Trajectory E-Filter Zero Phase Error Tracking Controller for Non-Minimum Phase XY Table System

Michael Jackson Patrick, Norlela Ishak, Ramli Adnan

Abstract— Non-minimum phase (NMP) model is a discrete-time model that obtained from small or reducing sampling-time. The feed-forward design of this model using inverse of the system closed-loop transfer function would result an unstable tracking control. This is due to the phase and gain errors that caused by NMP zero. The zero phase error tracking controller (ZPETC), developed by Tomizuka has attracted attention many researchers as an effective and simple remedy to the problem due to NMP zero. Unfortunately, ZPETC cancels only the phase error, as the gain error which cannot be eliminated by ZPETC becomes larger for fast tracking control. This causes undesirable effect on the tracking performance. Fortunately, Haack and Tomizuka have developed a new approach by including additional zeros to reduce the gain error while preserve the zero phase error characteristics. The new approach referred to as an E-Filter ZPETC. Simulation and real-time controls on NMP XY Table system shows that the tracking performance of lemniscates movement is significantly improved.

Index Terms— XY Table, Non-minimum phase model, Discrete-time system, Feed-forward control, Feedback control, Tracking control, ZPETC, E-Filter ZPETC,

◆

1 INTRODUCTION

XY Table has been widely used and important part of many computer numeric controlled (CNC) processing facilities, e.g. the work feeder of CNC lathe, CNC milling and drill press, the work table of laser processing, welding, dispenser, bonding, packing, drilling, laser cutting and painting [1]. In general, XY Table is composed of X-axis and Y-axis motion mechanism where each motion axis is driven by individual actuator such as DC servomotor through high precision ball-screw.

Perfect tracking control (PTC) is desired and important element for XY Table control system, and has been widely studied in [1-12]. PTC can be achieved by introducing a feed-forward controller as shown in Fig. 1. This feed-forward act as an inverse of system closed-loop transfer function, cancelling all poles and zeros of the closed-loop system.

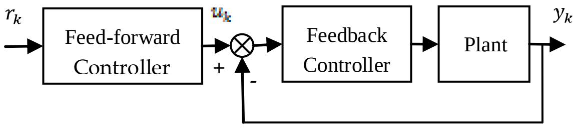


Fig. 1 Two-degrees-of-freedom Controller

PTC can provide the overall transfer function of unity between the desired output r_k and the actual output y_k. This will be true when the plant is minimum phase (MP). But when the plant is non-minimum phase (NMP), the NMP or unstable zero of the plant will become unstable pole to the feed-forward controller transfer function and thus, creating internal instability [9].

The phase and gain errors that caused by NMP zero are problematic, however, it can be resolved. The zero phase error tracking controller (ZPETC), developed by Tomizuka [10,11]has attracted attention many researchers as an effective and simple remedy to the problem due to NMP zero. Unfortunately, ZPETC cancels only the phase error, as the gain error which cannot be eliminated by ZPETC becomes larger for fast tracking control. This gain error causes undesirable effects to the tracking performance.

To compensate for the gain error, Haack and Tomizuka [12] have developed a new approach that refers to as E-Filter ZPETC by including additional zeros to reduce the gain error and preserve the zero phase error characteristics. The tracking performances for both ZPETC controllers are then been compared. Simulation and real-time controls on NMP XY Table system show that the tracking performance for lemniscates movement is significantly improved by applying the new approach. The simulation and real-time controls were done in MATLAB environment.

This paper was organized in the following manner: Section 2 describes plant model; Section 3 describes reference trajectory; Section 4 describes Trajectory E-Filter ZPETC; Section 5 describes feedback control; Section 6 is on results and discussion; and finally, Section 7 is the conclusion.

