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Study of Ion Sound Propagation Waves in Thermal Plasma

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Abstract— Plasma can support a great variety of wave motion. Both high frequency and low frequency, electromagnetic and electrostatic waves may propagate in plasma. The primary emphasis has been placed on the study of electrostatic waves because the ease with which such waves may be excited and detected and because the collisionless damping of waves predicted by Landau can be conveniently studied. In this paper we will discuss the propagation of ion sound waves in the collisionless thermal plasma by calculating the dispersion relation using fluid theory. The kinetic treatment shows clear that these waves are subject of strong Landau damping for weak temperature of ions compared to the one of electrons. The interest of this study can be applied to some astrophysical phenomena more precisely in the study of the generation of these waves in the topside ionosphere at low latitude sunrise and sunset.

Index Terms— Ion sound waves, thermal plasma, dispersion, fluid, kinetic, Landau, damping, temperature.

1 Introduction

IN Plasmas, there are a variety of waves which propagate with a variety of nonlinear dispersion relations. We will examine one type of these waves and the approximations needed to find its dispersion relation. Ion sound wave is a longitudinal electrostatic wave in unmagnetized plasma arises from the motion of the ions by assuming that the frequency is low enough that ion can participate in motion. For small wave number k, it has the linear form of a normal sound wave. In this study we will discuss the propagation of these waves in non-collisional plasma, taking into account thermal effects. Thermal changes in wave propagation are not well described by fluid equations. To do this we will use the kinetic description of plasma and the appropriate equation is the *Vlasov equation*.

In hot plasma, the dispersion function and its derivatives have a wide range of applications in the descriptions of waves of small amplitude. It is also widely used in the description of polarization strongly inhomogeneous media. Accurate assessment of this function is important in various fields of science. *Fried* and *Conte* (1961) have presented interesting work on the main properties of the dispersion function of hot plasma.

2 Basic Properties of Hot Plasmas (Thermal), Ionisation, Saha Law

In thermal plasmas, collisions between particles can cause ionization if the energy difference between the particles is enough large (of the order of a few eV), or the recombination, if the energy difference is quite low.

As in the same ionized gas, the two forms of collisions can occur, a balance can be established. Just to maintain this balance that the plasma is hot enough. It must even have a temperature of several tens of thousands of degrees [1], [2] eg stars and nuclear explosions.

The state of ionization of plasma is related to its temperature T and density n and the degree of ionization which is defined by

$$\propto = \frac{n_e}{n_0+n_e} = \frac{Zn_i}{n_0+Zn_i} \qquad (1)$$

Where n_e is the electron density, n_i the ion density and n_0 neutral density. Because of collisions, atoms, molecules, or ions in the plasma can be ionized if the temperature is such that

$$k_B T > U_i/10 \qquad (2)$$

U_i is the ionization potential. If plasma is in thermodynamic equilibrium, the ionization is balanced by recombination. This balance is described by *the Saha equation* [3].

$$\frac{n_i}{n_0} = \frac{(2\pi m_e k_B)^{3/2}}{h^3}\frac{T^{3/2}}{n_i} exp[-U_i/k_B T] \qquad (3)$$

Where n_0 is the neutral density, n_i the density of ions et $n_i = n_0$ is the balance between the ionization rate (depending on T) and the rate of recombination (depending on density) h is Planck's constant ($h = 6.62\ 10^{-34} J.s$) and $(2\pi m_e k_B T)^{3/2}/h^3$correspond to the thermal wavelength of an electron ($= 2.4.10^{21}$).

The term that contributes the most is $exp[-U_i/kBT]$

- If $Ui >> k_B T$ low ionisation, $\propto \rightarrow 0$ (industrial plasmas and ionosphere).

- If $Ui << k_B T$ high ionisation, $\propto \rightarrow 1$ (thermonuclear plasmas and stellar).

Typically, $\propto$ begins to be meaningful when $k_B T > U_i = 10$ and allows distinguishing between weakly and strongly ionized plasmas.

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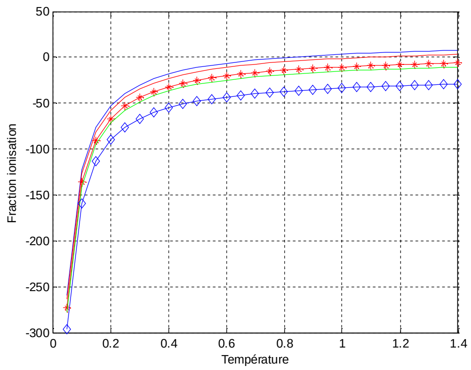

Fig.1. (a) The ionization of a fraction of hydrogen ($U_i = 13{,}6\, eV$) in function of temperature and density (log scale).

