

TRANSFORM A DIGITAL FILTER TO ANOTHER DIGITAL FILTER USING PASCAL'S TRIANGLE

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Abstract— Two matrix equations, Pascal matrix equation and inverse Pascal matrix, are derived and demonstrated in this work. These matrix equations are used for making a conversion and inversion between the coefficients of an analog low pass filter transfer function H(s) and a digital transfer function H(z). The involving of the Pascal's triangle in both matrix equations is helpful, easier to transform between analog filter and digital filter and therefore, the procedure of design a digital filter form another digital filter is presented in this paper.

Index Terms— Bilinear z-transform, Pascal's triangle, low pass to low pass, low pass to high pass, low pass to band pass, low pass to band stop, digital to digital filter, Pascal matrix equation, pre-warping frequency.

1 INTRODUCTION

There are the procedures of design a digital filter (low pass, high pass, band pass and band stop), one of them is starting from a given analog low pass filter, transform it to analog filter which the same class of the desired digital filter using frequency transformations and then apply the bilinear z-transform with pre-warping frequency [2]. This method with the involving the Pascal's triangle derived the matrix equations, called "Pascal matrix equation" and "Inverse Pascal matrix equation", they are used to find the relationship between the coefficients of the transfer functions in s-domain and z-domain.

This paper introduces the procedure of designing a digital filter from a given digital filter by using Pascal matrix equation and inverse Pascal matrix equation.

2 THE MATRIX [T] PASCAL'S TRIANGLE WITH INSERTING ZEROS

A great application of the Pascal's triangle is for finding the coefficients and expansion of a binomial expression (U+L)ⁿ. The table.1 below, illustrates a converting an analog low pass filter to a low pass, high pass, band pass and band stop digital filter. Let insert zeros into Pascal's triangle to make a matrix [T] having a size of (n+1;N+1) as shown in figure.1 below

$$[T] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & U & 0 & L & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U^2 & 0 & 2UL & 0 & L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & U^3 & 0 & 3U^2L & 0 & 3UL^2 & 0 & L^3 & 0 & 0 & 0 \\ 0 & U^4 & 0 & \dots & 0 & \dots & 0 & \dots & 0 & \dots & 0 & 0 \\ U^n & 0 & \dots & 0 & \dots & 0 & \dots & 0 & \dots & 0 & L^n & 0 \end{bmatrix}$$

figure.1 The matrix [T] Pascal's triangle with zeros

Where n is the nth-order of the analog low pass filter and N is the Nth-order of the digital filter.

If the cut off frequency of the low pass and the upper cut off frequency of the band pass, band stop filter is f_U. The cut off frequency of the high pass and the lower cut off frequency of the band pass, band stop filter is f_L. The sampling frequency is f_s, then

$$\begin{cases} c = \cot(\pi \frac{f_U}{f_s}) \\ t = \tan(\pi \frac{f_L}{f_s}) \end{cases} \Rightarrow \begin{cases} U = \frac{c}{1-ct} \\ L = \frac{t}{1-ct} \end{cases} \quad (1)$$

This matrix [T] is used to derive the Pascal matrix equation in the next sections.

3. THE PASCAL MATRIX EQUATION

A digital filter (low pass, high pass, band pass and band stop) of the transfer function H(z) can be obtained from the transfer function H(s) of an designed analog low pass filter as the block diagram in fig.2 shows below

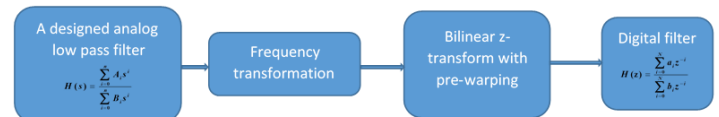


fig.2 Design a digital from a designed analog low pass filter

The table.1 illustrates more details for converting an analog low pass filter to a low pass, high pass, band pass and band stop digital filter.

