The Physical Properties of Halley Comet Tail using Mid-point Leapfrog Explicit Method

S. Z. Khalaf, A. A. Selman and E. E. Abd_Al lattef

Abstract— The physical properties of the interaction region between the solar wind and Halley comet have been taken a considerable atten-tion in this work. The cometary ion tail that formed and shaped by the existence of the interplanetary magnetic field (IMF) is also depends on the comet type. This interaction can be described by plasma and magnetohydrodynamic (MHD) laws which based to continuities equations. Simulations are achieved by developing computer codes using the MATLAB 7.8 to study the present model and verifying a solution of the continuities equations using mid-point Leapfrog explicit method in three dimen-sions. This research show good results where the interaction near the cometary nucleus which is mainly affected by the new ions added to the plasma of the solar wind, which increased the average molecular weight and result in many unique characteristics of the cometary tail. These characteristics were explained in the presence of the IMF. The physical properties that studied in this research were: mass density, magnetic field and velocity. The previous set was used for Halley comet under study. The behavior was explained on the base of newly added ions and it was shown that as more ions are added, the physical behavior changed.

Index Terms— Comet tail interactions- Halley comet- magnetohydrodynamic.

◆

1 INTRODUCTION

Halley studied the comet that appeared in 1531, 1607 and 1682 and he found that its orbit being an ellipse and closed, so Halley infers to return in 1758 and it appeared in the same year [1]. In (1951), Biermann discovered the solar wind as the main agent to shape cometary atmospheres, and since then, considerable progress has been made in the understanding of the interaction between the solar wind and cometary tail [2]. After Alfven (1957) predicted that the IMF plays an important role in the interaction process, Axford (1964) showed that it may be well described by the magnetohydrodynamical theory (MHD) [2]. This assumption made possible the first simplified one dimensional hydrodynamicl models of Biermann (1967), which demonstrated the basic physics of cometary atmospheres. Within the following two decades this type of modeling has been improved by Brosowski and Wegmann (1973) as well as Wallis (1973) [5]. In (1980, 1982), Schmidt and Wegmann found modified MHD model and applied it in two and three dimensions. They applied the theoretical models and compared the result with practical measurements of the spacecraft which sent to the comets Giacobinizinner 1985 and Halley 1986, and the results were in a good agreement [2]. In 1987, Wegmann et al. applied the chemical comet-coma and MHD models to describe and analyze the plasma flow, the magnetic field and ion abundance in comet Halley in a consistent manner. They assumed that the volatile composition to consist of 80% water and 20 % carbon, nitrogen, oxygen and sulfur component [3]. The result of self-consistent gas dynamic model of the interaction between the solar wind and comet ionosphere was developed by Baranov and Lebedev in 1993, and they compared it with the data of onboard measurements performed during spacecraft encounters with comet Halley in March 1986. Theoretical profiles of plasma density and velocity as well as magnetic field strength, were in good agreement with the experimental ones[4].

The response of cometary plasma tail to interplanetary shocks was studied by Wegmann in 1994 using time dependent three dimensional MHD model simulations [5]. In 1998, K. Murawski et al. applied the numerical simulations performed in the framework of nonlinear two-dimensional magnetohydrodynamics to investigate the solar wind interaction with Comet Halley at 0.83 A.U. corresponding to the Vega 2 encounter [6]. The observation made by Sakigake revealed the asymmetric distribution of the interaction region of Halley's Comet, and the center of the interaction region was shifted toward the post-encounter side on the spacecraft trajectory. Oya (1993) proposed a model that this asymmetric distribution is explained by the field aligned motion of picked up ions. Based on this model, Katoh et al. in 2003 examined spatial and time scale of field aligned motion of cometary ions in the ion pickup process by using simulation and the shift of the interaction region was estimated [7]. D. Bodewits et al. in 2004 reported experimental stat-selective cross sections of electrons capture and used these to predict cometary line emission, and they showed that helium line ratios are direct diagnostic of the solar wind velocity, while their absolute intensities are linked to the local density of the solar wind [8]. The fast ions in the solar wind interact with neutral molecules and atoms in the coma of comets through charge exchange processes. The resulting excited species decay through ultraviolet or X-ray emission. Conversely, observations of cometary far ultraviolet and X-ray emissions can provide detailed insight in the interaction between comets and the solar wind. Based on an extensive interaction model, Bodewits et al. in 2005 identified the principal experimental and observational needs for the interpretation of cometary XUV emission [9].