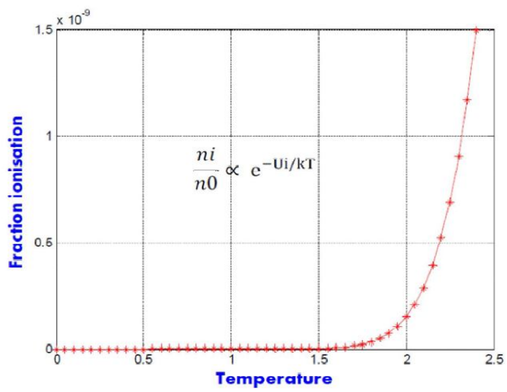

Fig. 1. (b) The ionization of a fraction of hydrogen ($U_i = 13{,}6\, eV$) for a constant density (Saha law).

3 VLASOV MAXWELL SYSTEM

The analysis of the behaviour of particles moving in a hot plasma is based on the Boltzmann equation of the function of distribution $f_s(\vec{r}, \vec{v}, t)$), also known as the Vlasov equation [4], [5]. This equation characterizes the evolution in time and space distribution of particles of non-collisional plasma in kinetic description. For the species s, we can write a kinetic equation of Vlasov in the form

$$\frac{\partial f_s}{\partial t}(\vec{r}, \vec{v}, t) + v.\vec{\nabla}_r f_s(\vec{r}, \vec{v}, t) + \frac{q_s}{m_s}\left(\vec{E} + \vec{v_s} \wedge \vec{B}\right)\vec{\nabla}_r f_s = 0 \quad (4)$$

This latter which is coupled to the Maxwell equations allows to describe the evolution of electric and magnetic fields. In the presence of electromagnetic field $(\vec{E}, \vec{B})$, Maxwell's equations are

$$\vec{\nabla}.\vec{E} = \frac{1}{\varepsilon_0}\sum_s q_s \int f_s d^3\vec{v} \quad (5)$$

$$\vec{\nabla}.\vec{B} = 0 \quad (6)$$

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (7)$$

$$\frac{1}{\mu_0}\vec{\nabla} \wedge \vec{B} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \sum_s q_s \int \vec{v} f_s d^3\vec{v} \quad (8)$$

The system consists of the Vlasov equations (4) and Maxwell (5) - (8) is closed and called Vlasov-Maxwell system. In the absence of magnetic field applied from outside, the field $\vec{B}$ will be zero (non relativistic case), the isotropic medium is called electrostatic and Lorentz force is reduced to an electrical force $q.\vec{E}(\vec{r}, t)$) and the system of Vlasov-Poisson [6] and the equation is written

$$\frac{\partial f_s}{\partial t} + v.\vec{\nabla}_r f_s + \frac{q_s \vec{E}}{m_s}\vec{\nabla}_v f_s = 0 \quad (9)$$

4 DISPERSION FUNCTION IN A HOT PLASMA

It is easy to see that, according to an appropriate scale variable as the dispersion relation for electrostatic waves is expressed by the dispersion function Z:

$$Z(\eta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{e^{-\xi}}{\xi - \eta} d\xi \quad ,Im(\eta) > 0 \quad (10)$$

Moreover, Z (η) is the Hilbert transform of a Gaussian [7],[8]. With $\eta = \frac{\omega}{...}, \xi = u/v_s$ and $v_{th} = \sqrt{2k_B T_s/m_s}$ and $v_{th} = \sqrt{2k_B T_s/m_s}$

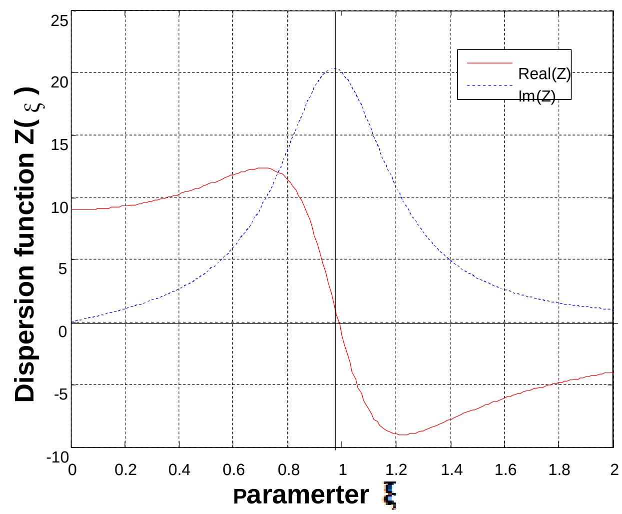

Fig. 2. Real (full curve) and imaginary (dashed curve) parts of plasma dispersion function Z (η)