	s- domain	Frequency transformations	z-domain
LP to LP	H(s)	$H(\frac{s}{\omega_c})$	$H(c \frac{1-z^{-1}}{1+z^{-1}})$ (2)
LP to HP	H(s)	$H(\frac{\omega_c}{s})$	$H(t \frac{1+z^{-1}}{1-z^{-1}})$ (3)
LP to BP	H(s)	$H(\frac{s^2 + \omega_0 \omega_c}{(\omega_1 - \omega_2)s})$	$H(U \frac{1-z^{-1}}{1+z^{-1}} + L \frac{1+z^{-1}}{1-z^{-1}})$ (4)
LP to BS	H(s)	$H(\frac{\omega_1 - \omega_2}{s^2 + \omega_0 \omega_c})$	$H(\frac{1}{U \frac{1-z^{-1}}{1+z^{-1}} + L \frac{1+z^{-1}}{1-z^{-1}}})$ (5)

From table.1, the relationship between the coefficients [A_i], [B_i] of the transfer function H(s) and [a_i], [b_i] can be expressed as the matrix equation below

$$\begin{cases} [a] = [P][\Delta(A,U,L)] \\ [b] = [P][\Delta(B,U,L)] \end{cases} \quad (6)$$

The matrix [P] contains the positive and negative binomial coefficients of the Pascal's triangle in the first, last row and the first, last column corresponding to the edge size and the nth row of the Pascal's triangle and another elements in the matrix [P] can be calculated from its left, diagonal and above elements. It has a size of (N+1, N+1). There are two different matrices respectively for low pass filter P_{LP} and P_{HBS} for P_{HP} (high pass), P_{BP} (band pass), P_{BS} (band stop) and they can be found as:

$$Lp \rightarrow Lp: P_{LP} = \begin{bmatrix} (P_{LP})_{i=1,j=1 \rightarrow n+1} = 1 \\ (P_{LP})_{i=1,j=1 \rightarrow n+1} = (-1)^{j-1} \\ (P_{LP})_{i=1 \rightarrow n+1,j=1} = \binom{n}{i-1} \\ (P_{LP})_{i=1 \rightarrow n+1,j=n+1} = (-1)^{i-1} \binom{n}{i-1} \\ (P_{LP})_{i,j} = (P_{LP})_{i,j-1} - (P_{LP})_{i-1,j-1} - (P_{LP})_{i-1,j} \end{bmatrix}_{n+1;n+1}$$

$$Lp \rightarrow Hp, Bp, Bs: P_{HP}, P_{BP}, P_{BS} = \begin{bmatrix} (P_{HBS})_{i=1,j=1 \rightarrow n+1} = 1 \\ (P_{HBS})_{i=n+1,j=1 \rightarrow n+1} = (-1)^{j-1} \\ (P_{HBS})_{i=1 \rightarrow n+1,j=1} = (-1)^{i-1} \binom{N}{i-1} \\ (P_{HBS})_{i=1 \rightarrow n+1,j=n+1} = \binom{N}{i-1} \\ (P_{HBS})_{i,j} = (P_{HBS})_{i,j-1} + (P_{HBS})_{i-1,j-1} + (P_{HBS})_{i-1,j} \end{bmatrix}_{n+1;n+1}$$

The matrix [Δ] in equation (6) has a size of (N+1, 1) and it can be found by multiplication between the coefficients matrix of the analog low pass filter and the matrix T of the Pascal's triangle with zeros as shown in the equation (7).

$$\begin{cases} [(A_{j-1}(A, U, L))_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} = [(A_{j-1})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} [(T_{ij})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \\ [(A_{j-1}(B, U, L))_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} = [(B_{j-1})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} [(T_{ij})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \end{cases} \quad (7)$$

The table 2 below illustrated the matrix [Δ] for low pass, high pass, band pass and band stop filter.