2 PLANT MODEL

The XY Table used in this paper is shown in Fig. 2. The NMP plant model was derived from open-loop input-output test. The open-loop transfer function of the plant is approximated using MATLAB system identification toolbox in the form of ARX331 with input-output signals sampled at $80ms$. The X-

---

- M.J. Patrick is currently pursuing master in Electrical Engineering in faculty of Electrical Engineering, UiTM, Shah Alam, Malaysia mj_29jack@yahoo.com.sg
- N. Ishak and R. Adnan are with faculty of Electrical Engineering, UiTM, Shah Alam, Malaysia
norlelaishak@salam.uitm.edu.my
ramli324@salam.uitm.edu.my

axis plant model is given by Eq. (1) and for Y-axis plant model is given by Eq. (3).

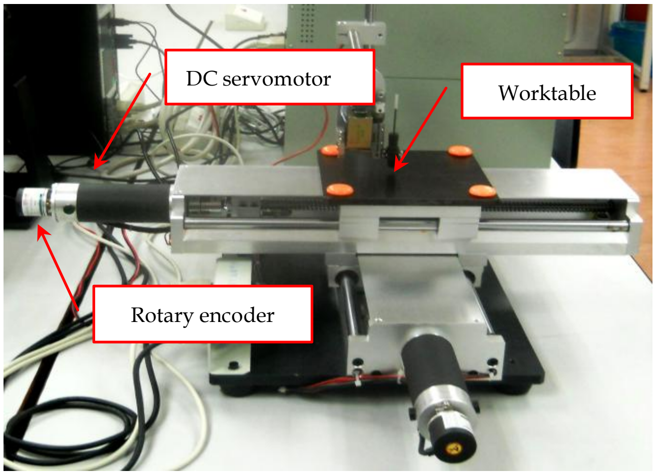


Fig. 2 XY Table system

2.1 X-axis Plant Model

$$\frac{B_o(z^{-1})}{A_o(z^{-1})}=\frac{0.03698z^{-1}+0.1217z^{-2}+0.05178z^{-3}}{1-0.6479z^{-1}-0.5451z^{-2}+0.1933z^{-3}} \qquad (1)$$

From Eq. (1), the zeros polynomial obtained are

$$B_c(z^{-1})=1+\ 3.2910z^{-1}+1.4002z^{-2} \qquad (2)$$

$$B_c(z)=1+\ 3.2910z^{1}+1.4002z^{2}$$

When Eq. (2) is factorized, the locations of zero are at $z=-0.5021$ and $z=-2.7889$. This means that the obtained model is a NMP model with one zero situated outside and far away from the unit circle.

2.2 Y-axis Plant Model

$$\frac{B_o(z^{-1})}{A_o(z^{-1})}=\frac{0.03978z^{-1}+0.1054z^{-2}+0.03507}{1-0.7495z^{-1}-0.4557z^{-2}+0.2056z^{-3}} \qquad (3)$$

From Eq. (3), the zeros polynomial obtained are

$$B_c(z^{-1})=1+2.6496z^{-1}+0.8816z^{-2} \qquad (4)$$

$$B_c(z)=1+2.6496z^{1}+0.8816z^{2}$$

When Eq. (4) is factorized, the locations of zero are at $z=-0.3902$ and $z=-2.2594$. This means that the obtained model is also a NMP model with one zero situated outside and far away from the unit circle.

3 REFERENCE TRAJECTORY

A lemniscates movement of $30mm$ in radius is shown in Fig. 3. It also shows the reference trajectory of respective individual axis used for plant worktable to move in lemniscates. For X-axis, the reference trajectory is a sinusoidal wave signal with maximum amplitude of $+30mm$, minimum amplitude $-30mm$ and frequency of 0.1 radians per second. For Y-axis, the reference trajectory is also a sinusoidal wave signal with maximum amplitude of $+30mm$, minimum amplitude $-30mm$ and frequency of 0.3 radians per second.