As is shown on the figure 2, for $\mathrm{Im}(\eta) > 0$, this function is defined in the upper half complex plane, the analytic continuation in the lower half-plane is obtained by writing

$$Z(\eta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{e^{-\xi}}{\xi - \eta} d\xi + i\sqrt{\pi}\, e^{-\eta^2} \quad (11)$$

and therefore the dispersion relation becomes

$$1 - \sum_s \frac{\omega_{ps}^2}{k^2 v_{th,s}^2} Z'\left(\frac{\omega}{k.v_{th,s}}\right) = 0 \quad (12)$$

With [8] $Z'(\eta) = -2[1 + \eta\, Z(\eta)]$. And the expression of Landau damping is given by:

$$1+\frac{e^2}{\varepsilon_0 m_e k^2}\int\frac{k\nabla_v f_0}{\omega-kv}d^3v=0 \qquad (13)$$

5 STUDY OF ION SOUND WAVES PROPAGATION

5.1 Kinetic theory

The general kinetic dispersion relation for electrostatic waves takes the form

$$1+\frac{e^2}{\varepsilon_0 m_e k}\int_{-\infty}^{\infty}\frac{\partial F_{0e}/\partial u}{\omega-ku}du+\frac{e^2}{\varepsilon_0 m_i k}\int_{-\infty}^{\infty}\frac{\partial F_{0i}/\partial u}{\omega-ku}du=0 \qquad (14)$$

And the Landau Damping is given by:

$$1+\frac{e^2}{\varepsilon_0 m_e k^2}\int\frac{k\nabla_v f_0}{\omega-kv}d^3v=0 \qquad (15)$$

Where

$$F_{0e/i}(u)=\frac{n}{(2\pi T_{e/i}/m_{e/i})^{1/2}}\exp\left(-\frac{m_{e/i}u^2}{2T_{e/i}}\right) \qquad (16)$$

The wave with a phase velocity, ω/k, is much less than the electron thermal velocity, but much greater than the ion thermal velocity. We may assume that $\omega \gg k\,u$ for the ion term. It follows that, to lowest order, this term reduces to $-\omega_{pi}^2/\omega^2$. Conversely, we may assume that $\omega \ll k\,u$ for the electron term. Thus, to lowest order we may neglect ω in the velocity space integral. Assuming F_0 e to be a Maxwellian with temperature T_e, the electron term reduces to

$$\frac{\omega_{pe}^2}{k^2}\frac{m_e}{T_e}=\frac{1}{(k\,\lambda_D)^2} \qquad (17)$$

With λ_D is the Debye length [9] and ω_{pe} the electron plasma frequency. Thus, to a first approximation, the dispersion relation can be written

$$1+\frac{1}{(k\,\lambda_D)^2}-\frac{\omega_{pi}^2}{\omega^2}=0 \qquad (18)$$

$$\omega^2=\frac{\omega_{pi}^2(k\,\lambda_D)^2}{1+(k\,\lambda_D)^2}=\frac{T_e}{m_i}\frac{k^2}{1+k^2\lambda_D^2} \qquad (19)$$

For $k\,\lambda_D \ll 1$, we have $\omega=(T_e/m_i)^{1/2}k$, a dispersion relation which is like that of an ordinary sound wave, with the pressure provided by the electrons, and the inertia by the ions. As the wave-length is reduced towards the Debye length, the frequency levels off and approaches the ion plasma frequency.

In the long wave-length limit, we see that the wave phase velocity $(T_e/m_i)^{1/2}$ is indeed much less than the electron thermal velocity [by a factor $(m_e/m_i)^{1/2}$], but that it is only much greater than the ion thermal velocity if the ion temperature, T_i, is much less than the electron temperature, T_e.

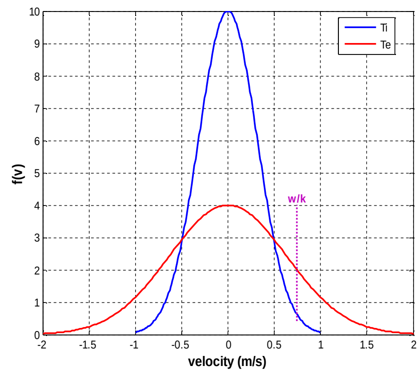

Fig. 3. Ion and electron distribution function f(v) with $T_i \ll T_e$

In fact, if $T_i \ll T_e$ then the wave phase velocity can lie on almost flat portions of the ion and electron distribution functions, as shown in Fig. 2, implying that the wave is subject to very little Landau damping. Indeed, an ion sound wave can only propagate a distance of order its wave-length without being strongly damped provided that T_e is at least five to ten times greater than T_i.