Table .2 The matrix [Δ] for Low pass, High pass, Band pass and Bans stop filter

n	[A]		The matrix [T]												
	Lp, Hp, Bp	Bs													
0	A ₀	A _n	0	0	0	0	0	1	0	0	0	0	0	0	
1	A ₁	...	0	0	0	0	U	0	L	0	0	0	0	0	
2	A ₂	A ₃	0	0	0	U ²	0	2UL	0	L ²	0	0	0	0	
3	A ₃	A ₂	0	0	U ³	0	3U ² L	0	3UL ²	0	L ³	0	0	0	
...	...	A ₁	0	...	0	...	0	...	0	...	0	...	0	0	
n	A _n	A ₀	U ⁿ	0	...	0	...	0	...	0	...	0	L ⁿ	0	
			Δ _{M-n}	...	Δ ₃	Δ ₂	Δ ₁	Δ ₀	Lp to Lp	N=n	t=0	U=c			
			Lp to Hp	N=n	c=0	L=t		Δ ₀	Δ ₁	Δ ₂	Δ ₃	...	Δ _{M-n}		
			Lp to Bp, Bs	N=2n	Δ ₀	Δ ₁	Δ ₂	...	Δ _{M-1}	Δ _{M-n}	Δ _{M+1}	Δ _{M+2}	...	Δ _{2n-1}	Δ _{2n}

From table.2, for low pass filter, let t=0, then L=0, U=c, the matrix [T] becomes a matrix [T_c] has a size of (n+1; n+1) and the same for high pass filter, c=0, U=0, L=t, the matrix [T] is the matrix [T_t] as shown

$$[T_c]_{n+1;n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 \\ 0 & 0 & c^2 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & c^n \end{bmatrix} \quad [T_t]_{n+1;n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 \\ 0 & 0 & t^2 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & t^n \end{bmatrix}$$

For low pass to band pass and low pass to band stop, the matrix [T] is used.

The subscripts of the coefficients [A] and [B] in the transfer function H(s) low pass filter is writing in ascending order for low pass, high pass, band pass and in descending order for band stop.

Base on the equation (6) and equation (7), one new formula can derived and it is called "The Pascal Matrix Equation" as is described in equation (8):

$$\begin{cases} [(a_{i-1})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} = [(P)_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \left([(A)_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} [(T_{ij})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \right) \\ [(b_{i-1})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} = [(P)_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \left([(B)_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} [(T_{ij})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \right) \end{cases} \quad (8)$$

The Pascal matrix equation is a general formula to transform the coefficients of an analog filter transfer function H(s) to the coefficients of a digital filter transfer function H(z).

The procedure of converting an analog low pass filter to a digital filter were studied. The next sections will introduce how to invert a digital filter to an analog low pass filter using the Pascal matrix equation.

4. THE INVERSE PASCAL MATRIX EQUATION

The inverting a digital low pass, high pass, band pass and band stop to an analog filter is a method to transform the coefficients [ai] and [bi] of the transfer function H(z) to the coefficients [Ai] and [Bi] of the transfer function H(s). This method can be done by using the Pascal matrix equation (8), the matrix [Ai] and [Bi] can be found as:

$$\begin{cases} [A]_{1:n+1} = ([P]_{N+1;N+1}^{-1} [a]_{N+1;1})_{1:n+1} [T]_{n+1;n+1}^{-1} \\ [B]_{1:n+1} = ([P]_{N+1;N+1}^{-1} [b]_{N+1;1})_{1:n+1} [T]_{n+1;n+1}^{-1} \end{cases} \quad (9)$$

The equation (9) is called "the Inverse Pascal Matrix Equation" and it can be used to find the coefficients of the analog low pass filter from a digital filter.

In this equation, the involving of the inverse matrix [P]⁻¹ and [T]⁻¹ will not make easy computing and hand-calculation for lager matrix size. To overcome this problem, consider some features of the matrix [P] and [T] in the next section.

