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# An Inventory Model for Deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging

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**Abstract** - The Paper presents an inventory model for deteriorating products with demand as linear function of time and time dependent holding cost. A two-parameter weibull distribution is used to represent the distribution of the time to deterioration. In which shortages are allowed and partially backlogged, backlogging rate is variable and is dependent on the length of the next replenishment.

**Index Terms**— Deteriorating Products, Partial backlogging, shortage, time varying holding cost, Weibull distribution.

## 1 INTRODUCTION

Inventory is defined as an idle resource which helps us to run the business successfully and effectively. The inventory products can be classified into three categories based on their shelf life. They are obsolescence (b) Deterioration (c) Without deterioration. Deterioration is damage caused due to Spoilage, Dryness, etc. The equation for Inventory model for Deteriorating products is $\frac{dI(t)}{dt} + \theta I(t) = -f(t)$ where $\theta$ is the constant decay rate, I(t) the inventory level at time t, and f(t) the demand rate at time t.

In the classical inventory model the demand rate was assumed to be constant. But it is not always possible. For example seasonal goods (like greetings cards, Umbrella etc) demand rate is not a constant throughout the year. The demand rate may be time dependent, price dependent and stock dependent. Ajantha Roy developed an inventory model where demand rate is a function of selling price. Vipin Kumar, Sr Singh, Sanjay Sharma [7] have developed a production inventory model for linear time the demand is with which implies an uniform change in the demand rate of the product per unit time. Time – proportional demand was developed by Dave and Patel .S.K.Ghosh and K.S.Chaudhuri[4] had discussed Quadratic time demand in their inventory model. Similarly the constant deterioration rate was relaxed by Covert and Philip. A two – parameter weibull distribution to represent the distribution of time to deterioration were considered by Zhao Pei-xin [15], S.K.Ghosh and K.S Chaudhuri [4], Azizul Baten and Anton Abdulbasah kamil [3] etc.


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In general holding cost is assumed to be known and constant. But in realistic holding cost may not always be constant. Many researchers like C.K.Tripathy and U.Mishra [1], V.K.Mishra and L.S.Singh[12], Ajantha Roy[2] etc have discussed with time dependent holding cost.

In real life some times all the demand cannot be satisfied with the existing inventory. When shortages occur, all customers may wait until the arrival of the next order, all customers may leave the system or some customers are willing to wait for the next replenishment. V.K.Mishra and L.S.Singh[12 ] have developed an inventory model with variable as backlogging rate. Inventory model with partial Backlogging rate was developed by Liang-Yuh OUYAHG, K.S.W.U, M.C. Chany [9], G.P Samanta, Ajanta Roy [ 11], etc.

In this paper, an inventory model for deteriorating products is developed with time dependent on demand and holding cost. Shortages are allowed and partially backlogged. Backlogging rate is variable and is dependent on the length of the next replenishment. We use the same notation and assumptions as C.K.Tripathy, L.M.Parthan [14] except some.

## 2 MATERIALS AND METHODS

### Assumptions and Notations

The following are the assumptions and notations applied in the proposed model.

- The inventory system deals with single item
- The lead time is zero
- Shortages are allowed and are partially backlogged. During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative

inventory is $\beta(t) = e^{-\lambda(T-t)}$, $\lambda$ is backlogging parameter and (T-t) is waiting time ($t_1 \le t \le T$)

- $\theta$ the deterioration rate. The deterioration of units follows the two parameter weibull distribution $\theta(t)=\alpha\beta t^{\beta-1}$ where $0< \alpha <1$ is the scale parameter and $\beta>0$ is the shape parameter.
- Holding cost h(t) per item-unit is time dependent and is assumed to be h(t) = c+dt when c >0, d >0.
- T is the length of the cycle.
- A is the cost of placing an order
- Replenishment is instantaneous at an infinite rate.
- The demand rate is D(t) = a +bt, where a > 0, b > 0.
- The planning horizon is finite
- A,$C_1$,$C_2$, $C_3$ & $C_4$ denote the set up cost, inventory carrying cost, deterioration cost per unit time, shortage cost for backlogged items and the unit cost of lost sales respectively. All of the cost parameters are positive constants.
- Deteriorating take s places after the life time of items.

