

# Soret effect on unsteady free convection flow of a dusty viscous fluid between two infinite flat parallel plates filled by a porous medium with inclined magnetic field

R.Panneerselvi, P.Moheswari

**Abstract:** In this paper, unsteady laminar free convective flow of a dusty viscous fluid through porous medium of non-conducting walls in the presence of inclined magnetic field with volume fraction and heat source is considered. Governing equations are solved by perturbation technique and the results are obtained for fluid and particle velocity, temperature of the dusty fluid at the inclined magnetic field and concentration with various parameters such as  $t$  (time),  $M$  (Magnetic parameter),  $Pr$  (Prandtl number),  $Gr$  (Grashof number),  $S$  (Heat source parameter),  $\theta$  (Inclined magnetic field angle),  $\epsilon_3$  (Porous parameter),  $\phi$  (Volume fraction of dusty particles),  $p$  (Pressure gradient). From these it is observed that increase in inclined magnetic field angle causes the decrease of velocity in the fluid and fluid temperature increases by increase in heat source parameter.

**Key Words:** Convective flow, inclined angle, Porous medium, Heat and Mass Grashof number Schmidt number, Soret number, Heat source, MHD, Dusty fluid.

## 1 INTRODUCTION

The concept of heat transfer of a dusty fluid has a wide range of applications in air conditioning, refrigeration, pumps, accelerators, nuclear reactors, space heating, power generation, chemical processing, filtration and geothermal systems etc. The good example of heat transfer is the radiator in a car, in which the hot radiator fluid is cooled by the flow of air over the radiator surface. Chakraborty [8] studied MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variable viscosity. Unsteady hydromagnetic flow and heat transfer from a non isothermal stretching sheet immersed in a porous medium was discussed by Chamkha [1].

Daniel Simon [3] studied the effect of heat transfer on unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field. They concluded that the temperature distribution decreases as Prandtl number increases. Sandeep. N, Sugunamma. V [5] studied the effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. They concluded that increase in inclined magnetic field angle causes the decrease of velocity of the fluid and fluid temperature decreases by increase in heat source

parameter.

Idowu. A.S. and Olabode. J.O. [2] investigated an unsteady MHD poiseuille flow between two infinite parallel plates in an Inclined Magnetic field with Heat Transfer. Bhavana. M, Chenna Kesavaiah. D, Sudhakaraiyah. A [4] studied the Soret effect on free Convective Unsteady MHD flow over a vertical plate with Heat source. P. Mohan Krishna, Sugunamma. V, Sandeep. N [6] has investigated Magnetic field and chemical reaction effects on convective flow of dusty viscous fluid. They concluded that Temperature Profile decreases with increases of Prandtl number. Srikanth Rao. P and Mahendar. D [7] studied the Soret effect on unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate. Singh. N. P and Atul kumar singh [10] studied the MHD effects on heat and Mass transfer in flow of a dusty viscous fluid with volume fraction. Hazem A. Attia [9] studied the Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Malashetty. M.S, Umavathi. J. C, and Prathap Kumar. J [11] studied the Convective magnetohydrodynamic fluid flow and heat transfer in an inclined channel.

In the present paper the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of inclined magnetic field with volume fraction, heat source and Soret effect are considered. Here, the effect of velocity, velocity of the particle phase, temperature of the dusty fluid at the inclined magnetic field with various parameters as  $t$  (time),  $M$  (Magnetic parameter),  $Pr$  (Prandtl number),  $Gr$  (Grashof number),  $Gc$  (Mass Grashof number),  $Sr$  (Soret number),  $Sc$  (Schmidt number),  $S$  (Heat source parameter),  $\theta$  (Inclined magnetic field angle),

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$K_1$  (Chemical reaction parameter),  $\varepsilon_3$  (Porous parameter),  $\phi$  (Volume fraction of dusty particles),  $p$  (Pressure gradient) are analyzed graphically.