5. THE FEATURES OF THE MATRIX [P] and [T]

There are some features of the matrix [P] and they can be used to find the inverse matrix [P]⁻¹. For the matrix low pass [P_{LP}], if multiply the matrix by itself will give a diagonal matrix with all the numbers in the diagonal are equal to 2ⁿ [3], and from this the inverse matrix [P_{LP}] can be found as:

$$\begin{cases} [(P_{LP})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} [(P_{LP})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} = 2^n [I] \\ \therefore [(P_{LP})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1}^{-1} = \frac{1}{2^n} [(P_{LP})_{i=1 \rightarrow n+1, j=1 \rightarrow n+1}]_{i;n+1} \end{cases} \quad (10)$$

The inverse of the matrix [P_{HBS}] can be found similar way and

can be written as

$$\left[(P_{HBS})_{i=1 \rightarrow N+1; j=1 \rightarrow N+1} \right]_{N+1; N+1}^{-1} = \frac{1}{2^N} \left[\left((-1)^{i+j} P_{HBS \ i; j} \right)_{i=1 \rightarrow N+1; j=1 \rightarrow N+1} \right]_{N+1; N+1}^{P_{HBS} \rightarrow N \times N} \quad (11)$$

The matrix $[T_c]$ and $[T_t]$ are the diagonal matrix, so the inverse of them can be obtained by replacements each element in diagonal with its reciprocal as illustrated below

$$[T_{c-inv}] = [T_c]^{-1} = \left[\left(\frac{1}{T_c} \right)_{i=j} \right]_{i=1 \rightarrow n+1; j=1 \rightarrow n+1} \quad [T_{t-inv}] = [T_t]^{-1} = \left[\left(\frac{1}{T_t} \right)_{i=j} \right]_{i=1 \rightarrow n+1; j=1 \rightarrow n+1} \quad (12)$$

Let a matrix $[T_h]$ has a size of $(n+1; n+1)$ and it can be written like a left-half of the Pascal's triangle with zeros $[T]$ figure.1 as shown below:

$$[T_h]_{n+1; n+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & U & 0 \\ 0 & 0 & 0 & U^2 & 0 & 2UL \\ 0 & 0 & U^3 & 0 & 3U^2L & 0 \\ 0 & \dots & 0 & \dots & 0 & \dots \\ U^n & 0 & \dots & 0 & \dots & 0 \end{bmatrix}_{n+1; n+1}$$

The inverting of the matrix $[T_h]$ is an upper-left triangular matrix $[T_{h-inv}]$ of the size $(n+1; n+1)$ and it can be found as following steps:

- If the main anti-diagonal is the first, then all the odd anti-diagonal above it are 3rd, 5th, 7th... and corresponding to $m=1, 2, 3, 4, \dots$

- Each element in the m^{th} anti-diagonal can be expressed in the formula as:

$$(T_{h-inv})_{(n+1-2(m-1)-j+1); j} = K_{(n+1-2(m-1)-j+1); j} \left(\frac{L}{U} \right)^{m-1} U^{-j+1} \quad (13)$$

- The coefficients K can be calculated as below:

- . All the K in main anti-diagonal are equal to 1.
- . From $m=2, 3, 4, 5, \dots$ all the K in the first column are replaced with -2, +2, -2, +2... accordingly.
- . All another K in all anti-diagonals can be found by

$$K_{(n+1-2(m-1)-j+1); j} = K_{(n+1-2(m-1)-j+2); j} - K_{(n+1-2(m-1)-j+3); j} \quad (14)$$

- All another elements in the matrix $[T_{h-inv}]$ equal to zero.

The matrix $[T_{h-inv}]$ can be used to inverse form a digital band pass and band stop to an analog low pass filter.

From the equation (9), (10), (11), (12) and the matrix $[T_{h-inv}]$, the inverse Pascal matrix equation for low to low pass, high pass to low pass, band pass to low pass and band stop can be expressed as

. Inverse a digital low pass filter to an analog low pass filter

$$\begin{cases} [A]_{1; n+1} = \frac{1}{2^N} \left([P_{LP}]_{n+1; n+1} [a]_{n+1; 1} \right)_{1; n+1} [T_{c-inv}]_{n+1; n+1} \\ [B]_{1; n+1} = \frac{1}{2^N} \left([P_{LP}]_{n+1; n+1} [b]_{n+1; 1} \right)_{1; n+1} [T_{c-inv}]_{n+1; n+1} \end{cases} \quad (15)$$