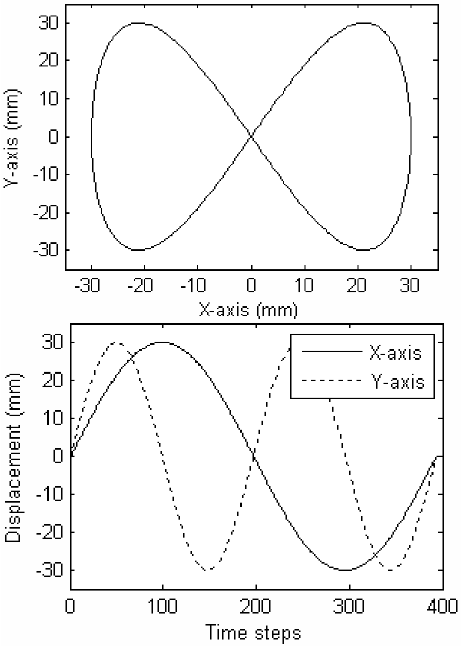


Fig. 3 Lemniscates movement and its individual axis trajectory signals

4 TRAJECTORY E-FILTER ZPETC

Referring to Fig. 1 let the closed-loop transfer function (without feed-forward control) is given as

$$G_c(z^{-1})=\frac{z^{-d}B_c(z^{-1})}{A_c(z^{-1})} \qquad (5)$$

Where z^{-d} represent a d-step delay and

$$B_c(z^{-1})=b_0+b_1z^{-1}+\cdots+b_{n_b}z^{-n_b}$$

$$A_c(z^{-1})=1+a_1z^{-1}+\cdots+a_{n_a}z^{-n_a}$$

$$b_0\neq 0$$

Let factorized function $B_c(z^{-1})$ into minimum phase $B_c^{+}(z^{-1})$ and non-minimum phase $B_c^{-}(z^{-1})$ factors as

$$B_c(z^{-1})=B_c^{+}(z^{-1}).B_c^{-}(z^{-1}) \qquad (6)$$

The ZPETC as feed-forward controller is based on poles, minimum phase (MP) zero and phase cancellations reported in literature [10,11] can be illustrated as in Fig. 4, where z^d

represents a d-step ahead. Thus, ZPETC method utilizes the future desired output in order to compensate for the d-step delay in Eq. (5). The factor $[B_c^-(1)]^2$ is a scaling factor which normalizing the low frequency gain of the overall transfer between desired outputs to actual output toward unity. As a result, only gain error that caused by NMP zero remain. However, at low frequency, ZPETC gave acceptable tracking performance.

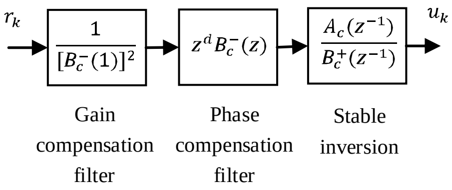


Fig. 4 ZPETC structure

To compensate for the gain error, Haack and Tomizuka [12] have developed a new approach that refers to E-Filter by including additional zeros to reduce the gain error and preserve the zero phase error characteristics as illustrated in Fig. 5. Adding zeros to ZPETC structure can enlarge the frequency region of unity gain.

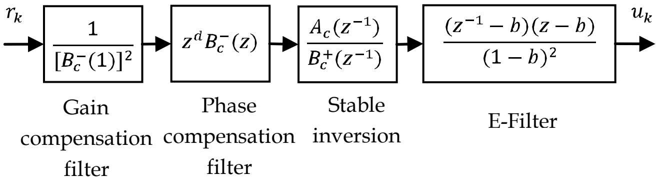


Fig. 5 E-Filter ZPETC structure

Due to the effect of poles, MP zero and phase cancellations between ZPETC structure of Fig. 4, and feedback control of Fig. 7, the control structure for simulation purposes can be simplified as given in Fig. 6. Thus, the implementation of tracking control by simulation does not require the whole plant model transfer function. What was needed here was only the gain compensation filter, zeros polynomial of the plant model and E-Filter.