Reyes-Ruiz and Mauricio in 2008 studied the effects of viscous momentum transport and energy dissipation in the interaction of the solar wind with the plasma environment of a comet.

- *S. Z. Khalaf is currently lecturer Assist Prof. has PhD in Astronomy Dep. in Baghdad University, Iraq, PH-+9647707819819. E-mail: salman.zkhalaf@yahoo.com*
- *A. A. Selman is currently lecturer Assist Prof. has PhD in Astronomy Dep. in Baghdad University, Iraq.*
- *E. E. Abd_Al lattef is currently MsC student in Astronomy Dep. in Baghdad University, Iraq.*
(*This information is optional; change it according to your need.)*

Analytical calculations were carried out to estimate the rate of viscous dragging of material from the comet's ionosphere to the plasma tail [10]. Mauricio et al. in 2009 compared the results of the numerical simulation of the viscous-like interaction of the solar wind with the plasma tail of comets, with velocities of H2O ions in the tail of comet Swift-Tuttle determined by means of spectroscopic, ground based observations. They found that the flow rapidly evolves from an arbitrary initial condition to a quasi-steady state for which there is a good agreement between the simulated tail ward velocity of H2O ions and the kinematics derived from the observations [11]. In the same year, Jia et al. studied and examined the tail disconnection event on April 20, 2007 on comet 2P/Encke, caused by a coronal mass ejection (CME) at a heliocentric distance of 0.34 AU. During their interaction, both the CME and the comet were visible by the STEREO-A spacecraft [12]. The effect of a cometary jet on the solar wind interaction is studied by Wiehle et al. in 2010 using comet 67P/Churyumov-Gerasimenko as case study. This comet was the target of the Rosetta-mission which will arrive in 2014 [13].

2 PHYSICAL MODEL

One of the major methods to determine the structure of the solar coronal and solar wind is the time-dependent multi-dimensional magnetohydrodynamic (MHD) model. This model is preferred because it's very difficult to analytically calculate the solution of non linear MHD equations. So the MHD model is the study of the motion of an electrically conducting fluid in the presence of magnetic field [14]. MHD model is used to calculate the plasma fluid and magnetic field properties near the comet. The flow is similar to that of solar wind past planet, but in comet several peculiarities appear due to the large atmosphere extending to distance more than 106 Km and to prevailing role of ionization processes [14],[15]. MHD model applied to study the effect of solar wind and its interaction with the ionized particles of comet. The solar wind plasma and the produced ions from the comet move inside the solar magnetic field. The state of plasma with mass density (ρ), number of particle (n), momentum (q), pressure (p), velocity (υ) and magnetic field (B) at space x and time t is determined by the conservation law of ideal MHD. The equations of the ideal MHD describe the dynamics of plasma under the influence of a magnetic field, and are given by [14].

2.1 Conservation of mass density:

The conservation of mass density is written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = \rho^{\cdot}_c \qquad (1)$$

where P^0c the mass source of cometary ions, and is giving by:

$$\rho^{\cdot}_c = \frac{G\sigma m_c}{4\pi r^2 v_c} e^{-\frac{\sigma}{v_c} r} \qquad (2)$$

where G, r and v are the production rat, the ionization rate, the distance from the nucleus and the Solar wind particles velocity respectively.