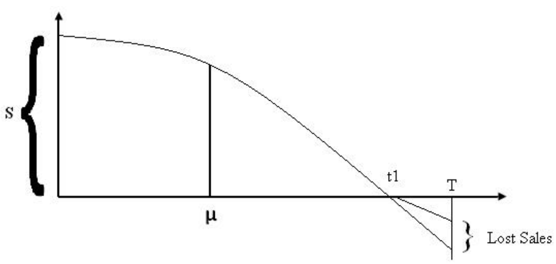


Fig1:Graphical Representation of Inventory System

## 3 MATHEMATICAL MODEL

During the period (o,μ) the inventory level is decreasing and at time $t_1$ the inventory reaches zero level, where the shortages starts, and in the period ($t_1$ ,T) some demands are backlogged.

The rate of Change of inventory during positive stock period (o,$t_1$) and Shortage period ($t_1$,T) is governed by the differential equations

$$\frac{dI(t)}{dt} = -(a+bt),\ 0\le t\le \mu \quad (1)$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a+bt),\ \mu \le t \le t_1 \quad (2)$$

$$\frac{dI(t)}{dt} = -(a+bt)\, e^{-\lambda(T-t)},\ t_1 \le t \le T. \quad (3)$$

With boundary condition I (0)= S, I ($t_1$) =0.

Solving the equations (1),(2) and (3) and neglecting higher powers of t

$$I(t) = S - \left(at+\frac{bt^2}{2}\right) \quad 0\le t\le\mu \quad (4)$$

$$I(t) = a(t_1-t)+\frac{b}{2}(t_1^2-t^2)+\frac{a\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})+\frac{\alpha b}{\beta+2}(t_1^{\beta+2}-t^{\beta+2}) \quad \mu\le t\le t_1 \quad (5)$$

$$I(t) = a(t_1-t)+\frac{b}{2}(t_1^2-t^2) - a\lambda T(t_1-t)+\frac{a\lambda}{2}(t_1^2-t^2)-\frac{b\lambda T}{2}(t_1^2-t^2)+\frac{b\lambda}{3}(t_1^3-t^3) \quad t_1\le t\le T \quad (6)$$

From equation (4) and (5)

$$I(t) = S-\left(at+\frac{bt^2}{2}\right)$$

$$I(\mu) = S-\left(a\mu+\frac{b\mu^2}{2}\right) \quad (7)$$

$$I(\mu) = a(t_1-\mu)+\frac{b}{2}(t_1^2-\mu^2)+\frac{\alpha a}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})+\frac{\alpha b}{\beta+2}(t_1^{\beta+2}-\mu^{\beta+2}) \quad (8)$$

Equating (7) and (8)

$$S = a\mu+\frac{b\mu^2}{2}+a(t_1-\mu)+\frac{b}{2}(t_1^2-\mu^2)+\frac{\alpha a}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})+\frac{\alpha b}{\beta+2}(t_1^{\beta+2}-\mu^{\beta+2})$$

$$S = at_1+\frac{bt_1^2}{2}+\frac{\alpha a}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})+\frac{\alpha b}{\beta+2}(t_1^{\beta+2}-\mu^{\beta+2}) \quad (9)$$

Using (9) in (4)

$$I(t) = S-\left(at+\frac{bt^2}{2}\right)$$

$$I(t) = a(t_1-t)++\frac{b}{2}(t_1^2-t^2)+\frac{\alpha a}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})+\frac{\alpha b}{\beta+2}(t_1^{\beta+2}-\mu^{\beta+2}) \quad (10)$$

Total amount of lost sales $I_L$ during the period (0, T) is

$$I_L = \int_{t1}^{T}\left(1-e^{-\lambda(T-t)}\right)(a+bt)\,dt$$

$$I_L = \frac{a\lambda T^2}{2}+\frac{b\lambda T^3}{6}-a\lambda Tt_1+\frac{a\lambda t_1^2}{2}-\frac{b\lambda Tt_1^2}{2}+\frac{b\lambda t_1^3}{3} \quad (11)$$