. Inverse a digital high pass filter to an analog low pass filter

$$\begin{cases} [A]_{1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{n+1; n+1} [a]_{n+1; 1} \right)_{1; n+1} [T_{t-inv}]_{n+1; n+1} \\ [B]_{1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{n+1; n+1} [b]_{n+1; 1} \right)_{1; n+1} [T_{t-inv}]_{n+1; n+1} \end{cases} \quad (16)$$

. Inverse a digital band pass filter to an analog low pass filter

$$\begin{cases} [A]_{1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{N+1; N+1} [a]_{N+1; 1} \right)_{1; n+1} [T_{h-inv}]_{n+1; n+1} \\ [B]_{1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{N+1; N+1} [b]_{N+1; 1} \right)_{1; n+1} [T_{h-inv}]_{n+1; n+1} \end{cases} \quad (17)$$

. Inverse a digital band stop filter to an analog low pass filter

$$\begin{cases} [A]_{n+1-i; 1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{N+1; N+1} [a]_{N+1; 1} \right)_{1; n+1} [T_{h-inv}]_{n+1; n+1} \\ [B]_{n+1-i; 1; n+1} = \frac{1}{2^N} \left(\left[(-1)^{i+j} P_{HBS \ i; j} \right]_{N+1; N+1} [b]_{N+1; 1} \right)_{1; n+1} [T_{h-inv}]_{n+1; n+1} \end{cases} \quad (18)$$

The methods covert and invert between the coefficients of an analog low pass filter and a digital filter were studied. The Pascal matrix equation and inverse matrix equation are derived and they are easy to use for hand-calculation and computing. One application of them is transformation a digital to another digital filter.

6. TRANSFORM A DIGITAL FILTER TO ANOTHER DIGITAL FILTER

This section introduces a new method to design a digital from another filter. Given a digital filter, invert it to an analog low pass filter using inverse Pascal matrix equation and from analog low pass filter converts it to a desired digital filter using Pascal matrix equation. A block diagram in figure.3 illustrates the transformation from a digital to another digital filter.



fig.3 Transformation a digital filter to another digital filter

Given the coefficients $[a_g]$ and $[b_g]$ of a digital filter, invert them to the coefficients $[A_g]$ and $[B_g]$ of an analog low pass filter. The equation (9) is rewritten as

$$\begin{cases} [A_g]_{1; n+1} = \left([P_g]_{N+1; N+1}^{-1} [a_g]_{N+1; 1} \right)_{1; n+1} [T_g]_{n+1; n+1}^{-1} \\ [B_g]_{1; n+1} = \left([P_g]_{N+1; N+1}^{-1} [b_g]_{N+1; 1} \right)_{1; n+1} [T_g]_{n+1; n+1}^{-1} \end{cases} \quad (19)$$

Then convert $[A_g]$ and $[B_g]$ to the coefficients $[a_d]$ and $[b_d]$ of a desired digital filter. The equation (8) is rewritten as

$$\begin{cases} [a_d]_{N+1; 1} = [P_d]_{N+1; N+1} \left([A_g]_{1; n+1} [T_d]_{n+1; n+1} \right)_{N+1; 1} \\ [b_d]_{N+1; 1} = [P_d]_{N+1; N+1} \left([B_g]_{1; n+1} [T_d]_{n+1; n+1} \right)_{N+1; 1} \end{cases} \quad (20)$$

From equation (19) and (20), a new equation, called transform Pascal matrix equation, is found, this equation is used for transform a digital to another digital filter.

$$\begin{cases} [a_d]_{N+1; 1} = [P_d]_{N+1; N+1} \left(\left([P_g]_{N+1; N+1}^{-1} [a_g]_{N+1; 1} \right) [T_g]_{n+1; n+1}^{-1} [T_d]_{n+1; n+1} \right)_{N+1; 1} \\ [b_d]_{N+1; 1} = [P_d]_{N+1; N+1} \left(\left([P_g]_{N+1; N+1}^{-1} [b_g]_{N+1; 1} \right) [T_g]_{n+1; n+1}^{-1} [T_d]_{n+1; n+1} \right)_{N+1; 1} \end{cases} \quad (21)$$