**1.2 Mathematical Formulation**

In Cartesian co-ordinate system, consider unsteady laminar flow of a dusty, incompressible, Newtonian, electricaly conducting, viscous fluid through a porous medium of uniform cross section  $h$ , when one wall of the channel is fixed and the other is oscillating with time about a constant non-zero mean. Initially at  $t \leq 0$ , the channel wall as well as the fluid is assumed to be at the same temperature  $T_0$ . When  $t > 0$ , the temperature of the channel wall is instantaneously raised to  $T_w$  which oscillate with time and is thereafter maintained constant. Let  $x$ -axis be along the fluid flow at the fixed wall and  $y$ -axis perpendicular to it. An inclined magnetic field is applied to the flow along  $y$  direction with the heat source.

**1.2.1 Assumptions:**

The governing equations are written based on the following assumptions:

- (i) The dust particles are solid, spherical, non conducting and equal in size and uniformly distributed in the flow region.
- (ii) The density of dust particles is constant and the temperature between the particles is uniform throughout the motion.
- (iii) The interaction between the particles, chemical reaction between the particles and liquid has not been considered to avoid multiple equations.
- (iv) The volume occupied by the particles per unit volume of the mixture, (i.e., volume fraction of dust particles) and mass concentration have been taken into consideration.
- (v) The dust concentration is so small so that it does not disturb the continuity and hydro magnetic effects. This means that the continuity equation is satisfied.

**1.2.2 Governing equations of the flow**

The fluid flow is governed by the momentum, energy and concentration equation under the above assumptions

$$\frac{\partial u}{\partial t} = \left[ -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta^+(T - T_0) + g\beta^*(C - C_0) \right] + \frac{1}{(1-\phi)} \left[ \frac{KN_0}{\rho} (v-u) - \frac{KN\sigma\mu_c^2 H_0^2}{\rho} u \sin^2 \theta - \frac{\mu}{K_1} u \right] \quad (1)$$

$$\frac{\partial v}{\partial t} = \frac{\phi}{N_0 m} \left[ \begin{matrix} -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \\ \rho g \beta^+(T - T_0) + \\ \rho g \beta^*(C - C_0) \end{matrix} \right] + KN_0 (u - v) \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_0) \quad (3)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} + K_1 (C - C_0) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The boundary conditions of the problem are:

$$\begin{aligned} t \leq 0; \quad u(y,t) = v(y,t) = 0, \\ T(y,t) = C(y,t) = 0 \quad \text{for } 0 \leq y \leq 1 \\ t > 0; \quad u(y,t) = v(y,t) = 0, \\ T(y,t) = C(y,t) = 0 \quad \text{for } 0 \leq y \leq 1 \\ u(y,t) = v(y,t) = 1 + \varepsilon e^{int}, \\ T(y,t) = C(y,t) = 1 + \varepsilon e^{int} \quad \text{at } y = 1 \end{aligned} \quad (5)$$

Where  $u(y,t)$  is the velocity of the fluid and  $v(y,t)$  velocity of the dust particles,  $m$  is the mass of each dust particle,  $N_0$  is the number density of dust particles,  $T$  is the temperature.  $T_0$  is the initial temperature,  $T_w$  is the raised temperature,  $C$  is the concentration,  $C_0$  is the initial concentration,  $C_w$  is the raised concentration,  $\beta^+$  is the volumetric coefficient of thermal expansion,  $K_1$  is the porous parameter,  $K$  is the Stoke's resistance coefficient,  $\sigma$  is the electrical conductivity of the fluid,  $\mu_c$  is magnetic permeability,  $Q_0$  is heat source,  $H_0$  is the magnetic field induction,  $C_p$  is the specific heat at constant pressure,  $\phi$  is the volume fraction of dust particles and  $k$  is thermal conductivity.

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be  $h$  and characteristic velocity is  $v$ . We introduce the following non-dimensional variables:

$$\begin{aligned} x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad p^* = \frac{h^2 p}{\rho v^2}, \quad t^* = \frac{vt}{h^2}, \quad u^* = \frac{uh}{v}, \\ v^* = \frac{vh}{v}, T^* = \frac{T - T_0}{T_w - T_0}, \quad \text{and} \quad C^* = \frac{C - C_0}{C_w - C_0} \end{aligned} \quad (6)$$

Substituting the above non-dimensional parameters in the governing equations (1 - 4) and then removing asterisks, we get