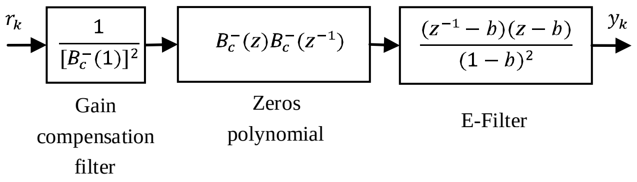


Fig. 6 Trajectory E-Filter ZPETC structure for simulation control

The implementation of proposed real-time control for Trajectory E-Filter ZPETC is based on the combination of E-Filter ZPETC structure of Fig. 5, and feedback control of Fig. 7.

5 FEEDBACK CONTROL

The feedback control in Fig. 7 was designed using pole-placement method.

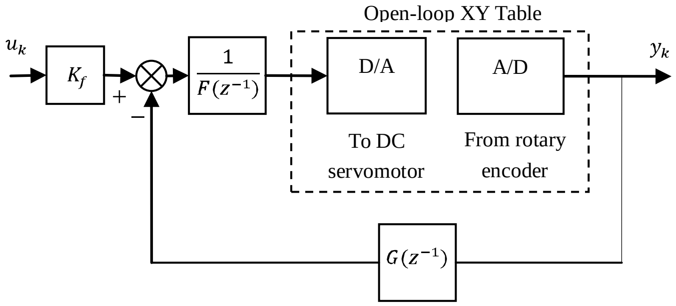


Fig. 7 Feedback control structure

This method enables all poles of the closed-loop system to be placed at desired location and providing good and stable output performance. The closed-loop transfer function of the feedback system is given by

$$\frac{y_k(z^{-1})}{u_k(z^{-1})} = \frac{K_f B_o(z^{-1})}{A_o(z^{-1})F(z^{-1}) + B_o(z^{-1})G(z^{-1})} \tag{7}$$

$$K_f = \frac{\sum T}{\sum B_o}$$

Where

$$A_o(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ \ldots + a_n z^{-n}$$

$$B_o(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ \ldots + b_m z^{-m}$$

$$F(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-2} + \ \ldots + f_m z^{-m-1}$$

$$G(z^{-1}) = g_0 + g_1 z^{-1} + g_2 z^{-2} + \ \ldots + g_{n-1} z^{-n-1}$$

The feedback control parameters were obtained by solving the Diophantine equation of Eq. (8) to solve for $F(z^{-1})$ and $G(z^{-1})$. The literature on this material can be obtained in [13].

$$A_o(z^{-1})F(z^{-1}) + B_o(z^{-1})G(z^{-1}) = 1 + t_1 z^{-1} \tag{8}$$

t_1 : Desired location of pole inside unity circle

In this paper, $t_1 = -0.2$ were used since the tracking required fast response. Let $E.M = D$ where E is a Sylvester Matrix given by

Thus, vector F and G can be computed from $M = E^{-1}.D$. Using developed MATLAB m-file, the following parameters were computed

5.1 X-axis Plant Model

$$T = 1 - 0.2z^{-1} \tag{9}$$

$$K_f = 3.8012$$

$$F(z^{-1}) = 1 + 0.0203z^{-1} - 0.4938z^{-2}$$

$$G(z^{-1}) = 11.5633 - 9.6061z^{-1} + 1.8433z^{-2}$$

5.2 Y-axis Plant Model

$$T = 1 - 0.2z^{-1} \tag{10}$$

$$K_f = 1.4863$$

$$F(z^{-1}) = 1 + 0.2289z^{-1} + 0.0817z^{-2}$$

$$G(z^{-1}) = 0.8060 + 1.1579z^{-1} - 0.4787z^{-2}$$

6 RESULTS AND DISCUSSION

The simulation and real-time controls were done in MATLAB environment. The simulation and real-time results for the system using Trajectory ZPETC and Trajectory E-Filter ZPETC to move in lemniscates shape is shown in Figs. 8-11.

It can observed that the resulting lemniscates movement for the system using E-Filter ZPETC is better than the system that using ZPETC. This is explained by rough lemniscates movement of Fig. 8 and Fig. 10. Note that, ZPETC cancels only the phase error. However, at low frequency, ZPETC gave acceptable tracking performance as shown in Table 1 and Table 2 for X-axis. Fortunately, the system that using E-Filter ZPETC produces better lemniscates movement as shown in Fig. 9 and Fig. 11.