Example 1: Convert a digital 4th-order Chebyshev type 2 low pass filter $H_{LP}(z)$ at cut off frequency at 400Hz, 20dB stop band ripple to a digital high pass filter at sampling frequency 2 kHz.

$$H_{LP}(z) = \frac{2.6744 + 1.7645z^{-1} + 3.9222z^{-2} + 1.7645z^{-3} + 2.6744z^{-4}}{17.6412 - 18.2293z^{-1} + 16.3414z^{-2} - 4.0816z^{-3} + 1.1282z^{-4}}$$

From equation (21), the conversion of the digital low pass filter to a digital high pass is expressed as

$$\begin{cases} [a_d]_{5; 1} = \frac{1}{2^4} [P_{HBS}]_{5; 5} \left(\left([P_{LP}]_{5; 5} [a_g]_{5; 1} \right) [T_{c-inv}]_{5; 5} [T_t]_{5; 5} \right)_{5; 1} \\ [b_d]_{5; 1} = \frac{1}{2^4} [P_{HBS}]_{5; 5} \left(\left([P_{LP}]_{5; 5} [b_g]_{5; 1} \right) [T_{c-inv}]_{5; 5} [T_t]_{5; 5} \right)_{5; 1} \end{cases}$$

$$n = 4; c = \cot\left(\pi \frac{400}{2000}\right) = 1.3764; t = \tan\left(\pi \frac{400}{2000}\right) = 0.7265$$

$$\begin{cases} [a_g] = [2.6744; 1.7645; 3.9222; 1.7645; 2.6744] \\ [b_g] = [17.6412; -18.2293; 16.3414; -4.0816; 1.1282] \end{cases}$$

From equation (21), the conversion of the digital band pass filter to a digital band stop is expressed as

$$\begin{cases} [a_d]_{s:1} = \frac{1}{2^i} [P_{HBS}]_{s:5} \left(([(-1)^{i+j} P_{BP}]_{s:5} [a_g]_{s:1})_{1:3} [T_{b-inv}]_{3:3} [T_r]_{3:5} \right)_{s:1} \\ [b_d]_{s:1} = \frac{1}{2^i} [P_{HBS}]_{s:5} \left(([(-1)^{i+j} P_{BP}]_{s:5} [b_g]_{s:1})_{1:3} [T_{c-inv}]_{3:3} [T_r]_{3:5} \right)_{s:1} \end{cases}$$

$$\begin{cases} ([P_{LP}] [a_g]_{s:1})_{s:1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 2 & 0 & -2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & -2 & 0 & 2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2.6744 \\ 1.7645 \\ 3.9222 \\ 1.7645 \\ 2.6744 \end{bmatrix} = \begin{bmatrix} 12.8 \\ 0 \\ 24.2484 \\ 0 \\ 5.7420 \end{bmatrix} \\ ([P_{LP}] [b_g]_{s:1})_{s:1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 2 & 0 & -2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & -2 & 0 & 2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 17.6412 \\ 37.7566 \\ 79.9336 \\ 94.3474 \\ 57.4217 \end{bmatrix} = \begin{bmatrix} 12.8 \\ 37.7566 \\ 79.9336 \\ 94.3474 \\ 57.4217 \end{bmatrix} \end{cases}$$

$$\begin{cases} ([P_{LP}] [a_g]_{s:1} [T_{r-inv}]_{s:1})_{s:1} = [12.8 \ 0 \ 24.2484 \ 0 \ 5.7420] \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.7265 & 0 & 0 & 0 \\ 0 & 0 & 0.5279 & 0 & 0 \\ 0 & 0 & 0 & 0.3835 & 0 \\ 0 & 0 & 0 & 0 & 0.2786 \end{bmatrix} \\ = [12.8 \ 0 \ 12.8 \ 0 \ 1.6] \end{cases}$$