2.2 Conservation of particles number

Because of the production of heavy cometary ions the mean molecular weight varies throughout the cometary atmosphere. To determine ion, proton and electron densities as well as ion temperature the equation of continuity must be solved for the particle number density:

$$\frac{\partial n}{\partial t} + \nabla.(nv) = n^{\cdot}_c \qquad (3)$$

The source term n^0c takes into account the production of particles by photoionization and electron impact ionization; recombination removes particles from the plasma. In the ideal model:

$$n^{\cdot}_c = {\rho^{\cdot}_c}/{m_c} \qquad (4)$$

Where m_c represents the constant molecular mass with typical value $m_c=20m_p$ (m_p is the proton mass).

2.3 Conservation of velocity

The conservation of velocity is written as

$$\frac{\partial v}{\partial t} - v\nabla v + \frac{1}{\rho}\nabla P + \frac{B}{4\pi\rho}\nabla B = 0 \qquad (5)$$

where P, B are the pressure and the magnetic field respectively.

The source term in this equation is neglected because it contributes with ~ 2% to the equation

2.4 Conservation of magnetic field

The conservation of magnetic field is written as,

$$\frac{\partial B}{\partial t} = [\nabla \times [v \times B]] \qquad (6)$$

2.5 Conservation of pressure

The conservation of pressure is written as

$$\frac{\partial P}{\partial t} + v\frac{\partial \rho}{\partial r} + \gamma\rho\frac{\partial v}{\partial r} = P^{\cdot}_c \qquad (7)$$

where

γ: Specific heat ratio.

Pc : The pressure source term, this related to the internal energy source term ( ) and is giving by [3]:

$$e_c = \frac{1}{2}\rho^{\cdot}_c v^{\cdot}_c \qquad (8)$$

The interaction of plasma with neutral gases is one of the important topics in space plasma physics and solar system. The interaction region of solar wind and outflowing cometary gas can be considered as an ideal laboratory to study the basic interaction processes which are occur between these two gases of different origins finally assimilate each other [16]. As the comet nears the sun the neutral gases are sublimated from the surface of cometary nucleus and escape freely from the coma because of the weak gravitational force of comets. Neutral molecules in the comet are eventually ionized in different processes, such as photoionization by solar ultraviolet photons, charge exchange with solar wind protons, and collision ionization by energetic electrons [16]. The atmosphere of a comet continuously interacts with the solar wind, which it varies in the frame of comet and cause the cometary plasma tail [16].

3 NUMERICAL METHOD

There are many numerical methods used to solve MHD continuity equations and partial differential equation (PDE) in general. The solution technique depends mainly on the type of the problem and its mathematical shape. Three type of PDE are well known in most physical systems, these are:
As demonstrated in this document, the numbering for sections upper case Arabic numerals, then upper case Arabic numerals, separated by periods. Initial paragraphs after the section title are not indented. Only the initial, introductory paragraph has a drop cap.

- ***Hyperbolic equation:***

$$\frac{\partial v}{\partial t} + a\frac{\partial v}{\partial x} = 0 \qquad a > 0 \qquad (9)$$

- ***Parabolic equation:***

$$\frac{\partial v}{\partial t} - a\frac{\partial v}{\partial x} = 0 \qquad a < 0 \qquad (10)$$

- ***Elliptic equation:***

$$\frac{d^2 v}{dx^2} + \frac{d^2 v}{dy^2} = 0 \qquad (11)$$

where a is constant.

Thus comparing MHD continuity equations, it can be seen that the present physical system is described by a hyperbolic equation [17].

Numerical solutions are further divided into two main groups:

1- Explicit Methods: Generally known of their ease and simplicity. However, these methods are with modest accuracy and the solution of time coordinates requires a new run [18].

2- Implicit Methods: These methods have the advantage of higher accuracy and ability to solve for the time coordinate along spatial ones. Yet, the programming effort required is much more complicated then explicit methods and the programs generally require much more time to finish [18].

In this paper it has been used mid-point leapfrog explicit method which is widely used to solve numerical problem with initial-boundary values. It's attractive because it is simple with accuracy of second order, but most of all it has very good stability when computing oscillatory solutions [19].This method is 2nd order accurate in space and time, i.e. with accuracy ((Δt)2 ,(Δx)2).