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + GrT + GcC + \varepsilon_1 v - \lambda u \quad (7)$$

Where  $\lambda = \varepsilon_1 + \varepsilon_2 M + \varepsilon_3 = \text{constant}$

$$f \frac{\partial v}{\partial t} = \phi \left[ -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + GrT + GcC \right] + \beta(u - v) \quad (8)$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + ST \quad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + K_1 C + Sr \frac{\partial^2 T}{\partial y^2} \quad (10)$$

Where

$$Gr = \frac{g\beta^+(T_w - T_0)h^3}{\gamma^2} \quad (\text{Grashof number}),$$

$$Gc = \frac{g\beta^*(C_w - C_0)h^3}{\gamma^2} \quad (\text{mass Grashof number}),$$

$$\varepsilon_1 = \frac{f}{\sigma_1(1-\phi)}, \quad \sigma_1 = \frac{m\gamma}{Kh^2}, \quad \varepsilon_2 = \frac{1}{(1-\phi)},$$

$$M' = \mu_c^2 h^2 H_0^2 \frac{\sigma}{\mu}$$

$$M = M' \sin^2 \theta \quad (\text{Magnetic parameter}),$$

$$f = \frac{mN_0}{\rho} \quad (\text{Mass concentration of dust particles}),$$

$$\varepsilon_3 = \frac{\mu h^2}{K_1(1-\phi)} \quad (\text{Porous Parameter}),$$

$$K_1 = \frac{K_1 h^2}{\gamma} \quad (\text{Chemical reaction parameter}),$$

$$\beta = \frac{f}{\sigma_1} \quad (\text{Concentration resistance ratio}),$$

$$\text{Pr} = \frac{\mu C_p}{k} \quad (\text{Prandtl number}),$$

$$\text{Sc} = \frac{\gamma}{D} \quad (\text{Schmidt number}),$$

$$S = \frac{Q_0 h^2}{\nu \rho C_p} \quad (\text{Heat source parameter}),$$

$$\text{Sr} = \frac{D_m K_T}{T_m} \frac{T_w - T_0}{C_w - C_0} \quad (\text{Soret number}).$$

The non-dimensional boundary conditions are:

$$\begin{aligned} t \leq 0; \quad u(y,t) = v(y,t) = 0, \quad T(y,t) = C(y,t) = 0 \quad \text{for} \quad 0 \leq y \leq 1 \\ t > 0; \quad u(y,t) = v(y,t) = 0, \quad T(y,t) = C(y,t) = 0 \quad \text{for} \quad y = 0 \\ u(y,t) = v(y,t) = 1 + \varepsilon e^{im}, \quad T(y,t) = C(y,t) = 1 + \varepsilon e^{im} \quad \text{at} \quad y = 1 \end{aligned} \quad (11)$$

### 1.3 SOLUTION OF THE PROBLEM

To solve the equations (7-10) we use the below equations introduced by Soundalgekar and Bhat(1984) when  $\varepsilon \ll 1$ .

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y) e^{int} \\ v(y,t) &= v_0(y) + \varepsilon v_1(y) e^{int} \\ T(y,t) &= T_0(y) + \varepsilon T_1(y) e^{int} \\ C(y,t) &= C_0(y) + \varepsilon C_1(y) e^{int} \end{aligned} \quad (12)$$

$$\frac{\partial p}{\partial x} = P = \text{Constant}$$

After substituting equation (12) in equations (7-10), we can write

$$u_0''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) u_0(y) + \varepsilon_1 v_0(y) = P - Gr T_0(y) - Gc C_0(y) \quad (13)$$

$$\beta v_0(y) = \beta u_0(y) + \phi \left[ u_0''(y) - P + Gr T_0(y) + Gc C_0(y) \right] \quad (14)$$

$$T_0''(y) + S \text{Pr} T_0(y) = 0 \quad (15)$$

$$C_0''(y) + K_1 \text{Sc} C_0(y) + \text{Sr} \text{Sc} T_0''(y) = 0 \quad (16)$$

$$u_1''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3 + in) u_1(y) + \varepsilon_1 v_1(y) = -Gr T_1(y) - Gc C_1(y) \quad (17)$$

$$(\beta + in) v_1(y) = \beta u_1(y) + \phi \left[ u_1''(y) + Gr T_1(y) + Gc C_1(y) \right] \quad (18)$$

$$T_1''(y) + [S - in] \text{Pr} T_1(y) = 0 \quad (19)$$

$$C_1''(y) + [K_1 - in] \text{Sc} C_1(y) + \text{Sr} \text{Sc} T_1''(y) = 0 \quad (20)$$

