Pakistan. This ancient five thousand year old city is the largest of the Indus Valley and is widely recognized as one of the most important early cities of South Asia and the Indus Valley Civilization. Mohenjo Daro was one of the world`s first cities and contemporaneous with ancient Egyptian and Mesopotamian civilizations. It is sometimes referred to as `An Ancient Indus Valley Metropolis`

A very sophisticated and technologically advanced urban culture is evident in the Indus Valley Civilization. The quality of municipal town planning suggests knowledge of urban planning and efficient municipal governments which placed a high priority on hygiene. The streets of major cities such as Mohenjo-daro or Harappa were laid out in perfect grid patterns. The houses were protected from noise, odors, and thieves.

Figure: Eurocentric chronology of mathematics history.



Some historians made some concessions, by acknowledging the work of Egyptian, Babylonian, Indian and Arabic mathematicians (and occasionally the work of the Far East and China). Modified versions of the Eurocentric model commonly took the form seen below.

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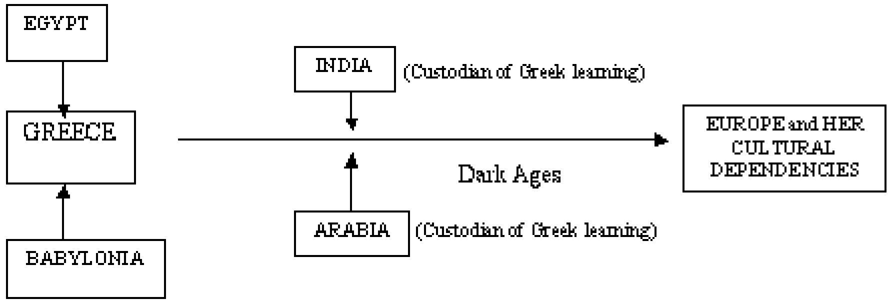
Figure: Modified Eurocentric model.



The Indus Valley civilization is one of the world's three earliest urban Civilization along with Mosopotamia and Egypt .This civilization existed from about 2600 BCE to about 1700 BCE, which means it existed at about the same time as Egyptian and Sumerian civilization .Were these ancient civilizations aware of each other at that time?
In 1922, archaeologists found the remains of an ancient city called Harappa. They found another city, located 400 miles southwest of Harappa, called Mohenjo-Daro.
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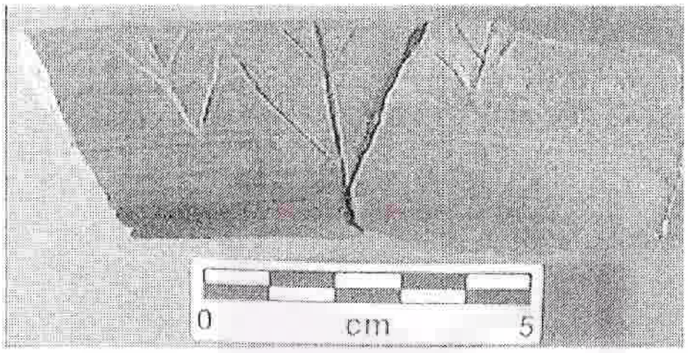
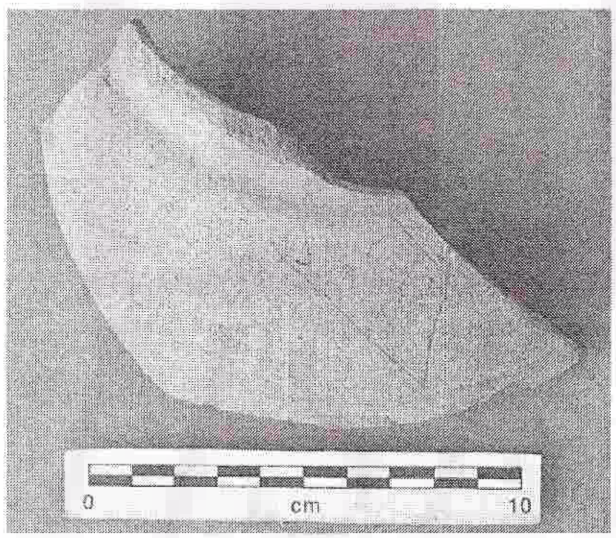
Harappa is a city in Punjab northeast of Pakistan and about 35 km southwest of Sahiwal.
This was a very advanced civilization .Scientists can not yet read their language. Did these people have a common language?
The quality of municipal town planning suggests knowledge of urban planning and efficient municipal governments which placed a high priority on hygiene. The streets of major cities such as Mohenjo-daro or Harappa were laid out in perfect grid patterns.
The houses were protected from noise, odors, and thieves.
The public buildings of these cities also suggest a high degree of social organization.

Both Mohenjo-daro and Harappa share relatively the same architectural layout. These cities do seem very developed indeed considering the time period they were built. Mohanjo-daro, Harappa and their civilization vanished without a trace from History until discovered in the 1920's.Where did these people come from, and where did they go? No body knows.



The Earliest Writing



Weights, Harappa.

The cubical weights conform to the standard Harappan binary weight system prevalent in the Bronze Age.

Seals, Mohenjo-daro.

The animal motifs appearing on Indus seals included totemic animals like the tiger, the rhinoceros, the water buffalo and the gharial (crocodile)

Bison seal

A two-sided seal with four script symbols inscribed above a bison with its head lowered

Narrative seal

A narrative seal with a deity, human head on a stool, kneeling worshiper, and seven figures

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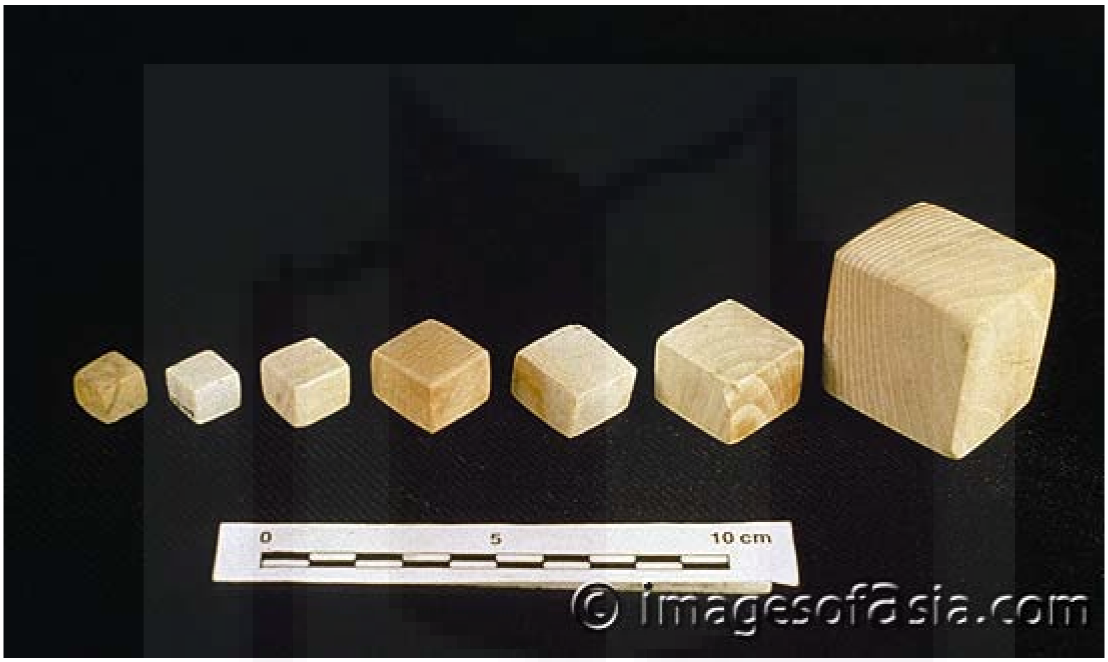

Cubical ancient Indus weights

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An Elephant seal

Seals, Mohenjo-daro.