Total amount of shortage units $I_s$ during the period (0,T) is

$$I_s = -\int_{t1}^{T} I(t)\,dt$$

$$I_s = -at_1T+\frac{aT^2}{2}-\frac{bt_1^2T}{2}+\frac{bT^3}{6}+a\lambda T^2t_1-\frac{a\lambda T^3}{3}-\frac{a\lambda Tt_1^2}{2}+\frac{b\lambda T^2t_1^2}{2}+\frac{\lambda bt_1^3T}{3}-\frac{b\lambda T^4}{3}+\frac{at_1^2}{2}+\frac{2bt_1^3}{3}-\frac{a\lambda Tt_1^2}{2}+\frac{2a\lambda t_1^3}{6}-\frac{b\lambda Tt_1^3}{3}-\frac{b\lambda t_1^4}{4} \quad (12)$$

Total amount of deteriorated items $I_D$, during the period (0,T) is

$$I_D = \int_{\mu}^{t1}\alpha\beta t^{\beta-1} I(t)\,dt$$

$$I_D = \alpha\beta\left\{\frac{at_1^{\beta+1}}{\beta(\beta+1)}+\frac{bt_1^{\beta+2}}{\beta(\beta+2)}+\frac{\alpha at_1^{2\beta+1}}{\beta(2\beta+1)}+\frac{\alpha bt_1^{2\beta+2}}{\beta(2\beta+2)}-\frac{at_1\mu^{\beta}}{\beta}+\right.$$

$$\frac{a\mu^{\beta+1}}{\beta+1} - \frac{bt_1^2\mu^{\beta}}{2\beta} + \frac{b\mu^{\beta+2}}{2(\beta+2)} - \frac{\alpha a t_1^{\beta+1}\mu^{\beta}}{\beta(\beta+1)} + \frac{\alpha a\mu^{2\beta+1}}{(\beta+1)(2\beta+1)} - \frac{\alpha b t_1^{\beta+2}\mu^{\beta}}{\beta(\beta+2)} + \frac{\alpha b\mu^{2\beta+2}}{(2\beta+2)(\beta+2)}\} . \quad (13)$$

During period (0,T) total number of units holding $I_H$ is

$$I_H = \int_0^{\mu}(c+dt)I(t)dt + \int_{\mu}^{t1}(c+dt)I(t)dt$$

$$= \int_0^{\mu}(c+dt)\{a(t_1-t) + \frac{b}{2}(t_1^2-t^2) + \frac{\alpha a}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1}) + \frac{\alpha b}{\beta+2}(t_1^{\beta+2}-\mu^{\beta+2})\}\,dt +$$

$$= \int_{\mu}^{t1}(c+dt)\{a(t_1-t) + \frac{b}{2}(t_1^2-t^2) + \frac{\alpha a}{\beta+1}(t_1^{\beta+1}-t^{\beta+1}) + \frac{\alpha b}{\beta+2}(t_1^{\beta+2}-t^{\beta+2})\}\,dt$$

$$= \frac{-\alpha a c\mu^{\beta+2}}{\beta+2} - \frac{\alpha a d\mu^{\beta+3}}{2(\beta+3)} - \frac{\alpha b c\mu^{\beta+3}}{\beta+3} - \frac{\alpha b d\mu^{\beta+4}}{2(\beta+4)} + \frac{act_1^2}{2} + \frac{adt_1^3}{6} + \frac{2bct_1^3}{6} + \frac{bdt_1^4}{8} + \frac{\alpha act_1^{\beta+2}}{\beta+2} + \frac{\alpha adt_1^{\beta+3}}{2(\beta+3)} - \frac{\alpha bct_1^{\beta+3}}{\beta+3} - \frac{\alpha bdt_1^{\beta+4}}{2(\beta+4)} . \quad (14)$$

Therefore total unit cost per unit time is given by

$P = \frac{1}{T}$ [ordering cost + Carrying cost + backordering cost + lost sale cost + purchase cost]