4 RESULT AND DISCUSSION

Three dimensional MHD system was assumed to simulate cometary plasma of Halley comet interaction with solar wind. The main results produced in this research are the physical properties of the cometary plasma which change due to the interaction with the solar wind. This interaction is described by the main conservation laws (equations (1 to 8)). The velocity equation has a source term but it mentioned because it contributes with ~2% to the equation therefore, this term has no effect on the actual results. As for the magnetic field, there is no source term.

TABLE 1
PHYSICAL PARAMETERS USED IN THE MAIN CALCULATIONS

Symbol	***Name***	***Arbitrary Value***
v_c (Km.s^{-1})	Solar wind particle velocity	10
m_c	Constant molecular mass	0.5
***G* (s^{-1})**	Production rate	10^{28}
σ (s^{-1})	Ionization rate	0.01
n_i, n_j, n_k	The time coordinates	20
k	Iteration number	150
γ	Specific heat ratio	1.4
Δx, Δy, Δz	The spatial coordinates	30
Δt (s)	The change in iteration of time	0.001

TABLE 2
INITIAL AND BOUNDARY CONDITIONS OF THE CODE.

Quantity	**Initial value**	**Boundary value**
Mass density, (ρ)	10^{-11}	1.5
Particles velocity, (u_x, v_y, w_z)	0	0.5
Magnetic field, (B_x, B_y, B_z)	10^{-3}	5.5

The results are presented in three-dimensional surface plot. During the basic calculations, The physical properties that are used for the solution of PDE are presented in table (1).

The code that solves a boundary-condition problem are a set of initial and boundary conditions . This set was chosen in order to have convergent solutions with physical meanings. The set of initial conditions were applied at the first step of the program and it is required to set-up the various matrices used during the calculations. The boundary conditions, on the other hand, were repeatedly applied during each loop of the code so that the limits of the calculated physical properties are always under control. Initial and boundary conditions used in this research are listed in Table (2).
The results of Halley comet in figures (1-3).

Equation (1), has been used in order to calculate the mass density of the comet and it was found that this value increasing in the interaction region between the solar wind particles and cometary ions which is formed in front of the cometary nucleus.

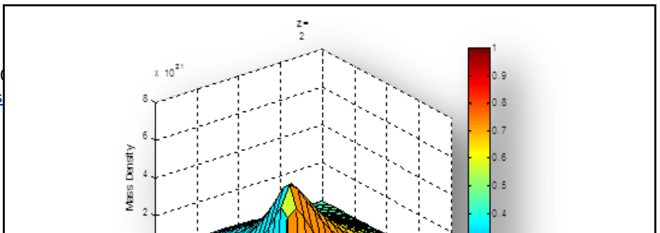


Figure (1) demonstrates that the peak or the maximum value which is equal to 1.35 × 1021 at z=2 would becomes 6.5×1020 at z=19, where z represents the distance outward from the nucleus. The nucleus is an active at large heliocentric distance where it spends most of its time in its orbit around the sun. When the heliocentric distance of the nucleus become less than a few A.U., its surface is heated by solar radiation and the ice sublimates producing a coma of nearly radially out flowing neutral gas and dust.

Photoionization of the neutral gas produces ions that are picked up by solar wind and mass load its flow. The mass loading slows down the solar wind because of conservation of linear momentum and ultimately leads to the formation of structures such as a bow shock, contact surface and plasma tail, and plasma pick up and depletion region, from these has been inferred that the mass density is decreases with increasing the distance from the nucleus of the tail. In the other hand the mass density of the solar wind plasma increases because the heavy cometary ions are continually added.