The animal motifs appearing on Indus seals included totemic animals like the tiger, the rhinoceros, the water buffalo and the gharial (crocodile)

Inscribed objects

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A set of seals and objects with writing and other figures

Rectangular Seals


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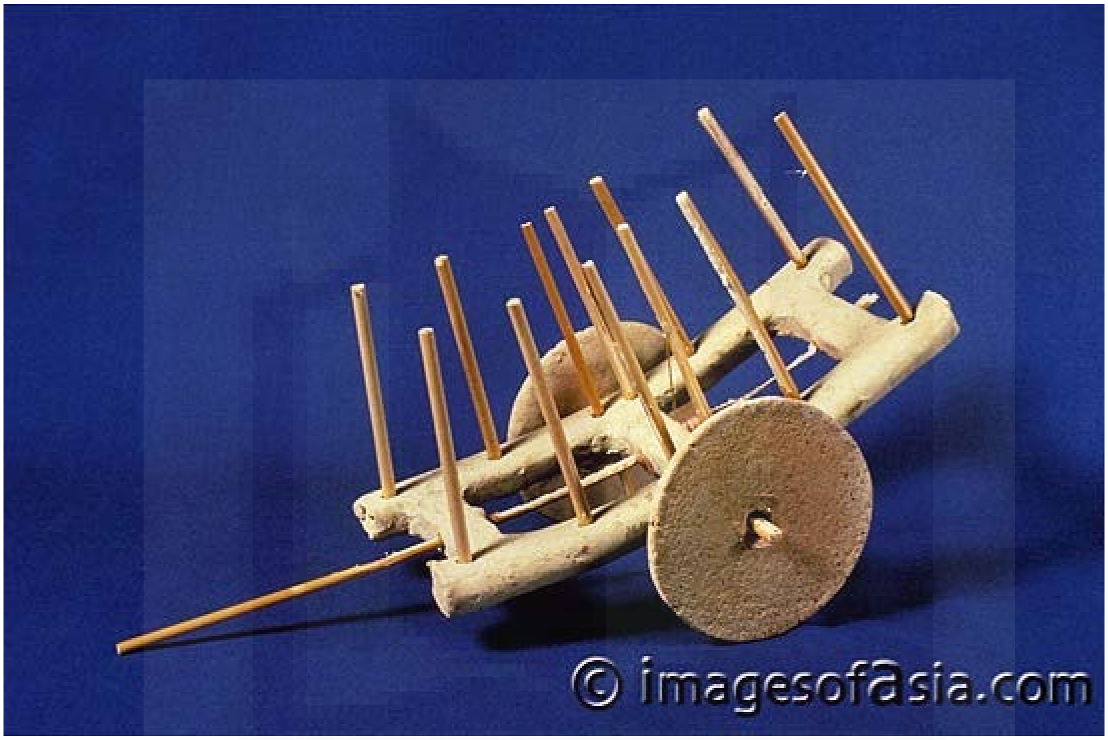
Swastika Seals

Tiger seal

An Indus seal showing the pictorial motif of a tiger and two signs



Indus toy cart

A terra cotta toy cart found at the ancient Indus settlement of Nausharo in Baluchistan

Unfired boat seal

An unfired steatite seal with a flat-bottomed boat incised on it

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A unicorn seal

A so-called unicorn seal, one of the most common ancient Indus motifs whose significance remains unknown

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Water buffalo seal

A seal showing a water buffalo from Mohenjo-daro with Indus inscription

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Indus deity seal

One of the best known ancient Indus seals shows a male deity in yogic position


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Chapter 2

Introduction

In this chapter we will describe some interesting hidden facts about natural number system in the light of iterative methods, which provide better insight to understand the role of the structural units of data in generating prime numbers. Further this approach can be applied to other sets of data to systematize the knowledge of primes that today is scattered.

Proper understanding of the number system situation is an essential prerequisite for tackling the distribution of primes .Analytic represents shows that Li(x) is not a good approximation of π(x) because it asks to approximate at exponential scale instead of linear or at quadratic scale
it reminds me saying of Hardy
"The mathematician's pattern's, like the painter's or the poet's must be beautiful; the ideas like the colours or the words must fit together in a harmonious way .Beauty is the first test. There is no permanent place in this world for ugly mathematics"
A limit during the development of the formula hides actual geometric structures, symmetries produced
And then the results on the basis of incomplete information are misleading.

Key Ideas

- *Prime numbers also represent a perfectly accurate counting system as their deep symmetries representation is nothing more than their asserted latest **positions** after some chosen iteration (repeated application of a function) which are reducible in period-1*

- *It very important to avoid limit through the construction of formula because of limit ,The erased part remains hidden hence limit eliminates the heart of the process e.g in Euler Equation*
- *Iterative method shows that symmetric structures are present in Euler equation*
- *Because of limit in Euler equation symmetric structures remains hidden meaning Euler equation hides its analytic structures because of limits*
- *Limit breaks the kernel of the problem and hence we get broken symmetry of primes*
  So cannot see the real structures because of branches cut off
- ***Complex number S in Riemann Zeta function** has no relation with distribution of primes or with the zeros of zeta function*
- *Zeta function is nothing more than the analytic representation of **solar eclipse***

- *zeros in critical line of Riemann Zeta function start coming after **6^{th} iteration***

- *Even numbers are not self-consistent as even numbers need proportionality in their construction*

- *Prime numbers are self-consistent ; because there is no need of proportionality in their construction*

- ***Euler formula is analytic representation of the iterative process***

- ***There is no need of repetition of structural units in relation to the central axis in generation of prime numbers b**ecause primes don't*

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- *satisfy the law of proportionality during their generation but still they are perfectly regular indicating that primes are generated in self-sustainable structural units ways ; for example sky is without pillars but not falling on us; there is no need of proportionality in this type of construction, birds Can be seen managing their flights; dive , flying without any visible hold thus indicating transformations in non-sustainable structures produces noise. Leading us that* ***Distribution of primes is underlying unity of mathematics and interconnection among its diverse branches***

- *We should be clear about the interconnection of distribution of prime with respect to their mediums; all materials have different geometric shapes; structures e.g. distribution of primes numbers with respect to real number line, distribution of primes with respect to human throat , distribution of primes on the layers of drum with respect to human hand meaning that beat of human hand can touch all layers of the drum in specific manners and then can develop a sequence on the basis of the basic tone(frequency)of that material such that difference of any two tones of sequence is not zero and then can establish functions accordingly,* ***as function frequencies are multiple of the fundamental tone and many over tone***

- *An infinite series cannot be split into various parts unless all the parts converge*

- *In linear systems iterates end up decreasing in length by a factor which is equal to spectral radius*

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- *and hence parallel to dominant eigenvector*

for proof or example: e-mail(sna716@hotmail.com)

- *Natural number 1 has one angle,2 has 2 angles, 3 has 3 angles,4 has 4 angles,5 has 5 angles,6 has 6 angles,7 has 7 angles,8 has 8 angles,9 has 9 angles, and then a zero angle.*
- *When all the conditions of non-Euclidean metric theory are satisfied with respect to different geometrical shapes the system has exactly one solution for example in case with opposite goals when the wind blows, Sail tense and make the boat go, while branches bend and help the tree stay put .Constants values occurring in minimization problems in Sails and trees have an important role In the construction of energy forms on fractal models(geometric objects ,physical elastic bodies) and on effective non-Euclidean metric theory;*
- *Issue of twin primes is related to the analysis of its simple function because prime numbers become sparse as whole numbers increase as all functions consists of sums of simple functions*
- *Stochastic linear algebra can model the behavior of the system because it involves the most structural equations*

- *All non-trivial zeros have real part almost at 1/2 to guarantee the stable degeneration of the complex plane;* ***zeta function has zeros and poles at same place;*** *first non- trivial zero which is at 1/2+i13.135 represents the value of fundamental frequency tone of Zeta function and then second non-trivial zero which is at 1/2+i19 is a multiple of fundamental frequency tone and of over tone; similarly the third , and so on to infinity.*