5.2 Fluid theory

Of course, it is possible to obtain the ion sound wave dispersion relation, $\omega^2/k^2 = T^2/m_i$, using fluid theory. The kinetic treatment used here is an improvement on the fluid theory to the extent that no equation of state is assumed, and it makes it clear to us that ion sound waves are subject to strong Landau damping (*i.e.*, they cannot be considered normal modes of the plasma) unless $T_i \ll T_e$.

A. Fluid sound waves

If we perturb and linearize the momentum equation and the continuity equation for field free plasma [10], we get:

$$-\omega\rho_0\vec{v_1}=-i\vec{k}\gamma\frac{p_0}{\rho_0}\rho_1 \qquad (20)$$

And

$$-i\omega\rho_1+i\vec{k}(\rho_0.\vec{v_1})=0 \qquad (21)$$

Dotting the first equation (23) with $\vec{k}$ and substituting into the second (24), we get:

$$-\omega\rho_1+\rho_0\frac{k^2}{\omega\rho_0}\gamma\frac{p_0}{\rho_0}\rho_1=0 \qquad (22)$$

and the dispersion relation is

$$\omega^2=k^2\gamma\frac{p_0}{\rho_0}=k^2c_s^2 \qquad (23)$$

With $v_\varphi = v_g = c_s$. This relation is shown on the Fig. 4. as a line which passed by the origin makes into account the proportionality between ω and c_s.

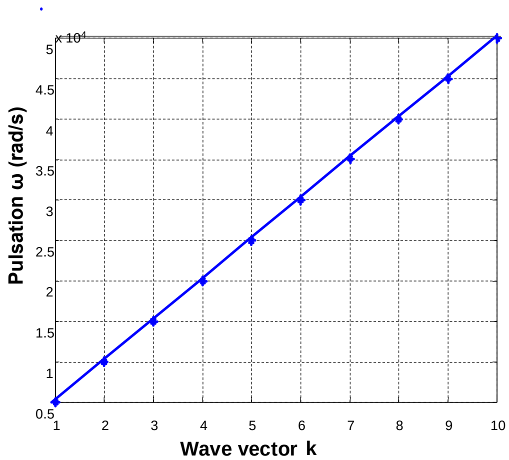

Fig. 4. The dispersion relation $\omega^2 = k^2 c_s^2$

B. Ion sound waves

Even if collisions are unimportant, sound waves, being longitudinal waves, generate density fluctuations which in turn generate electric fields that can provide the necessary restoring force.

When ion motion is involved, we know that the waves must be low frequency, so we can use the *plasma approximation,* ne ≈ ni≈ n0. We are still assuming that there is no magnetic field [9], [10].

$$\frac{\partial}{\partial t}(n_e - n_i) = -n_0 \vec{\nabla}.(\overrightarrow{v_{e1}} - \overrightarrow{v_{i1}}) \quad (24)$$

Thus if the ion and electron velocities differ, the densities will become different too. Thus the plasma approximation also requires $\overrightarrow{v_{e1}} = \overrightarrow{v_{i1}}$

$$\omega^2 = k^2 \left(\frac{\gamma_i k_B T_i + \gamma_e k_B T_e}{m_e + m_i}\right) \quad (25)$$

1. It is essentially identical to the result for fluid sound waves even though at a microscopic level there are profound differences. The coupling here is electrostatic non collisional.
2. The electrons move very rapidly, and the distribution may be assumed to be isothermal, γe = 1.
3. The electron mass is negligible compared with the ion mass in the denominator.