The corresponding boundary conditions becomes

$$\begin{aligned} u_0(y) = u_1(y) = v_0(y) = v_1(y) = 0, \quad \text{at} \quad y = 0 \\ T_0(y) = T_1(y) = C_0(y) = C_1(y) = 0, \quad \text{at} \quad y = 0 \\ u_0(y) = u_1(y) = v_0(y) = v_1(y) = 1, \quad \text{at} \quad y = 1 \\ T_0(y) = T_1(y) = C_0(y) = C_1(y) = 1, \quad \text{at} \quad y = 1 \end{aligned} \quad (21)$$

On solving equation (15) and (16) with the help of boundary conditions (21), we get

$$T_0(y) = \frac{\sin L_1 y}{\sin L_1} \quad (22)$$

$$C_0(y) = (1+w) \frac{\sin L_2 y}{\sin L_2} - w \frac{\sin L_1 y}{\sin L_1} \quad (23)$$

$$\text{Where } L_1 = \sqrt{S \text{Pr}}, \quad L_2 = \sqrt{\text{Sc} K_1}, \quad w = \frac{\text{Sr} \text{Sc} L_1^2}{L_1^2 - K_1 \text{Sc}}$$

Substituting equations (22) and (23) in equations (13) and (14)

$$\begin{aligned} u_0''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) u_0(y) + \varepsilon_1 v_0(y) = P - \\ Gr \frac{\sin L_1 y}{\sin L_1} - Gc(1+w) \frac{\sin L_2 y}{\sin L_2} + Gcw \frac{\sin L_1 y}{\sin L_1} \end{aligned} \quad (24)$$

$$\beta v_0(y) = \beta u_0(y) + \phi \left[ u_0''(y) - P + Gr \frac{\sin L_1 y}{\sin L_1} + Gc(1+w) \frac{\sin L_2 y}{\sin L_2} - Gcw \frac{\sin L_1 y}{\sin L_1} \right] \quad (25)$$

Substituting equation (25) in equation (24), we obtain

$$u_0''(y) - A^2 u_0(y) = P - (Gr - Gcw) \frac{\sin L_1 y}{\sin L_1} - Gc(1+w) \frac{\sin L_2 y}{\sin L_2} \quad (26)$$

$$\text{Where } A^2 = \frac{\beta(\varepsilon_2 M + \varepsilon_3)}{\beta + \varepsilon_1 \phi}$$

By solving equation (26) with the boundary conditions (21), we get

$$u_0(y) = \frac{P(e^{Ay} - 1)}{A^2} + \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} - A_3 - A_4 \right] \frac{\sinh Ay}{\sinh A} + \quad (27)$$

$$A_3 \frac{\sin L_1 y}{\sin L_1} + A_4 \frac{\sin L_2 y}{\sin L_2}$$

$$\text{where } A_3 = \frac{Gr - Gcw}{L_1^2 + A^2}, \quad A_4 = \frac{Gc(1+w)}{L_2^2 + A^2}$$

The first and second order partial derivatives of  $u_0(y)$  are

$$u_0'(y) = \frac{Pe^{Ay}}{A} + \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} - A_3 - A_4 \right] \frac{\text{Acosh} Ay}{\sinh A} + A_3 L_1 \frac{\cos L_1 y}{\sin L_1} + A_4 L_2 \frac{\cos L_2 y}{\sin L_2} \quad (28)$$

$$u_0''(y) = Pe^{Ay} + \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} - A_3 - A_4 \right] \frac{A^2 \sinh Ay}{\sinh A} - \quad (29)$$

$$A_3 L_1^2 \frac{\sin L_1 y}{\sin L_1} - A_4 L_2^2 \frac{\sin L_2 y}{\sin L_2}$$