- *All non-trivial zeros of zeta function forms an open set to degenerate C*

- *In graph of Riemann Zeta function the sequence*−3,−1, 1, 3, 9, 11, 17, 23, 29, 35, 41, 47, 53, 59, 69, 75, 81, 91, 97, 103, 113,123, 129, 135, 145, 155, 161, 171, 181, 187, 197, 207, 217, 223, 237, 247, 253 263, 273, 283, 293, 307, 313, 323, 329, 343, 353, 359, 373, 383, 393, 403, 417423, 437, 451, 457, 467, 481, 491, 501, 511, 525, 535, 545, 559, 569, 579……, . *formed by the number of lines that escape to the right is due to the fact* ***that symmetries*** *following analytic representation can be represented in(preferred scale(fundamental scale)& power of fundamental scales) that* ***merge in geometric structures*** *of function so escape from the process; when P and r are not relatively prime in the iterative process; meaning new non-primes in iterative process* ***produces same pattern*** *in this way*

Facts about prime numbers

- *The longest known arithmetic sequence of prime numbers contains 26 terms with a difference of 5,283,234,035,979,900 between successive number*

- *Functions frequencies are multiple of the fundamental tone's frequency*
- *The sound or harmonic of a violin string is composed of a fundamental tone and many overtones*
- *Prime numbers represent music in a noisy recording*
- *Prime number is made of a unique infinite sequence such that no two consecutive number have difference zero*
- *Prime numbers crop up randomly among all the whole numbers*
- *Prime numbers accumulate on defined spiral graph which run through the square Root spiral*
- *Spiral graphs are result of exclusively quadratic polynomials for example Polynomial* x^2+x+41 *appears on the square root spiral in the form of three spiral graphs defined by three different quadratic polynomials*
- *All natural numbers divisible by a certain prime factor*

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lies on spiral graph on the square root spiral

- *4,9,16,25,36,49,……………..form a three symmetrical system of three spiral graphs*
- *4,9,16,25,36,49,………… divides the square root spiral into three equal areas*
- *Fibonacci sequence plays a role in the structure of the Square Root Spiral*
- *Positive real numbers has polar coordinates;* $r = n = \sqrt{\Theta}$ *in rotation of 360 degrees*
- *Rational angle(measurement in rotation is a rational number 1/2 rotation;1/3 rotation etc.) has composite offset curves*
- *Non-prime numbers march north and south from the center of the spiral*
- *Offset curves exhibit patterns based on the relationship between their angles, their offsets, the factorizations, and the functions that generate them.*
- *Offset curves are real-valued quadratic functions in which the coefficient of x^2 is perfect square*
- *Patterns of offset curves depend on the factorizations of integers found on them, their angles, their offset, and the functions that generate offset curves.*

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- *If the features of the iterative process can be reduced to the first period (fundamental tone) due precisely to the intrinsic periodicity of the chosen iterative results. Then we establish new processes based in the symmetries from chosen iterative process constructed with finite sequence as ARGUMENT?*
- *If in an iterative process a new interval is generated from previous iteration. Can the new symmetries of iterative process behave the same way on new interval as before?*
- *Fundamental frequency (period one) is obtained analysing segregation of Old symmetries of iterative system and new symmetries of newly generated interval*
- *If all the remaining primes(prime periodicity's p) make the same pattern structures on new interval of elements n after K repeated applications &*
- *If all the remaining non-primes (mirror-symmetries r and if r is prime to p)there is no specific scale but if r is not prime to P; Preferred scale (basic tone) ; and invariant scale of powers of fundamental scale (over tone) produces the same pattern.*
- *Iterative method can hold infinite sequence of numbers*

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Conclusion:

Existence of second order differential equation by secondary sieving map for producing primes brings into light the values of constants occurring in the differential equations. Diverse behaviour of fractal analysis suggests intrinsic different spectral properties. Geometric-analytic model can be helpful in this regard which may indicate analytical methods and point out difficulties that deserves further investigation

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Chapter 3

Eigen Values, Field of Vales &Random Matrices

Introduction

The roots of the word " Eigen" can be found in the language respectably German and Dutch, which means to be not possessed by others in the field of values. Eigen values are in fact energy values.Eigen vectors are the soul of a matrix whose direction the matrix cannot change after interactions and pulls and finally reach the equilibrium state. The equilibrium state is the Eigen vector state

Eigen vector is very important to determine Eigen values. Since Hermitian matrices have important spectral properties and because of their deep mathematical structure can be understood as the complex extension of real symmetric matrices. Eigenvalues and Eigen vectors provide insight into the geometry of linear transformations. Hermitian matrices even out eigenvalues and have nice fundamental analytic properties. Spectrum of any self-adjoint operator is nonempty. Self-adjoint operators are multiplication operators on general measure space.

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Key Ideas

- *Change in energy scale results in new effective degrees of freedom*
- *Spectrum is periodic, upon lowering the energy scale we get nothing because of the change in collective phenomena*
- *We can calculate the energy of the photon by using the wave length and then see which the energy differences between electron levels. If it is equal then it is the right transition*
- *The energy of a photon is proportional to its frequency*
- *The functional renormalization group is a flexible and unbiased for dealing with scale dependent variables*
- *Starting point of a function is an exact functional equation yielding the gradual evolution from fundamental model to final stage as a function of a continuously decreasing energy scale*
- *We get things exact arranged in order of rank by expanding in powers of the fields for vertex function.*
- *The functional renormalization approximations provide the better understanding of correlated systems*

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- *As time passes Eigenvalues with negative imaginary parts(not very small) die out in the wave function*

- *The restriction of the Hamiltonian H to the subspace of the wave functions that survive after long time will have approximately real eigenvalues*

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Random Matrix Theory

Introduction:

Random matrix theory means a theory which involves the most structural equations e.g equations from stochastic linear algebra (can model the behavior of the systems). We start by selecting a very carefully theory based tested familiar matrices from very large size matrices with elements from independent, normal, uniform distribution. We test the general rules for these test matrices when $n \to \infty$*. These types of matrices catch every type of programming errors .We get required results from very appropriately chosen check matrices. If these check matrices give unspecified probabilities then suggesting some bounds on condition number*

$K = \sigma_1 / \sigma_n$ *of nxn of matrix* A *as* $n \to \infty$

Key Ideas

- *Fluctuation properties of Eigen values of large random matrices have limits that are independent of the probabilities distribution on the matrix ensembles*
- *Random matrix theory gives with high accuracy the distribution of the zeros of L-functions and Structure of average values of L-functions*
- *For all problems there is an option of considering random matrices over real or complex*
- *Free probability(branch of operator Algebra) has a deep connection* to random *matrix theory*
- *Random matrix theory and L-Function has potential to solve open problems of mathematics*

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In this section we will analyze Riemann Zeta Function based on field of values, spectrum, symmetries, coefficient of generating sequences, chaotic system, disconnected spaces, characteristics polynomials of matrices and Hamiltonians .We will also describe describe the limitations and the analysis of continuous part of the spectrum. Our main purpose is to draw attention that functional equation of zeta function is a solar eclipse and to the continuous part of the spectrum of the matrices give more information which discrete part of spectrum can't give alone .We indicate the importance of the locations of nontrivial zeros of Zeta functions and will show that Riemann Zeta is rightly the most important of all Zeta functions.

Since imaginary axis keeps track record of numbers and Mathematics of vibrations is forcing these imaginary numbers onto a straight line providing the reason to the sun to follow the specific path otherwise it will slip from its specified position required for the existence of solar system.