$$= \frac{1}{T}[A + C_1I_H + C_2I_D + C_3I_S + C_4I_L]$$

$$= \frac{1}{T}[A + \frac{C_1\alpha bdt_1^{\beta+4}}{2(\beta+4)} + \frac{C_1bct_1^{\beta+3}}{\beta+3} + \frac{C_1ad\alpha t_1^{\beta+3}}{2(\beta+3)} + \frac{C_1\alpha act_1^{\beta+2}}{\beta+2} + \frac{C_1bdt_1^4}{8} + \frac{C_12bct_1^3}{6} + \frac{C_1adt_1^3}{6} + \frac{C_1act_1^2}{2} - \frac{C_1\alpha bd\mu^{\beta+4}}{2(\beta+4)} - \frac{C_1\alpha bc\mu^{\beta+3}}{\beta+3} - \frac{C_1\alpha ad\mu^{\beta+3}}{2(\beta+3)} - \frac{C_1\alpha ac\mu^{\beta+2}}{(\beta+2)} + \frac{C_2\alpha\beta at_1^{\beta+1}}{\beta(\beta+1)} + \frac{C_2\alpha\beta at_1^{\beta+2}}{\beta(\beta+2)} + \frac{C_2\alpha^2\beta at_1^{2\beta+1}}{\beta(2\beta+1)} + \frac{C_2\alpha^2\beta bt_1^{2\beta+1}}{(2\beta+2)\beta} - \frac{C_2\alpha\beta at_1\mu^{\beta}}{\beta} + \frac{C_2\alpha\beta a\mu^{\beta+1}}{\beta+1} - \frac{C_2\alpha bt1^2\mu^{\beta}}{2} + \frac{C_2\alpha\beta b\mu^{\beta+2}}{2(\beta+2)} - \frac{C_2\alpha^2\beta at_1^{\beta+1}\mu^{\beta}}{\beta(\beta+1)} + \frac{C_2\alpha^2\beta a\mu^{2\beta+1}}{(\beta+1)(2\beta+1)} - \frac{C_2\alpha^2\beta bt_1^{\beta+2}\mu^{\beta}}{\beta(\beta+2)} + \frac{C_2\alpha^2\beta b\mu^{2\beta+2}}{(2\beta+2)(\beta+2)} + \frac{C_3aT^2}{2} - C_3at_1T - \frac{C_3bt_1^2T}{2} + \frac{C_3bT^3}{6} + C_3a\lambda T^2t_1 - \frac{C_3a\lambda T}{3} - \frac{C_3a\lambda t_1^2T}{1} + \frac{C_3b\lambda T^2t_1^2}{2} + \frac{C_3bt_1^3T}{3} - \frac{C_3b\lambda T^4}{4} + \frac{C_3at_1^2}{2} \frac{C_3bt_1^3}{3} + \frac{2C_3a\lambda t_1^3}{6} - \frac{C_3b\lambda Tt1^3}{3} - \frac{C_3b\lambda t1^4}{4} + \frac{C_4a\lambda T^2}{2} - \frac{C_4b\lambda T^3}{6} - \frac{C_4a\lambda Tt_1}{} + \frac{C_4a\lambda t_1^2}{2} - \frac{C_4b\lambda Tt_1^2}{2} + \frac{C_4b\lambda t_1^3}{3}]. \quad (15)$$

Optimal Value of $t_1$ can be obtained by solving the equation $\frac{\partial P}{\partial t_1} = 0$

$$\frac{\partial P}{\partial t_1} = \frac{1}{T}\{\frac{C_1abdt_1^{\beta+3}}{2} + C_1bct_1^{\beta+2} + \frac{C_1ad\alpha t_1^{\beta+2}}{2} + C_1\alpha\, act_1^{\beta+1} + \frac{C_1bdt_1^3}{2} + C_1bct_1^2 + \frac{C_1adt_1^2}{2} + C_1aCt_1 + \frac{C_2\alpha\beta at_1^{\beta}}{\beta} + \frac{C_2\alpha\beta bt_1^{\beta+1}}{\beta} + \frac{C_2\alpha^2\beta at_1^{2\beta}}{\beta} + \frac{C_2\alpha^2\beta bt_1^{2\beta+1}}{2} - \frac{C_2\alpha\beta a\mu^{\beta}}{\beta} - \frac{C_2\alpha\beta bt_1\mu^{\beta}}{\beta} - \frac{C_2\alpha^2\beta at_1^{\beta}\mu^{\beta}}{\beta} - \frac{C_2\alpha^2\beta bt_1^{\beta+1}\mu^{\beta}}{\beta} -$$

$$C_3aT - C_3bt_1T + C_3at_1 + C_3bt_1^2 + C_3a\lambda t_1^2 - C_3b\lambda Tt_1^2 - C_3b\lambda t_1^3 - C_4a\lambda T + C_4a\lambda t_1 - C_4b\lambda Tt_1 + C_4b\lambda t_1^2 + C_3a\lambda T^2 - 2C_3a\lambda t_1T + C_3b\lambda T^2t_1 + C_3bt_1^2T\} = 0. \quad (16)$$