Substituting equations (27) and (29) in equation (14), we obtain

$$v_0(y) = \frac{A_1 P(e^{Ay} - 1)}{A^2} + A_3 A_1 \frac{\sin L_1 y}{\sin L_1} + A_4 A_1 \frac{\sin L_2 y}{\sin L_2} + A_1 \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} - A_3 - A_4 \right] \frac{\sinh Ay}{\sinh A} \quad (30)$$

Where  $A_1 = 1 + \frac{\phi}{\beta} A^2$

By solving equations (19) and (20) with the boundary conditions (21), we get

$$T_1(y) = \frac{\sinh L_0 y}{\sinh L_0} \quad (31)$$

$$C_1(y) = (1+q) \frac{\sinh L_3 y}{\sinh L_3} - q \frac{\sinh L_0 y}{\sinh L_0} \quad (32)$$

Where  $L_3 = \sqrt{Sc(in - K_1)}$ ,  $L_0 = \sqrt{SPr}$ ,  $q = \frac{SrScL_0^2}{L_0^2 - L_3^2}$

Substituting equations (31) and (32) in equations (17) and (18), we obtain

$$u_1''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3 + in)u_1(y) + \varepsilon_1 v_1(y) = -Gr \frac{\sinh L_0 y}{\sinh L_0} - Gc(1+q) \frac{\sinh L_3 y}{\sinh L_3} + Gc q \frac{\sinh L_0 y}{\sinh L_0} \quad (33)$$

$$(\beta + inf)v_1(y) = \beta u_1(y) + \phi [u_1''(y) + \phi \left[ Gr \frac{\sinh L_0 y}{\sinh L_0} + Gc(1+q) \frac{\sinh L_3 y}{\sinh L_3} - Gc q \frac{\sinh L_0 y}{\sinh L_0} \right]] \quad (34)$$

Substituting equation (34) in equation (33) we obtain

$$u_1''(y) - B^2 u_1(y) = -(Gr - Gc q) \frac{\sinh L_0 y}{\sinh L_0} - Gc(1+q) \frac{\sinh L_3 y}{\sinh L_3} \quad (35)$$

On solving equation (35), with the boundary conditions (21), we get

$$u_1(y) = (1 + B_2 + B_3) \frac{\sinh By}{\sinh B} - B_2 \frac{\sinh L_0 y}{\sinh L_0} - B_3 \frac{\sinh L_3 y}{\sinh L_3} \quad (36)$$

The first and second order partial derivatives  $u_1(y)$  are given by

$$u_1'(y) = (1 + B_2 + B_3) \frac{B \cosh By}{\sinh B} - B_2 L_0 \frac{\cosh L_0 y}{\sinh L_0} - B_3 L_3 \frac{\cosh L_3 y}{\sinh L_3} \quad (37)$$

$$u_1''(y) = (1 + B_2 + B_3) \frac{B^2 \sinh By}{\sinh B} - B_2 L_0^2 \frac{\sinh L_0 y}{\sinh L_0} - B_3 L_3^2 \frac{\sinh L_3 y}{\sinh L_3} \quad (38)$$

Substituting equations (37) and (38) in equation (34), we obtain

$$v_1(y) = A_2 B_0 \left[ (1 + B_2 + B_3) \frac{\sinh By}{\sinh B} - B_2 \frac{\sinh L_0 y}{\sinh L_0} - B_3 \frac{\sinh L_3 y}{\sinh L_3} \right] \quad (39)$$

Where  $A_0 = 1 + \frac{\phi}{\beta} B^2$ ,

$$B_0 = \frac{\beta(\beta - nif)}{\beta^2 + n^2 f^2}, B_2 = \frac{(Gr - Gc q)}{L_0^2 - B^2}, B_3 = \frac{Gc(1+q)}{L_3^2 - B^2}$$

Substituting the equations (27) and (36) in the equation (12), we obtain

$$u(y,t) = \frac{P(e^{Ay} - 1)}{A^2} + \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} \right] \frac{\sinh Ay}{\sinh A} - \frac{\sinh Ay}{\sinh A} [A_3 + A_4] + A_3 \frac{\sin L_1 y}{\sin L_1} + A_4 \frac{\sin L_2 y}{\sin L_2} + \varepsilon \left[ (1 + B_2 + B_3) \frac{\sinh By}{\sinh B} - B_2 \frac{\sinh L_0 y}{\sinh L_0} - B_3 \frac{\sinh L_3 y}{\sinh L_3} \right] e^{int} \quad (40)$$