Main Observations

i).For very large n the random matrices do not tend to be normal matrices

i): We know field of values can be approximated for very large Hermitian positive definite (HPD) matrices .Since Riemann Zeta function indicates that the distribution of primes (pi) in the positions of its non-trivial zeros and Riemann Hypothesis says that these zeros are on the critical line and behaves like quasi-random function of t in critical line because the phases of the terms in the product of primes contribute as they were random. If we look at the maximum and minimum values of the zeta function; It is unknown where these values precisely sit. Super-symmetric quantum mechanics has provided some understanding of the exact solvability potentials in one dimensional quantum mechanics. Ideas based on shape invariance; a procedure for getting the spectrum, Eigen functions, reflection and transmission coefficients matrix algebraically for ***one particle system in one dimension so far.***

Correlations between the nontrivial zeros of the zeta function are believed to coincide asymptotically with those between the eigenvalues of large unitary or hermitian matrices and the value *distribution in critical line is believed to be related to that of the characteristics polynomials of these matrices.*

ii).Numerical computation can be successful if we concentrate our attention on distribution of maximum values of the characteristic polynomials; say for example of 4- matrices in the circular interval rather than with large number of matrices. Furthermore it makes the problem of numerical computation of the distribution easier than of large dimensions based on certain field of values and hence providing close connections based on field of values of Riemann zeta function and energy levels easier.

iii): We know that the correlation structure similar to those observed in the eigenvalues of Random unitary matrices and in the Riemann zeros is embedded in the Fermi gas system as well ***.Chaotic system described by Hamiltonian***
H=px
Has imaginary part of Riemann Zeta function nontrivial zeros as Eigen energies.
If the Spectrum of H consists of all of the Riemann Zeta function nontrivial zeros then Hamiltonian can prove Riemann hypothesis without analytic methods.
In Special theory of relativity in 3-dimensions representation, the potential driving A particle to move chaotically with energy H=px behaves as ***Gravitational potential; and hence linking theory of chaos to gravity and to Riemann Zeta function nontrivial zeros .***

*iv).****Riemann's zeros off the straight line will correspond to an imaginary frequency which indicates transition state (saddle point).Although all saddle points don't contribute the solutions.***

Example. The Sun is a spinning ball of gases kept together by its own gravity. If we give a kick to this ball; it will not destroy(explode) if imaginary numbers lie on a straight line as a result of a kick which maintains required gravitational potential.

v). The main steps of vibrational analysis in Gaussian are Mass weight(the Hessian), determination of the principal axes of Inertia(diagonal), generation of coordinates in the rotating and translating frame, transformation of the Hessian to internal coordinates and diagonalization, calculations of the frequencies, reduced mass, force constants and Cartesian displacements.

vi).When computations indicate that the spacing distribution is Poisson ; numeric suggest that the eigenvalue distribution is P

- *There is a similarity between the non-trivial zeros of Zeta function and eigenvalues of large matrices*

- *Asymptotically the nontrivial zeros of Zeta function obey logarithmic law in the critical strip*
- *Asymptotically Prime numbers obey logarithmic law along R; the real number line*
- *Asymptotically Zeros of Zeta function becomes denser while the primes become increasingly spaced out at a logarithmic rate.*

vii). In order to understand the universality of the probability distribution in Random matrix ; analysis of non-equilibrium dynamics ;Study of the long-time dynamics of quantum particle in random potential in 3 dimensions , fluctuations of the longest increasing sequence for a random permutation, Universality of the eigenvalue spacing distribution of ***Random Band matrices and of Random Sparse are important.***

- ***Random matrices describe highly correlated quantum system***

viii) Universality hypothesis states that the local statistics of disordered or chaotic systems depend on the underlying symmetry but are independent of further details because underlying symmetry are cardinality of C AND C IS CONTINUOUS.

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Link between Field of values; EigenValues and Random Matrices

Key Ideas

1).All of the main diagonal entries and eigenvalues of ***AЄMn*** *are in its field of values* ***F(A)*** *Which are contained in RHP*

2).Spectrum of any self-ad joint operator is non-empty; Hermitian matrices have orthogonal

Eigenvectors meaning eigenvectors can be chosen in the form of an orthonormal basis

3).Successive unitary similarities leaves eigenvalues and field of values F(A) invariant

4).Degenration happens for normal matrices. If solutions are real it means solutions are Oscillating while complex solutions exponentially decay or grow.

5).Any bounded convex set can be approximated by F (A) of some matrix. Real part of each Eigen is convex combination of the main diagonal entries.

6).The fact that the field of values F (A) of a square complex matrix is convex has an immediate Extension to the infinite-dimensional case

7).The chances for a group being an approximate invariance group is better the smaller, is its Effect in transforming the fields around

8). Eigenvalues (energy values) of a path dependant function can break magnatic field of values The ability of the eigenfunctions to form a basis for the space (expansion of function in terms Of them) for Hermitian operators is not true; Hermitian operators need to satisfy additional , properties for this property the spectral theorem to hold but is true for Hermitian matrices

9).Eigenvalues and eigenvectors and Hermitian matrices give us Fourier series when applying Them to functions and not to finite column vectors

10).The eigenfunctions are orthogonal and form a basis for arbitrary function depending on the Shape of the instrument chosen for performance. Hitting instrument at different points excites Different eigenfunctions with different frequencies, coefficients and, tones and expanding it . in the Eigen function.

Definition:

The field of values of n×n matrix A is a function from Mn into subsets of the complex plane and is denoted by

$$F(A) \equiv \{x_* Ax : x \in C_n, x_* x^* = 1\}.$$

F (A) can be continuum therefore can give information that the spectrum σ(.) set of eigenvalues of a matrix which is a discrete point set alone cannot give. As a function from Mn into subset of C The field of values F (.) has many functional properties
Compactness for all A∈Mn

Convexity property

Translation property

Scalar multiplication

Projection property

Positive definite indicator function

Positive semi definite

Indicator function

Spectral containment

Subadditivity for all A∈Mn

And combining the pleasant properties of eigenvalues of Hermitian and normal matrices and the field of values F (A) control quietly certain aspects of this nice structure for general purposes.

Thus F(A) may be thought of as

1). the normalized locus of the Hermitian form associated with A

2).The image of the surface of the Euclidean unit ball in C

3).The numerical range of A

*4).A compact set under the continuous transformation x→x*Ax*

5). A compact and hence bounded set in C

6). An unbounded analog of F (.)

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7). Sequence of conjugate transformation which converges to translation.

Now for $A = \begin{Bmatrix} a & 0 \\ 0 & b \end{Bmatrix}$

a, b ≥ 0 ; then field of values F(A) is ***ellipse with its interior, with center at origin.***

Minor axis is along imaginary axis, with length $|a-b|$*, major axis along the real axis with length a+b.*
Its ***foci are at*** $\pm\sqrt{ab}$ ***which are the Eigen values of A***

$$Z^*AZ = (e^{i\theta} Z)^* A (e^{i\theta} Z) \text{ for any } \theta \in \mathbb{R}$$

For F (A) we take unit vectors whose first component is real and non-negative, so we take two vectors

$$Z = [t \;,\; e^{i\theta}(1-t^2)^{.5}]^t$$

For t between 0 and 1 and θ between 0 and 2π

$$Z^*AZ = t(1-t^2)\,[(a+b)\cos\theta + i(a-b)\sin\theta]$$

As θ varies from 0 to 2π , the point (a+b)cosθ+i(a-b)sinθ ***traces a degenerate ellipse with center at origin with major axis from –(a+b) to (a+b) on the real axis and the minor axis from i(b-a) to i(a-b) on the imaginary axis in C***

as t varies from 0 to 1 ,the factor $t(1-t^2)^{.5}$ *varies from 0 to ½ and back to 0 verifying F(A) is the asserted* ***ellipse*** *with its interior which is* ***convex***

The two foci of the ellipse are located on the major axis at a distance

$$[\tfrac{1}{4}(a+b)^2 - \tfrac{1}{4}(a-b)^2]^{.5} = \pm\sqrt{ab}$$

From the center.
Therefore F(A) is convex subset of C for all A∈ Mn
Since F(A) is the range of the continuous function $x \rightarrow x^*Ax$
Over the domain $\{x:\; x \in C_n \;,\; x^*x = 1\}$