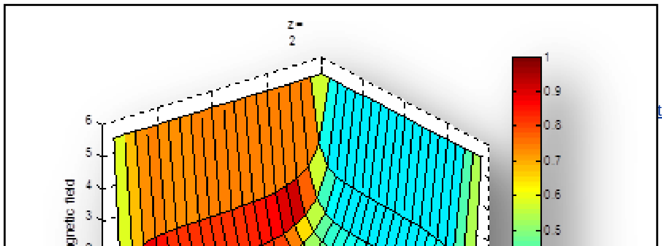
When the comet is near to the sun, it will be exposured to the electric and magnetic fields, where the comet is affected by two forces, the electric force and magnetic force

$$F_B=qv\times B$$

The resultant of these forces is called Lorentz force, as shown in the equation:

$$F=q(E+v\times B)$$

where F, q, E, v and B are the Lorentz force, the electric charge of particle, the electric field, linear velocity and The magnetic field respectively.



From the previous equation, it is noted that the Lorentz force is proportional to the magnetic field. So the magnetic field is increases as shown in figure (2). The net force depends on the magnitude and direction of both velocity and magnetic field. Therefore, when B increases, it is expected that, at constant velocity and direction, the force will also increases leading to more conferment of the plasma of the comet tail. This indicates that as the comet approaches the sun, there will more force exerted on the cometary plasma. This is because as the comet becomes near the sun, the magnetic field (IMF) will also increases in value since the sun generates intensive magnetic field.

The velocity is added between the comet nucleus and the un-perturbed solar wind region, since the ions are generated radialy from the comet, a net-velocity of ions will be found which behave as shown in figure (3). Also the Lorentz force depends on the velocity with fixing magnetic field and direction of both v and B, where the net force is increases by the velocity increasing. This is caused to interrelate among the plasma of comet tail.

Furthermore, the solar wind is slow down by shock wave which it is formed in front of the cometary nucleus and it properties depends on the velocity of comet.

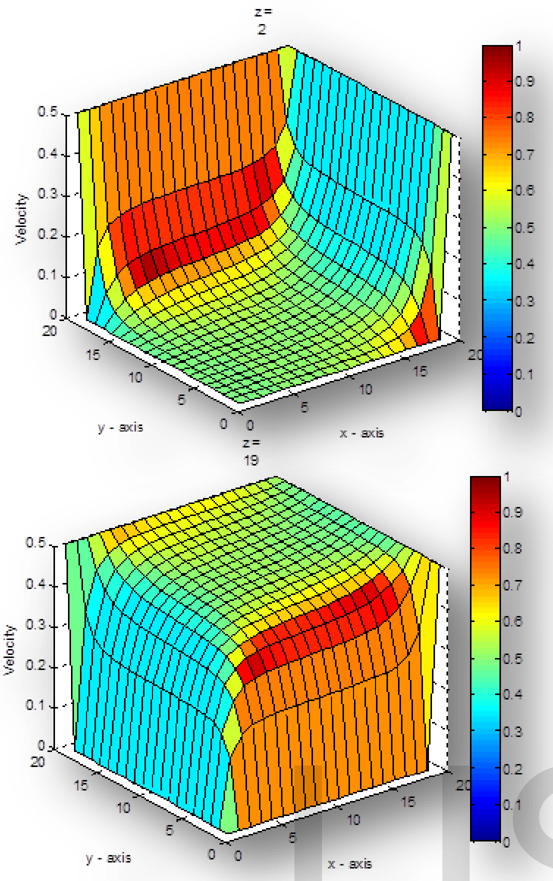


Fig 3. Relation between velocity (km.s-1) of Halley comet for three-dimension magnetohydrodynamic (MHD) simulation and space, using leapfrog method.

6 CONCLUSIONS

The results show that the mid-point leapfrog method has a higher (second) order of accuracy and it simple, stabile and more accurate from other approximations. Therefore, it has been used in this work.

The results of the mass density showed that as the source term increase, the peak of the total behavior will also increase. This was explained due to new added ions to the cometary coma. As the distance increase, the peak decreases and the behavior become broader. This was explained due to the large force exerted from the solar wind which increases as the comet approaches the sun.

Finally the general behavior was explained on the base of Lorenz force and it was concluded that the magnetic force changed from the comet nucleus outward as the distance increases.