Substituting the equations (30) and (39) in the equation (12), we obtain

$$v(y,t) = \frac{A_1 P(e^{Ay} - 1)}{A^2} + A_1 \left[ 1 - \frac{P(e^{Ay} - 1)}{A^2} \right] \frac{\sinh Ay}{\sinh A} - \frac{\sinh Ay}{\sinh A} [A_3 + A_4] + A_3 A_1 \frac{\sin L_1 y}{\sin L_1} + A_4 A_1 \frac{\sin L_2 y}{\sin L_2} + \varepsilon A_2 B_0 \left[ (1 + B_2 + B_3) \frac{\sinh By}{\sinh B} - B_2 \frac{\sinh L_0 y}{\sinh L_0} - B_3 \frac{\sinh L_3 y}{\sinh L_3} \right] e^{int} \quad (41)$$

Substituting the equations (22) and (31) in the equation (12), we obtain

$$T(y,t) = \frac{\sin L_1 y}{\sin L_1} + \varepsilon \frac{\sinh L_0 y}{\sinh L_0} e^{int} \quad (42)$$

Substituting the equations (23) and (32) in the equation (12), we obtain

$$C(y,t) = (1+w) \frac{\sin L_2 y}{\sin L_2} - w \frac{\sin L_1 y}{\sin L_1} + \varepsilon \left[ (1+q) \frac{\sinh L_3 y}{\sinh L_3} - q \frac{\sinh L_0 y}{\sinh L_0} \right] e^{int} \quad (43)$$

Hence the equations (40) - (43) represents velocity of the fluid, velocity of the dust particle, temperature and concentration respectively.

#### 1.4 Skin friction

The skin friction of the fluid ( $\tau_w$ ) at the plate  $y = 0$  is

$$\tau_l = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left( \frac{du_0}{dy} \right)_{y=0} + \varepsilon e^{\text{int}} \left( \frac{du_1}{dy} \right)_{y=0}$$

$$\tau_l = \frac{P}{A} + \frac{A[1 - A_3 - A_4]}{\sinh A} + \frac{A_3 L_1}{\sin L_1} + \frac{A_4 L_2}{\sin L_2} + \varepsilon \left[ \frac{(1 + B_2 + B_3)B}{\sinh B} - \frac{B_2 L_0}{\sinh L_0} - \frac{B_3 L_3}{\sinh L_3} \right] e^{\text{int}} \quad (44)$$

The skin friction of the fluid ( $\tau_p$ ) at the plate  $y = 0$  is

$$\tau_p = \frac{A_1 P}{A} + \left[ 1 - \frac{P(e^A - 1)}{A^2} - A_3 - A_4 \right] \frac{AA_1}{\sinh A} + \frac{A_3 A_1 L_1}{\sin L_1} + \frac{A_4 A_1 L_2}{\sin L_2} + \varepsilon A_2 B_0 \left[ \frac{(1 + B_2 + B_3)B}{\sinh B} - \frac{B_2 L_0}{\sinh L_0} - \frac{B_3 L_3}{\sinh L_3} \right] e^{\text{int}} \quad (45)$$

### 1.5 Rate of Heat and Mass transfer

The heat transfer in terms of Nusselt number (Nu) is

$$\tau_l = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left( \frac{du_0}{dy} \right)_{y=0} + \varepsilon e^{\text{int}} \left( \frac{du_1}{dy} \right)_{y=0}$$

$$Nu = \frac{L_1}{\sin L_1} + \varepsilon \frac{L_0}{\sinh L_0} e^{\text{int}} \quad (46)$$