The surface of the Euclidean unit ball, which is a compact set and we know the continuous image of the compact set is compact , ***Hence F(A) is compact.***

$A\in M_2 \to \longrightarrow \begin{matrix} 0 & a \\ b & 0 \end{matrix}$ a,b>0

$A \to A-(1/2trA)\ I \equiv A_0$ to get $trA=0$
ation

$A_0 \to UA_0U^* \equiv A_1$

$A_1 \to VA_1V^* \equiv A_2$ where $A_2 = e^{\wedge}i\Phi[\ 0\ \ a]$
$[\ \ b\ \ 0]$

$A_2 \to e^{-i\Phi} A_2 \equiv A_3$ unitary rotation

If $A_3 = \begin{matrix} 0 & a \\ b & 0 \end{matrix}$

and a=b

$A^*_3 A_3 = \begin{matrix} b^2 & 0 \\ 0 & a^2 \end{matrix}$ $A_3A^*_3 = \begin{matrix} a^2 & 0 \\ 0 & b^2 \end{matrix}$

Iff A_3 is normal

Now $A_3 \longrightarrow A$

By scalar multiplication (non-zero) , two unitary similarities, translation, all these properties preserve normality and non-normality so A_3 is normal iff A is normal and **Field of values of A_3 is an ellipse.**

$L_\theta \equiv$ the line { e^-iθ (λθ+ti) ; t∈R}

And half plane determined by L_θ is

F(A) is contained in $H_\theta \equiv$ the half plane e-i**θ{Z: ReZ<=λ}**

$p_\theta \equiv X_\theta{}^* A X_\theta$ iis a boundry point of F(A) and is a complex number.

Since F(A) is convex; each maximum point occurs as a P_θ

$P_\theta \in L_\theta \cap F(A)$; L_θ is a support line for F(A).

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For any scalar a does not belong to F(A) there is a line L_θ separating F(A) and

Scalar a meaning a does not belong to H_θ

Finite $F_{out}(A,\theta)$ is bounded and outer boundary of F(A) is a convex curve.

It is very important to note that for a convex set in a closed upper half-plane and if a strict convex combination of given points in C is real, then all of these points are real.

For $A \in M_2$ we transform, A to special form

$$\begin{matrix} 0 & a \\ b & 0 \end{matrix}$$

$a,b > 0$

by $A \longrightarrow A - (1/2\, tr\, A)\, I \equiv A_0$ to get $trA_0 = 0$

Then by unitary similarity $A_0 \longrightarrow UA_0U^* \equiv A_1$

to make both diagonal entries of A_1 zero then another unitary similarity $A_1 \longrightarrow VA_1V^* \equiv A_2$

$$A_2 = e^{i\phi} \begin{matrix} 0 & a \\ b & 0 \end{matrix}$$

$a, b > 0 \quad \phi \in R$

by unitary rotation $A_2 \longrightarrow e^{-i\phi}A_2 \equiv A_3$ so we get

$$A_3 = \begin{matrix} 0 & a \\ b & 0 \end{matrix}$$

$a,b > 0$

Note:

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The outer boundary of F(A) is a convex curve but we are not sure that the interior of outer boundary is completed filled out with points of F(A).Hence only the outer boundary of F(A) Doesn't prove that F(A) is convex.

Outer boundary of F(A) is a convex curve and F(A) is a convex set.

Since F(A) is non-empty, closed and bounded, so its complement has unbounded component.

Let $e^{i\theta} A = H + iK$

$\theta \in \{0, 2\pi\}$ *H and K are Hermitian,* $H, K \in M_n$

$\lambda_n(H)$ *be the largest Eigen value of H*

let $S_\theta \equiv \{ x \in C^n \quad x \neq 0 \quad Hx = \lambda_n(H)x \}$

$\dim(S_\theta) = K \geq 1$

then the intersection of $F(e^{i\theta} A)$ *with the vertical line* $\text{Re}Z = \lambda_n(H)$ *is the set*

$\lambda_n(H) + i\{ x^*Kx : x \in S_\theta \quad \|x\|_2 = 1 \}$

If we take K=1 then it is a single point and if K > 1 , then it is a finite interval which is a convexity property of field of values F(A) of Hermitian matrices which follows from spectral theorem.

Outer boundary of F(A) is convex which may contain straight lines; $\theta \in [0, 2\pi]$ *and any convex polygon is the field of values of a matrix of higher dimensions. In Normal Matrices Eigen values are the vertices of the polygon.*

Let $A = \begin{Bmatrix} \lambda_1 & \alpha \\ 0 & \lambda_2 \end{Bmatrix}$

F(A) is a point iff $\lambda_1 = \lambda_2$ *and* $\alpha = 0$ *; a line segment joining* λ_1 *and* λ_2 *iff* $\alpha = 0$

A disc of radius $1/2(|\alpha|)$ *iff* $\lambda_1 = \lambda_2$

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An ellipse with its interior with foci at λ_1 and λ_2 , but if A is of higher dimensions, then a very rich variety of shapes is possible for the field of values F(A)

Conclusion:

Any bounded convex set can be approximated by field of values of some matrix using Properties of field of values F(A) which are in some sense complete.

Area[convex[union I from n F[A(i)]]*

-- $\geq c_n$

Area [F(A)]

$\lim_{n\to\infty} C_n = 1$

$\lim_{x\to\infty} \pi(x) / (x/\ln x) = 1$ $\qquad C_n \equiv (n - 2) / 7n - 2 - 6 [n(n - 2)^.5]$

$C_{n^*} \equiv (2n - 5) / (2n + 7)$

$\lim_{n\to\infty} C_n / C^* = 1$

$C_n \approx C^*$ as $n\to\infty$

Hence for a convex set in a closed upper half-plane and if a strict convex combination of given points in C is real then all of these points are real.

Asymptotic Behavior of Field of Values F(A) and The Riemann Zeta function $\zeta(s)$

In this section we will discuss the asymptotic behavior of the field of values F(A) and the zeta function

$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$, and its connection to random matrix theory. We describe the essential aspects of both fields and how insights in one apply to the other. The goal is overview of these two fields. Matrix theory is a requirement to understand the conjecture for the distribution of the horizontal parts of the zeroes of ζ(s) using unitary matrix approach; i.e. distribution of zeroes of ζ(s) of characteristic polynomial.

Conclusion

Random matrix theory is a key to understand ζ(s). Zeta function controls the genus of more than one function. Pause (zero) and play position (functional equation) of this key are important to understand. Functional equation of ζ(s) is a solar eclipse and is therefore related to, past, present, and future calendar of the earth. Since solar eclipse is a straight line configuration of three Celestial bodies (sun, moon, earth)in a gravitational system so extension of solar eclipse is straight line configuration of N Celestial bodies and then extension to infinity .

Summary:

We know square and circle are hemimorphic and square indicates as sine waves. Quantum Mechanics

Is random but wave function is not Random .Analyticity properties of the Spectral Data (jump Matrix) prevents instability. Structural information buried in a matrix

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challenges in a path finding problem. Path dependent functions give eigenvalues while path independent functions deform to a point. Each entry on the matrix's diagonal represents a self-link of the co-responding vertex. We try to compare the path of the field of values F(A) with general complete-path matrix equations .Interval and length of information at certain time are part of scaling factors for the periodic function and both affect frequency; periodic orbits obey an analogue of the prime number theorem . Path dependent function's energy values (Eigen values) can break magnetic field of values but scaling factors for the periodic function on multi-dimension will not be periodic. Recurrence (frequency) spectrum is periodic and if we compare it against the spectral density of another process with higher (interval, sampling, frequency)

We get nothing because it is periodic; as periodic functions give structure to space of all possible orbits and periodic orbits obey an analogue of the prime number theorem. We can consider in the process with higher rate if we "limit the part of the spectrum" because when symmetry is broken coupling constants for all the factors have critical size at fundamental energy scale.