REFERENCES

[1] D. Morrison, T. Owen, 1996, "The planetary system", second edition.

[2] Schmidt_voigt: 1989, "Time dependent MHD simulation for cometary plasma ". Astronomy and Astrophysics, vol.210, pp.433-454.

[3] Tamas I. Gombosi, Kenneth C. Hansen, Darren L. DeZeeuw and Michael R. Combi, 1998, "MHd simulation of comets: The Plasma Enviroment of Comet Hale-Bopp", pp.1.

[4] V.B. Baranov and M.G. Lebedev, 1993, "The interaction between the solar wind and the comet p/Halley atmosphere: observation versus theoretical predictions ".Astronomy and Astrophysics, vol.294, pp.695-706.

[5] Rudolf Wegmann, 1995, "MHD modeling calculation for the effect of interplanetary shocks on the plasma tail of a comet".Astronomy and Astrophysics, vol.294, pp.601-614.

[6] K. Murawski, D.C. Boice, W.F. Huebner and C. R. Devore, 1998, "Two-Dimension MHD simulation of the solar wind interaction with comet Halley". ACTA ASTRONOMICA, vol. 48, pp.803-817.

[7] Yoto Katoh, Hirosh Oya, Masahide Iizima and Takayuki Ono, 2003,"Numerical study on the spatial extends of interaction region surrounding comet nucleus-Ion pick up process". Earth planets space, vol.55, pp.705-711.

[8] D. Bodewits, Z. Juhász, and R. Hoekstra, 2004,"Catching some sun: probing the solar wind cometary X-ray and far-ultraviolet emission". The Astrophysical journal, vol.606, pp.L81-L84.

[9] Bodewits D., Hoekstra R. and Tielens A. G. G. M., 2005, "Probing the interaction between comet and the solar wind", vol.774, pp.261-269.

[10] Reyes-Ruiz and Mauricio, 2008," Modeling the viscous interaction of the solar wind with comets", pp.2606.

[11] Mauricio Reyes-Ruiz, Roberto Vazquez and Hector Perez-de-Tejada, 2009, "Viscous-like Interaction of the Solar Wind with the Plasma Tail of Comet Swift-Tuttle "Earth and Planetary Astrophysics", pp.2.

[12] Y. D. Jia, C. T. Russell, L. K. Jian, W. B. Manchester, O. Cohen. A. Vourlidas, K. C. Hansen, M. R. Combi and T. I. Gombos, 2009, "study of the April 20, 2007 CME-comet interaction event with an MHD model". Astrophysics, vol. 696, pp. 56-60.

[13] Wiehle, Stefan; Motschmann,Uwe; Gortsas, Nikolaos; Mueller, Joachim; Kriegel, Hendrik; Koendrik, Christoph; Glassmeier, Karl-Heinz, 2010, "Cometary jets in interaction with the solar wind:a hybrid simulation study ", 38thCOSPAR Science Assembly, pp.8.

[14] Buti, B., 1988, "Cometary and Solar Plasma Physics", World Scintific Bublishing Company, Singapore, pp.1.

[15] Keiji Hayashi, 2006, "Three-dimensional time-dependent MHD simulation model of the solar corona and solar wind". ILWS Workshop.

[16] Toshio Terasawa and Shun Takahashi, 1993, "Plasma Enviroment of Comet Halley Observed by Suisei". Terra Scientific Publishing Company, pp.615-628.

[17] K. C. Hansen, M. R. Combi, F. J. Crary, D. L. De Zeeuw, T. I. Gombosi, D. T. Young: "Global MHD Simulations of Comet Borrelly's Plasma Environment: Effects of a Strong Neutral Jet", DPS 2001 meeting, November 2001, from "http://www.aas.org/publications/baas/v33n3/dps2001/615.htm".

[18] Potter, D. 1973, "Computational Physics", John Wily and sons.

[19] Klaus A. Hoffmann and Steven T. Chiang, 2000, "Computational Fluid Dynamics", Fourth edition, vol.1, ch.6.