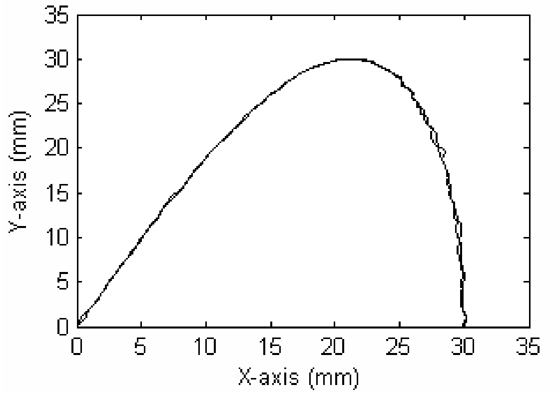


Fig. 8 Simulation result of lemniscates movement for Trajectory ZPETC

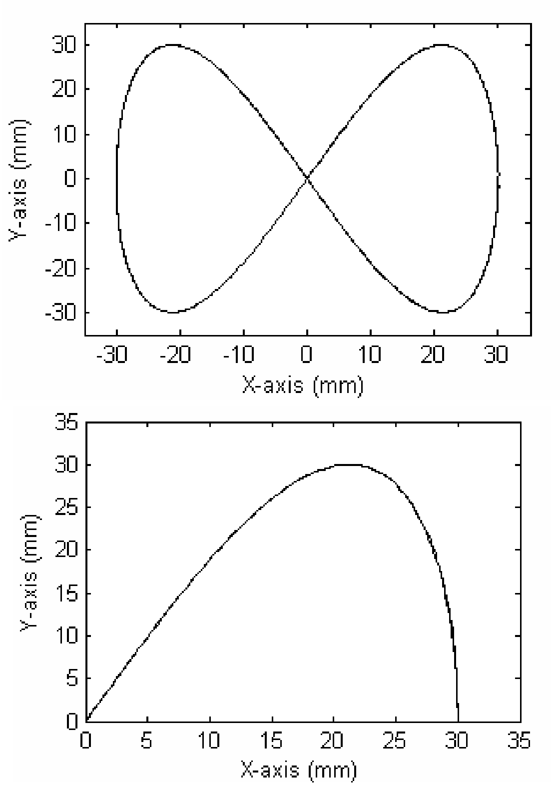
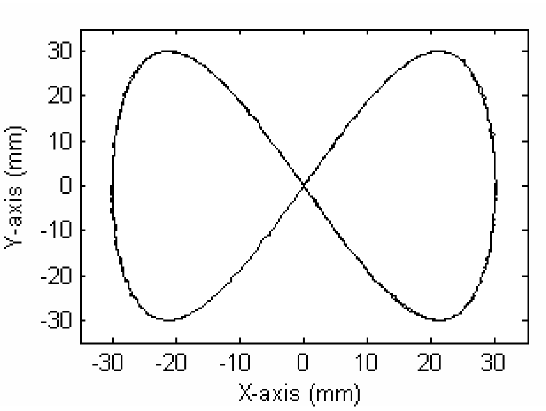
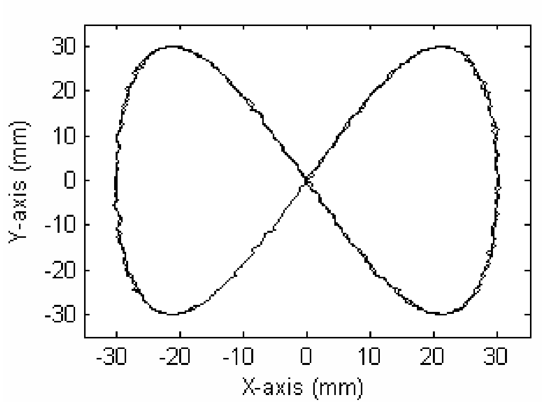