However, Vlasov theory (a detailed study of the effect of the particle velocity distributions) shows that the wave is strongly damped unless the electron temperature greatly exceeds the ion temperature. Thus the ion sound speed is determined by the electron temperature and the ion mass.

$$v_{is} = \sqrt{\frac{k_B T_e}{m_i}} \quad (26)$$

We may now consider the electric field necessary to affect the coupling. Poisson's equation is:

$$\vec{k}.\vec{E} = k^2\phi = \frac{e}{\varepsilon_0}(n_i - n_e) \quad (27)$$

Now we allow for small differences between the electron and ion densities. The ion density is given by the continuity equation:

$$n_i = n_0 + \frac{\vec{k}.\vec{v}}{\omega} n_0 \quad (28)$$

while the electrons respond rapidly to the electric field, and so follow the Boltzman relation:

$$n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right) \approx n_0 \left(1 + \frac{e\phi}{k_B T_e}\right) \quad (29)$$

Thus

$$k^2\phi = \frac{en_0}{\varepsilon_0}\left(\frac{\vec{k}.\vec{v}}{\omega} - \frac{e\phi}{k_B T_e}\right) \quad (30)$$

Rearranging, we get:

$$\phi\left(k^2 + \frac{en_0}{\varepsilon_0}\frac{e\phi}{k_B T_e}\right) = \frac{en_0}{\varepsilon_0}\frac{\vec{k}.\vec{v}}{\omega} \quad (31)$$

We should recognize the second term in the parentheses as $1/\lambda_D^2$.
Now we rewrite the momentum equation for the ions, substituting this expression for Φ in the electric field term $(\vec{E} = -\vec{\nabla}\phi = -i\vec{k}\phi)$.

$$-\omega m_i n_0 \vec{k}.\vec{v_1} = -en_0 \frac{en_0}{\varepsilon_0}\frac{\vec{k}.\vec{v_1}}{\omega}\left(\frac{1}{1+1/k^2\lambda_D^2}\right) - k^2 \frac{\gamma_i k_B T_i}{m_i} \quad (32)$$

$$\omega^2 = k^2\left(\frac{\gamma_i k_B T_i}{m_i} + \frac{\lambda_D^2 \omega_p^2}{1+k^2\lambda_D^2}\right) \quad (33)$$

where ω_{pi} is the ion plasma frequency.
The numerator in the second term is:

$$\lambda_D^2 \omega_p^2 = \frac{\varepsilon_0}{en_0}\frac{k_B T_e}{e} e^2 \frac{n_0}{m_i \varepsilon_0} = k_B \frac{T_e}{m_i} \quad (34)$$

Thus the new result is identical to the previous one except for the denominator $1 + k^2\lambda_D^2$.

Thus the correction is necessary only when $k\lambda_D$ is not small, that is when the wavelength is less than or equal to the Debye length. The full wavelength is within the region where we would expect the plasma approximation to fail. When $k\lambda_D \gg 1$ we find $\omega \approx \omega_p$ and we have oscillations at the ion plasma frequency.

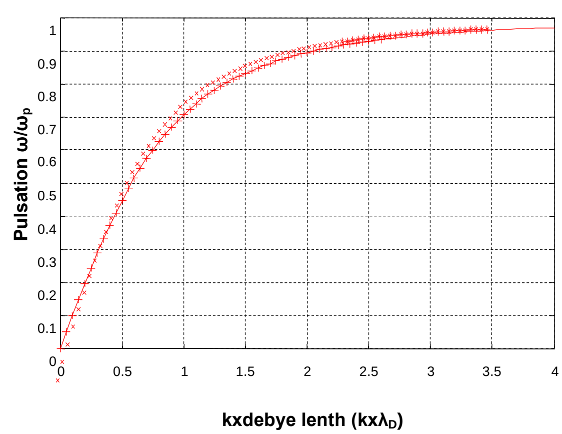

Fig 5: Dispersion relation for ion sound

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The wave reduces to plasma oscillations of the ions for which we obtain a graph of the dispersion relation as is shown on the Fig.5. similar to that of the Langmuir wave dispersion relation.

5 CONCLUSION

We have studied the ion-sound waves under the condition of $k\,\lambda_D \ll 1$, in a hot, isotropic, and unmagnetized plasma modeled with the generalized $f_s(\vec{r},\vec{v},t)$ distribution function. We have derived the dispersion relations for the ion-sound waves. It is possible to obtain the ion sound wave dispersion relation, $T_i \ll T_e .\, \omega^2/k^2 = T_e/m_i$, using fluid theory. The kinetic treatment used here is an improvement on the fluid theory to the extent that no equation of state is assumed, and it makes it clear to us that ion sound waves are subject to strong Landau damping unless $T_i \ll T_e$. For this condition the electrons are hot and an electrostatic wave in which ions do play a major role is found at lower frequencies. These waves are characterized by phase velocities lying between the thermal velocities of ions and that of the electrons and propagate only when $\omega \gg \omega_p$.

The present work can extended to study the generation of ion sound waves in the topside low latitude ionosphere at sunrise and sunset.

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