Random dynamics fits experimentally coupling well and Feynmann diagrams are Related to* Random Matrices*

$I - ((Sin(\pi x)/\pi(x))^2$ is a pair-co-relation function for the Eigen values of a random hermitian matrix and into exponential form imaginary numbers produce SineWave

For $A \epsilon M_n(R)$ the field of values F(A) is symmetric to real axis, $A \epsilon M_n$ outer boundary of F(A) is convex, existence of closed real convex set in C indicates link of F(A) to ζ (s)

Therefore in ζ (s) = ζ (1/2 + ir**j**) , r**j** are Eigen values of Random Matrix with any specific distribution.

f at ∂S= (ξ , 1 , 0 , ∞) = (0 ξ ∞ 1) = 1/2 ----------------(*)

Since f: square = S = { x + iy : lxl<1 ,lyl < 1}→H (U.H.Plane) = { x+iy : y>0}

ζ (s) is not defined for negative values of S therefore by more than one conformal map and different normalization at vertices, rotation of 90 degree, ad-bc = 1 in linear fractional transformation and by considering the ***Conformal map of the square***

{ - ϵ < x <1 , - ϵ < y < 1 }

We get *

Remarks:

Square and circle are hemimorphic and squares indicate as sine waves. The main approximation to a wave is a pure tone a sine wave of that same frequency and the collection of frequencies of the harmonics that compose the wave being synthesized is the spectrum of the wave.

Field of values is sequence of conjugate transformation which converges to translation $Z \rightarrow Z+1$

Collection of open set regenerates field

Renormalization is finding a sequence of parabolic transformation $T_{n(Z)} = Z + b_n$ for which the limit $(A_n T_n A^{-1}_n) = Z + 1$ *into exponential form imaginary numbers produce SineWave*

<u>Key ideas:</u>

- *Nature is chaotic; is nature a homeomorphism; meaning can we deform it to a unitary representation or be extended to a homomorphism?*
- *Compact manifold can be deformed to holomorphic equivariant harmonic maps.*
- *Primes go through properties of ζ(s); At higher order zeros of ζ(s) are regularly spaced*
- *Existence of Euler product of ζ(s) guarantees that the non-trivial zeros of the Zeta function lie on a line.*
- *ζ(s) gives the generalized form of all types residues of all poles*

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- *Analytic continuation of function of real variable can be reduced to holomorphic functions*
- *Convexity property of field of values F(A) of Hermitian Matrices follows from spectral theorem ;zero free region of zeta function are connected components even one removes critical strip*
- *Critical strip of ζ(s)^k is an open set possibly to regenerate* ***F(ζ)***
- ***Periodic orbits obey an analogue of the prime number theorem***
- ***In Normal matrices Eigen values are the vertices of the polygon***

Introduction:

The Riemann zeta function ζ(s), which is defined for Re(s) > 1 by $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$.

We can also write ζ(s) *as a product over primes by unique factorization, known as the Euler product of* ζ(s).

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

Since definition is only valid for Re(s) > 1 but Riemann Zeta function can be analytically continued to all of ***C*** *using complex analysis.*

Riemann Hypothesis explains that all complex zeros of the function in which the analytical continuation of $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$ *in the region o < Re(s) < 1 lie*

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on the line x =1/2 ; hence the role of imaginary part is very important as the east-west axis of two dimensional map of imaginary numbers keeps track of real numbers. Geometry of any graph is related to imaginary numbers and into exponential form imaginary numbers ***produce Sine Wave.***

$Z(s)$ converges absolutely when Re(s) > 1; ζ(z) has a pole at z = 1 whose residue is 1. Now we will differentiate $\zeta(s)$ to see the behavior of zeroes and poles of $\zeta(s)$, we know at s = 1 pole of $\zeta(s)$ and s = 0 mirror of pole of ζ (s) and we also know that logarithmic differentiation of a complex function yields information about zeroes and poles of that function because all types of poles and zeroes are in ζ(S) and not only the pole at s = 1 and ζ(s) gives the generalized form of all types of residues of all poles. In order to understand Riemann Hypothesis, it is very important to understand analytic continuation of ζ(s) meaning meromorphic structure of analytic continuation (pole structure of analytic continuation).We will address each issues one by one keeping in mind that analytic continuation of function of real variable can be reduced to holomorhphic function. Since singular point of an analytic function is an obstacle to the element of the function along the curve and a maximal analytic continues function f to the domain D but cannot be analytically continue to the boundary of D. The maximal analytic continuation of (D_0, f_0) in M complex many fold is unique. Continuation of f_0 to the

point Z: (D’, f’) ~ (D̋, f″)

for Z∈ $D' \cap D''$ and $f' = f''$ in the neighborhood of Z function f_0 is lifted to D_f in

a natural manner and its value on the equivalence class at Z containing (D_o , f_o) is set

Equal to $f_o(Z)$; which continues analytically to all of D_f and it does not continue to any boundary of D_f over compact manifold M.

***Analytic continued fraction** is related to three-term recurrence relations; since different types of zero free regions are obtained for sequence of polynomials having complex co-efficient and having three-term recurrence relations; in the form of sectors and stripes in **C** (which are obtained from Bessel function).*

Therefore different regions can be obtained by considering required conditions on co-efficient a_n, b_n so that polynomial $A_{n(z)}$ and $B_{n(z)}$ don't vanish simultaneously; and non-vanishing of A_n gives results concerning the non-vanishing of B_n.

For example Zero-free parabolic regions are obtained for the polynomials $B_n(z)$ by considering the special case $a_n = -a_n z$ where

$A_n/B_n = a_1/b_1 + a_2/b_2 + \ldots\ldots\ldots\ldots + a_n/b_n$ *is a continued fraction,* $a_n, b_n \in C$

$B_0 = 1$, $B_{-1} = 0$ *and* $B_n = b_n B_{n-1} + a_n B_{n-2}$

$A_n B_{n-1} - B_n A_{n-1} = (-1)^{n-1} a_1 a_2 \ldots\ldots a_n$; $n \in N$

$A_0 = 0$, $A_{-1} = 1$,

$A_n = b_n A_{n-1} + a_n A_{n-2}$

*We need $A_n/B_n = \infty$ holds, so A_n and B_n do no*t vanish simultaneously.

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$A_n/B_n \neq \infty$ *if* $B_n = 0$

If a seq B_n *holds true for*

For $a_1 \epsilon \mathbb{C}, a_1 \neq 0$

Arbitrarily B_n *can be taken as the* n^{th} *denominator of a sequence of continued fraction of* A_n/B_n

Want $A_n/B_n \neq$ *infinity, so require conditions on* a_n, b_n so $B_n \neq 0$. *For* $n \geq 2$ $n \epsilon N$

$a_2/b_2 + a_3/b_3 + \ldots. + a_n/b_n = A_{n-1}/B_{n-1}$

$A_n = 0$ *if* $B_{n-1} = 0$ *holds.*

Therefore non-vanishing of A_n *, gives results concerning the non-vanishing of* B_n

$a_n = -a_n z$

$B_n = B_n + z_{n+1}$ $\quad a_n > 0 , B_n > 0 \qquad n \epsilon N$

Since generating function $f(x) = {}^{i}\Sigma_{n=0}\, a_n x^n$*, where* a_n *form a sequence i.e.* a_n *is an ordered set of mathematical objects of polynomial sequence, e.g convergent sequence, fractal sequence, iteration sequence etc. Exact expression for the generating function in more than one variable, e.g. for generating function in two variables; we need to find conjugates in two directions (e.g. steps and contacts); analytic structure of this generating function, its transition corresponding to collapse etc.*

For example [Average value of n at a point (x,y) in plane]; generating function is

$G(x,y) = (F(x,y))/(1-e^{k(x)}y) \quad ; \quad \langle n \rangle = N$

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In order to understand the zeros of Riemann Zeta function we need to understand the characteristic of its derivatives.