Fig. 9 Simulation result of lemniscates movement for Trajectory E-Filter ZPETC





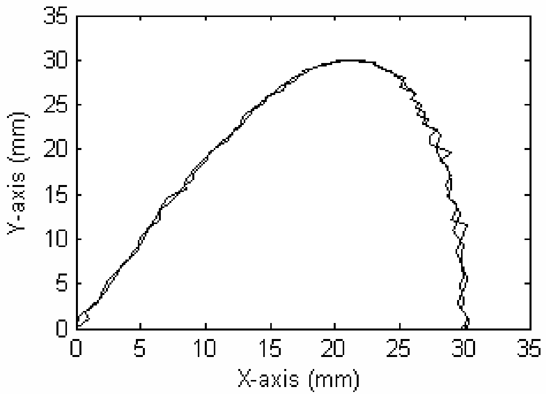


Fig. 10 Real-time result of lemniscates movement for Trajectory ZPETC

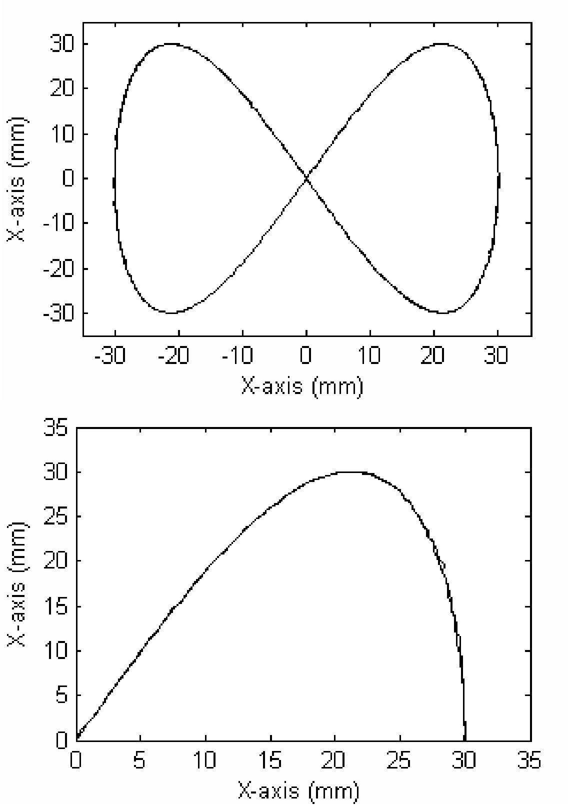


Fig. 11 Real-time result of lemniscates movement for Trajectory E-Filter ZPETC

From Table 1, by simulation control, it can be observed that by using the E-Filter ZPETC, the overall tracking root mean square error (RMSE) is better than using ZPETC. The E-Filter ZPETC shows better RMSE about 8.44% improvement for X-axis and 92.84% improvement for Y-axis.

TABLE 1: Tracking performance by simulation control

	X-axis RMSE (mm)	Y-axis RMSE (mm)
Trajectory ZPETC	0.1362	3.0521
Trajectory E-filter ZPETC	0.1247	0.2184

From Table 2, by real-time control, it can be observed that by using the E-Filter ZPETC, the overall tracking RMSE is also better than using ZPETC. The Trajectory E-Filter ZPETC shows better RMSE about 4.87% improvement for X-axis and 91.43% improvement for Y-axis.

TABLE 2: Tracking performance by real-time control

	X-axis RMSE (mm)	Y-axis RMSE (mm)
Trajectory ZPETC	0.2362	6.0521
Trajectory E-filter ZPETC	0.2247	0.5184

7 CONCLUSION

The simulation and real-time controls of NMP XY Table system using Trajectory E-Filter ZPETC are presented. The E-Filter ZPETC was successfully designed and implemented to NMP XY Table system. In simulation and real-time controls, tracking performance of system using E-Filter ZPETC shows better lemniscates movement compared to ZPETC. This is due to the effectiveness of the E-Filter ZPETC, which successfully preserve the zero phase error characteristics and reduces the gain error, by including additional zeros to the ZPETC structure. However, there is large difference between simulation and real-time results. This is due to the nature of ZPETC which is sensitive to plant-model mismatch that caused by external disturbance factor such as parameter variations.