The mass transfer in terms of Sherwood number (Sh) is

$$Sh = \left( \frac{\partial C}{\partial y} \right)_{y=0} = \left( \frac{dC_0}{dy} \right)_{y=0} + \varepsilon e^{\text{int}} \left( \frac{dC_1}{dy} \right)_{y=0}$$

$$Sh = (1+w) \frac{L_2}{\sin L_2} - w \frac{L_1}{\sin L_1} + \varepsilon \left[ (1+q) \frac{L_3}{\sinh L_3} - q \frac{L_0}{\sinh L_0} \right] e^{\text{int}} \quad (47)$$

### 1.6 Results and discussion

The numerical values in the following Figures were illustrated at  $\phi = 0.005$ ,  $n=1$ ,  $\varepsilon = 0.5$ . The numerical computations for the velocity of the fluid, velocity of the particle phase, temperature of the dusty fluid and concentration at an inclined magnetic field has been carried out for the various parameters  $t, M, f, Pr, Gr, S, \theta, \varepsilon_2, \phi, p$ .

Fig. 1 shows the velocity of the fluid and of the particle phase for different values of  $f$ . The velocity of the fluid and of the particle phase increases as  $f$  increases. It is evident that the velocity of particle phase is bit slower than the fluid phase. Fig. 2 shows the velocity of the fluid phase and of the particle phase for different values of  $M$ . It is clear that velocity decreases with increase of magnetic parameter  $M$  when  $Gr > 0$ .

Fig. 3 shows the velocity of the fluid phase and the particle phase for different values of  $\varepsilon_2$ . The velocity of the fluid and of the particle phase decreases as  $\varepsilon_2$  increases. Fig. 4 represents the velocity profiles of the fluid and of the particle phase for different values of  $\theta$ . The velocity of the fluid and of the particle phase decrease as increase in  $\theta$ , velocity of the both the fluid and of the particles decreases.

Fig.5 shows that there is a gradual decrease of fluid

and particle velocity with the increase of time. Fig.6 shows the velocity of the fluid and the velocity profiles of the particle phase for different values of  $S$ . The velocity for both the fluid and the particle phase decreases with the increase of  $S$ .

Fig. 7 shows that the increase in  $Gr$  causes gradual increase in fluid velocity and particle velocity. Fig. 8 shows the velocity of the fluid and of the particle phase for different values of  $Pr$ . The velocity of both the fluid and the particle phase decreases as  $Pr$  increases. It is evident that the velocity of particle phase bit faster than fluid phase.

Fig.9 shows the velocity of the fluid and the velocity profiles of the particle phase for different values of  $Gc$ . The velocity for both the fluid and the particle phase increases with the increase of  $Gc$ . Fig.10 shows the velocity of the fluid and the velocity profiles of the particle phase for different values of  $Sc$ . The velocity for both the fluid and the particle phase decreases with the increase of  $Sc$ .

Fig.11 shows the velocity profile of the fluid and the velocity profile of the particle phase for different values of  $Sr$ . The velocity for both the fluid and the particle phase decreases with the increase of  $Sr$ . Figs. 12 and 13 it is observed that an increase in  $\phi$  causes an increase in fluid velocity and particle velocity.

Fig.s.14 and 15 shows the temperature profile for various  $Pr$  and  $S$ . It is observed that the profile increases with the increase in  $Pr$  and  $S$ . Fig.16 shows the temperature profile for different values of  $t$ . It is observed that an increase of time causes the decrease in temperature of the fluid. Fig. 17 shows the concentration profile for various  $Sr$ . It is observed that the profile decreases with the increase in  $Sr$ . Fig. 18 and 19 shows the concentration profile for various  $S$  and  $Sc$ . It is observed that the profile decreases with the increase in  $S$  and  $Sc$ .

Fig.20 shows the Skin-friction of fluid and dust particles for different values of  $Gr$ . It is clear from the graph that the profile increases with the increase in  $Gr$ . Fig. 21 shows the Skin-friction of fluid and dust particles for different values of  $Gc$ . It is clear from the graph that the profile increases with the increase in  $Gc$ .

Fig.22 shows the Skin-friction of fluid and dust particles for different values of  $\varepsilon_2$ . It is clear from the graph that the profile decreases with the increase in  $\varepsilon_2$ . Fig. 23 shows the Skin-friction of fluid and dust particles for different values of  $M$ . It is clear from the graph that the profile decreases with the increase in  $M$ .