$$\begin{cases} ([P_{LP}] [b_g]_{s:1} [T_{r-inv}]_{s:1})_{s:1} = [12.8 \ 37.7566 \ 79.9336 \ 94.3474 \ 57.4217] \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.7265 & 0 & 0 & 0 \\ 0 & 0 & 0.5279 & 0 & 0 \\ 0 & 0 & 0 & 0.3835 & 0 \\ 0 & 0 & 0 & 0 & 0.2786 \end{bmatrix} \\ = [12.8 \ 27.4318 \ 42.1941 \ 36.1837 \ 16] \end{cases}$$

$$\begin{cases} (([P_{LP}] [a_g]_{s:1} [T_{r-inv}]_{s:1})_{s:1} [T_r]_{s:5})_{s:5} = [12.8 \ 0 \ 12.8 \ 0 \ 1.6] \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.7265 & 0 & 0 & 0 \\ 0 & 0 & 0.5279 & 0 & 0 \\ 0 & 0 & 0 & 0.3835 & 0 \\ 0 & 0 & 0 & 0 & 0.2786 \end{bmatrix} \\ = [12.8 \ 0 \ 6.7566 \ 0 \ 0.4458] \end{cases}$$

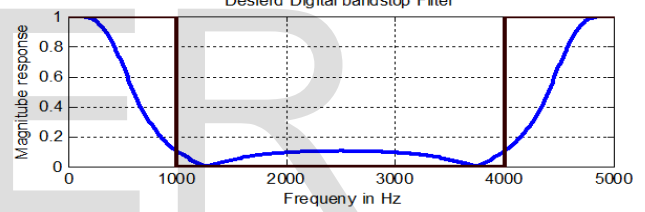
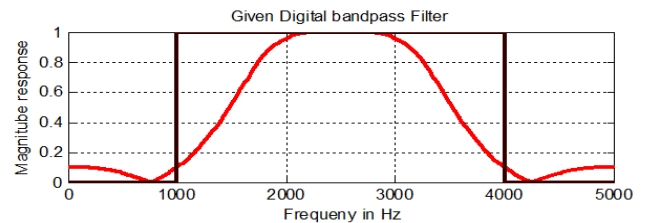
$$\begin{cases} (([P_{LP}] [b_g]_{s:1} [T_{r-inv}]_{s:1})_{s:1} [T_r]_{s:5})_{s:5} = [12.8 \ 37.7566 \ 79.9336 \ 94.3474 \ 57.4217] \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.7265 & 0 & 0 & 0 \\ 0 & 0 & 0.5279 & 0 & 0 \\ 0 & 0 & 0 & 0.3835 & 0 \\ 0 & 0 & 0 & 0 & 0.2786 \end{bmatrix} \\ = [12.8 \ 19.9304 \ 22.2727 \ 13.8770 \ 4.4582] \end{cases}$$

$$\begin{cases} [a_d]_{s:1} = \frac{1}{2^i} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ -4 & 2 & 0 & -2 & 4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 12.8 \\ 0 \\ 6.7566 \\ 0 \\ 0.4458 \end{bmatrix} = \begin{bmatrix} 1.2501 \\ -3.0885 \\ 4.1226 \\ -3.0885 \\ 1.2501 \end{bmatrix} \\ [b_d]_{s:1} = \frac{1}{2^i} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ -4 & 2 & 0 & -2 & 4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 12.8 \\ 37.7566 \\ 22.2727 \\ 13.8770 \\ 4.4582 \end{bmatrix} = \begin{bmatrix} 4.5836 \\ -2.8421 \\ 3.6877 \\ -1.3287 \\ 0.3577 \end{bmatrix} \end{cases}$$

$$n = 4; c = \cot(\pi \frac{1000}{10000}) = 0.3249; t = \tan(\pi \frac{4000}{10000}) = 0.3249$$

$$\begin{cases} [a_g]_{s:1} = [0.2528 \ 0 \ -0.2944 \ 0 \ 0.2528] \\ [b_g]_{s:1} = [1.1638 \ 0 \ 0.6557 \ 0 \ 0.2919] \end{cases}$$