ACKNOWLEDGMENT

Faculty of Electrical Engineering, Universiti Teknologi MARA, Shah Alam, Malaysia and all members of Distributed Control System Laboratory are greatly acknowledged.

REFERENCES

[1] M.J. Patrick, N. Ishak, M.H.F. Rahiman, M. Tajjudin, & R. Adnan, "Modeling and Controller Design for Non-Minimum Phase System with Application to XY-Table," *IEEE. 7th Control & System Graduate Research Colloquium (ICSGRC)*, pp. 113-118, 2011.

[2] N. Ishak, H. Ismail, M.H.F. Rahiman, M. Tajjudin, R. Adnan & Y.M. Sam, "Tracking Control for Electro-Hydraulic Actuator using ZPETC," *IEEE. 7th Control & System Graduate Research Colloquium (ICSGRC)*, pp. 94-97, 2011.

[3] H. Ismail, N. Ishak, M.H.F. Rahiman, M. Tajjudin & R. Adnan, "Positioning and Tracking Control of an XY Table," *IEEE. 7th International Conference on Control System, Computing & Engineering (ICCSCE)*, pp. 343-347, 2011.

[4] R. Adnan, A. Manan, Noritawati, M.H.F. Rahiman & M. Marzuki, "Trajectory Zero Phase Error Tracking Control using Comparing Coefficients Method," *IEEE. 5th International Colloquium on Signal Processing & Its Application (CSPA)*, pp. 385-390, 2009.

[5] R. Adnan, A. Manan & M. Marzuki, "Real-time Control of Non-minimum Phase Electro-Hydraulic System using Trajectory Adaptive ZPETC," *IEEE. 5th*

International Colloquium on Signal Processing & Its Application (CSPA), pp. 72-76, 2011.

[6] M.M. Mustafa, "Trajectory Adaptive Digital Tracking Controller for Non-minimum Phase Systems without Factorization of Zeros," *IEE Proc, Control Theory Application,* vol. 149, no. 2, pp. 157-162, 2002.

[7] S.S. Yen & P.L. Hsu, "An Optimal and Adaptive Design of the Feedforward Motion Controller," *IEEE/ASME J. Mechatronics*, vol. 4, no. 4, pp. 241-244, 1991.

[8] J.Z. Xia & C.H. Menq," Precision Tracking Control of Non-Minimum Phase System with Zero Phase Error," *Int. J. Control,* vol. 61, no. 4, pp. 791-807, 1995.

[9] D. Torfs, J. Swevers & D. Schutter,"Quasi-Perfect Tracking Control of Non-Minimal Phase Systems," *30th Conference on Decision and Control,* pp. 241-244, 1991.

[10] M. Tomizuka, "Zero Phase Error Tracking Controllers," *ASME J. of Dynamic, System, Measurement and Control,"* vol. 109, pp. 65-68, 1987.

[11] M. Tomizuka, "On the Design of Digital Tracking Controllers," *ASME J. of Dynamic, System, Measurement and Control,"* vol. 115, pp. 412-418, 1993.

[12] B. Haack & M. Tomizuka, "The Effect of Adding Zeros to Feedforward Controller," *ASME J. of Dynamic, System, Measurement and Control,"* vol. 113, pp. 6-10, 1991.

[13] M.T. Nasir, R. Adnan & M.H.F. Rahiman, "Practical System Identification," *Faculty of Electrical Engineering, UiTM*, ISBN: 978-967-5070-03-7, 2007.

[14] I.D. Landau, "System Identification and Control Design using P.I.M + software," *Prentice Hall*, Englewood Cliffs, N.J, 1990.