The minimum total average cost per unit time is obtained for those values of $t_1$ for which

$$\frac{\partial^2 P}{\partial t_1^2} > 0 \quad (17)$$

By solving equation (16) the value of $t_1$ can be obtained and then from equation (15) and (9), the optimal value of P and S can be found out respectively.

## 4 CONCLUDING REMARKS

In this paper, we developed a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. This model is useful not only for time dependent demand also for time dependent holding cost.

## REFERENCES

[1] C.K.Tripathy and U.Mishra(2010). An Inventory Model for Weibull Deteriorating Items with Price Dependent Demand and Time-varying Holding Cost, Applied Mathematical Sciences, Vol.4, no.44, 2171-2179.

[2] Ajantha Roy,(2008). An inventory model for deteriorating items with price dependent demand andtime-varying holding cost, AMO-Advanced Modeling and Optimization, Volume 10 Number 1.

[3] Azizul Baten and Anton Abdulbasah Kamil (2009). Analysis of inventory-production systems with Weibull distributed deterioration, International Journal of physical Sciences Vol.4(11), pp 676-682. [4] Zhao Pei-xin(2007). An EOQ Model for Items with Weibull Distribution Deterioration, IEEE, I4244-0737, pp-451-454.

[4] S.K.Ghosh and K.S.Chaudhuri (2004). An Order-Level Inventory Model for A Deteriorating Item with Weibull Distribution Deterioration, Time-Quadratic Demand and Shortages, AMO-Advanced Modeling and Optimization, Volume 6, Number 1

[5] Ruxian Li, Hongjie Lan and John R. Mawhinney(2010). A Review on Deteriorating Inventory Study, J.Service Science & Management, 3,117-129.

[6] S.K.Goyal and B.C.Giri (2001), Recent trends in modelling of deteriorating inventory, European Journal of Operational Research 134, 1-16

[7] Vipin Kumar, S R Singh and Sanjay Sharma (2010). Profit Maximization production inventory models with time dependent demand and partial backlogging, International Journal of Operations Research and Optimization, Volume 1, No.2, pp-367-375.

[8] Vinod Kumar Mishra and Lal Sahab Singh(2010). Deteriorating

Inventory Model with Time Dependent Demand and Partial Backlogging, Applied Mathematical Sciences, Vol. 4 No.72, 3611-3619.

[9] Liang-Yuh OUYANG, Kun-Shan WU and Mei-Chuan CHENG (2005). An Inventory Model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging, Yugoslav Journal of Operations Research 15, No 2, 277-288.

[10] R.Begum, S.K.Sahu and R.R.Sahoo(2010), An EOQ Model for Deteriorating Items with Weibull Distribution Deterioration, Unit Production Cost with Quadratic Demand and Shortages, Applied Mathematical Sciences, Vol.4, No.6, 271-288.

[11] G.P.Samanta and Ajanta Roy (2004), A Production Inventory Model with Deteriorating Items and Shortages, Yugoslav Journal of Operations Research 14, No 2, 219-230.

[12] Vinod Kumar Mishra and Lal Sahab Singh(2011), Deteriorating inventory model for time dependent demand and holding cost with partial backlogging, International Journal of Management Science and Engineering Management, 6(4): 267-271,

[13] P.K.Tripathy and S.Pradhan(2011). An Integrated Partial Backlogging Inventory Model having Weibull Demand and Variable Deterioration rate with the Effect of Trade Credit, International Journal Scientific & Engineering Research Volume 2, Issue 4,.

[14] C.K.Tripathy and L.M.Pradhan (2010). An EOQ Model foe Weibull Deteriorating Items with Power Demand and Partial Backlogging, Int.J.Contemp, Math.Sciences, Vol5 No.38, 1895-1904

[15] ZHAO Pei-Xin (2007), An Eoq Model for Items with Weibull Distribution Deterioration, IEEE.