***Derivatives** of Riemann zeta function have the following properties*

- *vertical strips in the right half-plane*
- *Critical strips tend to converge to their central critical lines*
- *Central critical lines possess vertical periodicities which give formulas for their exact number.*
- *All the zeros contained in central critical lines are simple*
- *Has poles of order K at the point s = 1 and can be extended to a meromorphic function on ℂ.*
- *Have no functional equation meaning zeros of derivatives don't lie on line*
- *Have no Euler product, meaning non-trivial zeros of derivatives do not lie on a line.*
- *Has zero free regions*
- *Has all simple zeros in critical line*
- *All zeros of derivatives (non-trivial) exhibit vertical periodicity.*
- $| (\log n)^k / (n^s) | > 0$
- *Critical strip of derivatives of Zeta is an open set to regenerate* $\boldsymbol{F}(\zeta(s))$
- *Zero free regions are the connected components that remain after one removes critical strip from the right half plane.*

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- *Differentiation (logarithmic) of Zeta yields information of complex function about zeros and poles; in graphs critical strips of derivaties are the narrow regions centered around the critical lines that separate regions of dominance (wedges) of the terms of critical strips.*

  Prime numbers go through like eigenvalues (energy values) and sequence of primes is non-holomorphic meaning generating function of primes admits near singularity asymptotic expansions in scale that involve logs and iterated logs i-e

  At least one term in expansion is iterated log or power of log with an exponent not in **Z** *(set of Integer). Integer recurrent sequence are one with rational generating functions which consists of only primes is a periodic, therefore is finite.*

  Since operators of complex order do not admit spectral cuts and zeta function has meromorphic structure. There are many curves in graph of Zeta which end up at the same point in the larger domain. In analytic continuation, continuation is area wise and not the point wise so paths are not prime in $\zeta(s)$ meaning paths have backtracking $a_{i+1} = 1/a_i$

  As $\zeta(s)$ goes twice in first quadrant. $Z(s)$ involves many different shaped curves therefore zeros need to address on an individual basis. .

  $\zeta(s)$ has simple pole at S=1 and there is not any homogeneous component of degree -2 so all ***residues, zeros, and poles of degree -2 and higher don't exist.***

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We know in Random Matrix entries are random numbers from some specified distribution for example finger print pattern, zebra's pattern etc.

Since $\zeta(s)$ is a certain super position of oscillators with scaling

$$P(w) \approx e^{-w(2\sigma-1)}$$

Because For $A \in M_n$ I with elements possessing a standard normal distribution the Real Eigenvalues are

$$\sigma(A_n) = \tfrac{1}{2} + \sqrt{2} \quad (\; 2F(1, -1/2; n ; \tfrac{1}{2}))/ (B (n, \tfrac{1}{2})$$

where `F(1, -1/2 ; n ; ½) hyper geometric function and B (n , ½) is Beta function

with asymptotic behavior

$$\sigma (A_n) \quad \sim \quad \sqrt{2n/\pi}$$

As scaling factors for the periodic function on multi dimension are not periodic, so

We take matrix A, hermition Random matrix whose entries are from very big population, continuous, unbounded, ≈ ***real*** *(so can be ordered)*

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Now $\zeta A(s)$ *for* $S = 0$ *; converges on* $\mathbb{C}$

In order to understand the spectrum of $\zeta A(s)$ *, it is important that det of*

$\zeta A(s) \neq 0$

Meaning, we should make sure that $\zeta A(s) = \lambda_{ii}^{-ss}$ *is defined for*

$\lambda i \in C$

*Since operators of complex order do not admit spectral cuts th*erefore it is *possible that spectrum is empty. Since* $\zeta(s)$ *has meromorphic structure and most often series are not convergent in meromorphic structure*

Since Field of values of a matrix A which is also denoted by

$$F(A) = C_o\{P\theta : 0 \leq \theta \leq 2\pi\} = \{x^*A\,x : x\epsilon C^k,\ xx^* = 1\}$$

$$= \bigcap (H\theta) \qquad 0 \leq \theta \leq 2\pi$$

$$= \bigcap (\text{ the half-plane } e^{-i\theta}\{Z : Re\,Z \leq \lambda_\theta\}$$

$$L\theta = \{\lambda_i \in C\}$$

Since the field of values F(A) is a convex subset of C^k $\forall$ $A \in Mn$ *as it is the continuous image of a connected set.*

$\forall$ $A \in Mn$ *and every angular mesh* θ *therefor*

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$$Max \quad |P\theta_i| \leq r(A) \leq Max \; |q\theta_i|$$

$1 \leq I \leq K \qquad\qquad 1 \leq I \leq K$

and $F^k(A) \subset F^{k-1}(A) \subset F^{k-2}(A) \subset \ldots\ldots\ldots\ldots\ldots \subset F^1(A)$

So $F[A(i')] \subset F(A)$

For $i = 1,2,3,4, \ldots\ldots\ldots, n$, where $A(i')$ is principal sub matrix of

$A \in Mn$ by deleting row and column i

Therefore $C^\circ [U_i \; F[A(i')]] \subset F(A)$

Now we want to check that how much of the right-hand side does the left-hand side fill up?

The answer is "ALL OF IT IN THE LIMIT" as $i \rightarrow \infty$

$$Area\,[C^\circ [U_i \; F[A(i')]] \,/\, Area\,[F(A)] \geq C_n$$

Ұ $A \in Mn$; there exists a sequence of constant $C_2, C3, C4, \ldots\ldots\ldots\ldots, \infty \in [0, 1]$

For $n = 2, 3, 4, \ldots\ldots\ldots\ldots\infty$ where

$C_n \,\Xi\; n - 2 \,/\, 7n - 2 - 6\,[n\,(n - 2)\verb|^|.5$

$C'_n \,\Xi\; 2n - 5 \,/\, 2n + 7$

$Limit\; n \rightarrow \infty (C_n \,/\, C'_n) = 1$

Limit $n \to \infty$ $C_n = 1$

Since Limit $x \to \infty$ $[\pi(x) / (x/\ln x)] = 1$

Conclusion:

Therefore F(A) and) *ζ(s) have same* **asymptotic behavior .**

Note: For $A \epsilon M_n(R)$ ***the field of values F(A) is symmetric to real axis,*** $A \epsilon M_n$ **outer** ***boundary of F(A) is convex, existence of closed real convex set in*** $\mathbf{C}$ ***indicates link of F(A) to*** $\zeta(s)$

Therefore in $\zeta(s) = \zeta(1/2 + ir_j)$, r_j *are Eigen values of Random Matrix* **with any specific distribution**.

F at $\partial S = (\xi, 1, 0, \infty) = (0 \;\; \xi \;\; \infty \;\; 1) = ½$

Note: f: square = S = { x + iy : lxl<1 ,lyl < 1}→H (U.H.Plane) = { x+iy : y>0}

$\zeta(s)$ *is not defined for negative values of S therefore by more than one conformal map and different normalization at vertices, rotation of 90 degree, ad-bc = 1 in linear fractional transformation and by considering the* **Conformal map of the square**

$\{-\epsilon < x < 1 \quad , \quad -\epsilon < y < 1\}$

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Chapter *5*

Need for deeper Development in Math Education

Activity and descriptive level analysis

Development of minds is need of the time and is central to student achievements. Not much could be achieved without advanced education systems linked in right direction with actual economic and political policies of the world. Undemanding faculty must be the driving forces of all these goals.
Students who are trained under advanced systems (e.g., Harward) to learn about a particular topic perform better than their formal systems peers.
According to Will Roger "Even if you are on the right track, you will get run over if you just sits there."
Collapsing economies, lack of scholars, lack of direction (e.g. political intolerance between different countries) wars, religious intolerance, increasing unemployment, rat race for nukes and injustice, certainly demands for deeper developments in our education systems that should be able to scratch the human minds. The passionately curious minds are need of the present time and since all creative principles resided in mathematics that can develop human understanding the most.
Researchers have pointed out that real issues of education systems are not rightly addressed to improve a student's performance and mindset worldwide. The following quotations confirm the holes of the systems.
For example, Quoted by Moszkowski in Conversations with

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Einstein (1920) 65.
1.“Most teachers waste their time by asking questions that

are intended to discover what a pupil does not know, whereas the true art of questioning is to discover what the pupil does know or is capable of knowing.”—Albert Einstein

2. “A human being is a part of a whole, called by us _universe_, a part limited in time and space. He experiences himself, his thoughts and feelings as something separated from the rest... a kind of optical delusion of his consciousness. This delusion is a kind of prison for us, restricting us to our personal desires and to affection for a few persons nearest to us. Our task must be to free ourselves from this prison by widening our circle of compassion to embrace all living creatures and the whole of nature in its beauty." Einstein

3. “I very rarely think in words at all. A thought comes, and I may try to express in words afterwards.”—Albert Einstein --- Quoted in M. Wertheimer, “Productive Thinking” (1959).