$$\begin{cases} [a_d]_{s:1} = \frac{1}{2^i} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ -4 & 2 & 0 & -2 & 4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ 4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.2528 \\ -0.2944 \\ 0.2528 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -2 & 0 & 7.5777 \\ 0 & 2.7528 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.3633 & 0 & 0.3633 & 0 \\ 0 & 0.2639 & 0 & 0.1320 & 0.1320 \end{bmatrix} \right)_{s:1} = \begin{bmatrix} 0.2056 \\ 0 \\ 0.0111 \\ 0 \\ 0.2056 \end{bmatrix} \\ [b_d]_{s:1} = \frac{1}{2^i} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ -4 & 2 & 0 & -2 & 4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ 4 & -2 & 0 & 2 & 4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1.1638 \\ 0.6557 \\ 0.2919 \end{bmatrix} \begin{bmatrix} -2 & 0 & 7.5777 \\ 0 & 2.7528 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.3633 & 0 & 0.3633 & 0 \\ 0 & 0.2639 & 0 & 0.1320 & 0.1320 \end{bmatrix} \right)_{s:1} = \begin{bmatrix} 1.5415 \\ 0 \\ -1.7888 \\ 0 \\ 0.6696 \end{bmatrix} \end{cases}$$

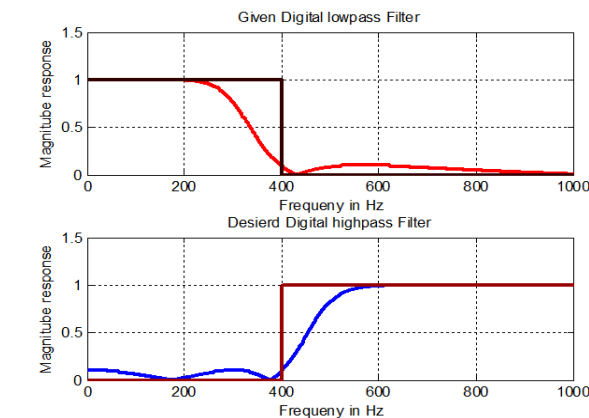


7. CONCLUSION

The new method were studied for converse and inverse between an analog low pass filter with the transfer function H(s) and a digital filter (low pass, high pass, band pass and band stop) with the transfer function H(z) and from that this method can be applied to transform a given digital filter into another digital filter. The involving of the Pascal's triangle uses in the Pascal matrix equation and inverse Pascal matrix equation, as presented and demonstrated in the examples, made the work easier for hand-calculation and computing when transforming between s-domain and z-domain. The features of the matrix [P] and [T] are so helpful to find the inverse matrix which are not easy to do with the larger matrix size. The algorithm of this method converse and inverse is so simple due to all operations imply the matrix multiplication and so it is more effective to program and calculation.

REFERENCES

[1] Emmanuel C. Ifeakor, Barrie W. Jervis. (1993). Digital Signal Processing. Addison-Wesley, USA
 [2] Konopacki J. (2005). The frequency Transformation by Matrix Operation and its Application in iir Filters Design. IEEE Signal Processing Letters, Vol. 12, No.1, pp 5-8, January.
 [3] F. J. Garcia-Ugalde. (2011). Z-transform by Pascal Matrix



Example 2: Convert a digital 4th-order Chebyshev type 2 band pass filter H_{BP}(z) at lower cut off frequency at 1Hz, upper cut off frequency 4kHz and 20dB stop band ripple to a digital band stop filter at sampling frequency 10kHz.

$$H_{BP}(z) = \frac{0.2528 - 0.2944z^{-2} + 0.2528z^{-4}}{1.1638 + 0.6557z^{-2} + 0.2919z^{-4}}$$

and its Applications in the Design of IIR Filters. IEEE, vol.9, N0.3, pp.355-366, December.

[4] Chivapreecha, S. (2005). Bilinear s-z frequency transformation using matrix Pascal operation, IEEE, Communications and Information Technology, Vol.1, pp 764-767.

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