4. “The ability to portray people in still life and in motion requires the highest measure of intuition and talent.” Einstein
5."We can't solve problems by using the same kind of thinking we used when we created them." Einstein
6. “Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world.” Quoted in interview by G.S. Viereck , October 26,1929. Reprinted in “Glimpses of the Great”(1930)

7. “I have no special talents. I am only passionately curious.” —Albert Einstein --- To Carl Seelig – March 11, 1952. AEA 39-013

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8.”The most beautiful experience we can experience is the mysterious. It is the source of all true art and science. . . ” From “The World as I See It” (1930), reprinted in Ideas and Opinions, 11.

9.”The aim [of education] must be the training of independently acting and thinking individuals who, however, see in the service to the community their highest life problem.”—Albert Einstein --- *From Address, October 15, 1936 – Reprinted in Ideas and Opinions, 60.*

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10.”We allow our ignorance to prevail upon us and make us think we can survive alone, alone in patches, alone in groups, alone in races, even alone in genders.”
Maya Angelou

Effective teachers can develop student thinking by linking instructions
Focus on recent related research, student existing knowledge base, undemanding believes, effective teaching, and by caring classroom environment. Teaching should be such that what is offered is perceived as a valuable gift and not as a business duty. Researchers have argued that many factors can improve our education system, knowledge (advanced scientist network based) is the main key source and Intellectual workers are the driving elements of the developed system.The ability to win hearts and minds in classrooms requires the highest measure of intuition and talent which have a greater impact on student achievement than any other factor which requires simple passionately curious teachers.

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"I have no special talents. I am only passionately curious." — Albert Einstein --- To Carl Seelig – March 11, 1952. AEA 39-013

"We learn something every day, and lots of times it's that what we learned the day before was wrong." Bill Vaughan

"Believe those who are seeking the truth, doubt those who find it". Andre Gide

"The third-rate mind is only happy when it is thinking with the majority. The second-rate mind is only happy when it is thinking with the minority. The first-rate mind is only happy when it is thinking."
A. A .Milne

There is need to identify forces influencing the development of system of education. Undemanding effective teachers can serve a variety of goals. It can deepen our understanding of our own education, society, assistance to policymakers and administrators. A review of correlates of academic achievement and its implication for educationists and policy maker would be fruitful.

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Objectives

The purpose of this study is to see teachers and students at their best in mathematics education.

The main objectives of the study are

1). Investigate the impact of teacher's curiosity and talent (quality of publications, Books, seminars, continuous link of the curriculum and research, strategies, courses, library, self-control and undemanding) on the academic achievements
(2). Investigate the academic knowledge of the university teachers in mathematics (# of publications, books, conferences, courses etc.) and student satisfaction level
(3) Examine the academic achievement of the university students after observing great teachers
(3) Investigate the causes which increase academic achievement the most
(4) Test the relationship between the teacher's philosophy and Einstein's and other scholar's ideas (on education)
(5) Recommend strategies for the improvement in mathematics education at university level

(6). Recommend strategies for the lack of direction in mathematics education

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Method

Working university teachers in mathematics along with the graduate students are the population of this study. A random sampling technique will be used for selection of the sample.

Ten Universities,
University of NU &UB (all math faculty and math grad students)

University of Harvard (1 faculty (the best), 1 top grad student)

University of Toronto Canada (1 faculty, 1 grad student)

University of Cambridge UK (1 faculty, 1 grad student)

University of Tokyo Japan (1 faculty, 1 grad student)

University of Melbourne Australia (1 faculty, 1 grad student)

University of Gottingen Germany (1 faculty, 1 grad student)
University of France
Peking University China
University of India
University of Punjab Pakistan or UET (1 faculty, 1grad student)

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Further 5 teachers and 25 students from 5 universities will be also randomly selected as the sample of the study.
Two questionnaires, one for university teachers and other for university students will provide data for the study. Further information regarding faculty qualifications will be obtained from website (counter verification from DGS)
Collected data will appear in tabular form in the light of objectives of the study by applying statistical tools (e.g., chi-square)

This study is very important because education is the main key of moral, cultural, political and economic development of any country.

This study is important for future research for other levels. The result of this study will provide information that would enable university administrators and the teachers to achieve skills, needed to succeed in universities. University students may use the results to increase their understanding of academic achievement. This research will provide guidance for future research studies in the same field.

Following objectives are kept forth for the study:

1. Lack of direction in Mathematics education
2. Examine the academic achievement of the university students as a result of teacher's higher qualifications
3. To find out the causes which effect the academic achievement the most
4. To sort out the relationship between the teacher's abilities and academic achievement.

Sample

The sample of the study consists of 5 faculty and 25 students of 5 universities

Using five-point scale in the light of objectives of study under the Supervisor guidance

(Quality of questions, filling, student ID, sample teachers and students, results of the students from controller, examination of the statements of questionnaires)

DATA

Rating the responses on the basis of following scoring procedure items of the questionnaires

Strongly Agree as 5

Agree as 4

Un-decided as 3

Disagree as 2

Strongly disagree as 1

Questionnaires items

After scoring the items, the scores of the individual items will be added to get the teacher's and student's scores.

ANALYSIS

Data tabulation, interpretation in the light of the objectives of the study.

by most suitable statistical tools like chi-square

And Pearson's Product- Moment Coefficient of Correlation(r) for the results.

RESULTS (for example)

The study aimed at investigating the implication of Einstein's thought

."Most teachers waste their time by asking questions that are intended to discover what a pupil does not know, whereas

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the true art of questioning is to discover what the pupil does know or is capable of knowing."—Albert Einstein

RESPONSES OF FACULTY

** df = 4 Table value X at 0.05 level = Y*

Strongly agree=5

Agree=4

UNdecided =3

Disagree=2

Strong Disagree=1

Formula for statistical treatment chi-square

Value of Chi square $\chi 2=\sum(f_0-f_e)^\wedge 2/f_e$

Formula for Pearson's Product –Moment Coefficient of Correlation r is as

R=NΣXY- (ΣX) (ΣY)/(N ΣX^2 (ΣX)^^2) ^.5-((N Σ Y^2- (ΣY)^2)^.5

- *Significant degree of freedom= 4*
- *Table value at 0.05 level*

Comparison of the calculated values, if this value is Greater than the table value at 0.05 level. Hence, the statement is accepted therefore bring it into knowledge of faculty recruitment offices of the university.

RESPONSES OF students

If accepted send to Directors of Graduate studies of universities.

Co-relation between the results of UB and 10 TOP Universities

Accepted or rejected faculty questionnaire

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Summary
Accepted or rejected items of students questionnaire
Summary of Sample study

Co-relation between the results of UB and 10 selected Universities and sample study

Conclusion of the study.

Reference:

www.asia images.com