Fig.24 shows the Skin-friction of fluid and dust particles for different values of  $f$ . It is clear from the graph that the profile increases with the increase in  $f$ . Fig.25 shows the Skin-friction of fluid and dust particles for different values of  $\theta$ . It is clear from the graph that the profile increases with the increase in  $\theta$ .

Fig.26 shows the Skin-friction of fluid and dust particles for different values of  $Sc$ . It is clear from the graph that the profile decreases with the increase in  $Sc$ . Fig.27 shows the



Skin-friction of fluid and dust particles for different values of  $S$ . It is clear from the graph that the profile decreases with the increase in  $S$ . Fig.28 shows the Skin-friction of fluid and dust particles for different values of  $Sr$ . It is clear from the graph that the profile decreases with the increase in  $Sr$ . Fig.29 shows the Skin-friction of fluid for different values of  $\phi$ . It is clear from the graph that the profile increases with the increase in  $\phi$ .

Fig.30 and Fig.31 shows the Nusselt number for different values of  $S$  and  $Pr$ . It is observed that the profile increases with the increase in  $S$  and  $Pr$ . Fig.32, Fig.33 and Fig.34 shows the Sherwood number for different values of  $Sr$ ,  $S$  and  $Sc$ . It is observed that profile decreases with the increase in  $Sr$ ,  $S$  and  $Sc$ .

### 1.5 CONCLUSION

In this work, unsteady laminar flow of a dusty, incompressible, Newtonian, electrically conducting, viscous fluid through a porous medium of uniform cross section  $h$ , when one wall of the channel is fixed and the other is oscillating with time about a constant non-zero mean. The flow is considered in the presence of inclined magnetic field with volume fraction, heat source and porous parameter. The following conclusions are made based on the result and discussion.

- i) The fluid velocity and particle velocity profile increases with the increase in Mass concentration of dust particle, Grashof number for Heat transfer, Grashof number for mass transfer, Volume fraction of dusty particles.
- ii) The fluid velocity and particle velocity profile decrease with the increase in Magnetic Parameter, Porous Parameter, Prandtl number, time, Soret number, Schmidt number, Heat source parameter, inclined magnetic field angle.
- iii) The fluid temperature profile increases with the increase in Prandtl number, Heat source parameter and decrease in time.
- iv) The fluid concentration profile decreases with increase in Heat source parameter, Soret number, Schmidt number.
- v) There is a fall in the skin-friction of fluid and dust particle due to the increase of Schmidt number, Porosity parameter, Magnetic parameter, Heat source parameter and Soret number. The profile increases with increase of thermal Grashof number, mass Grashof number, Mass Concentration of dust particles, Volume fraction of dust particles and inclined magnetic field.
- vi) Nusselt number increases with the increase of Prandtl number and Heat source parameter.
- vii) Sherwood number decreases with the increase of Soret number, Schmidt number and Heat source parameter.

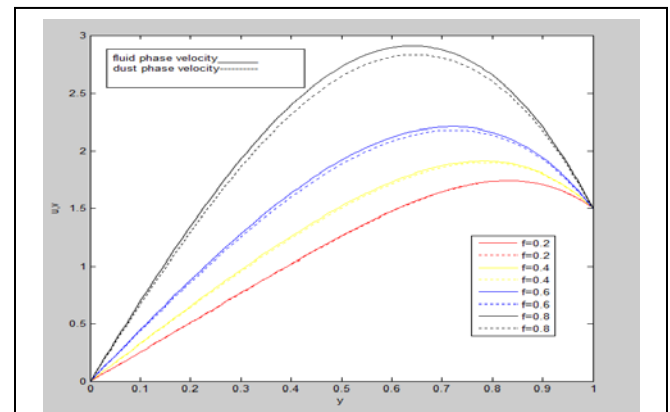


Fig. 1. Variation of velocity for different values of  $f$   
 When  $M = 2$ ,  $t = 0.001$ ,  $Pr = 0.71$ ,  $S = 2$ ,  $\epsilon_p = 1$ ,  $Sc = 0.22$ ,  
 $Gr = 5$ ,  $Sr = 2$ ,  $Gc = 1$

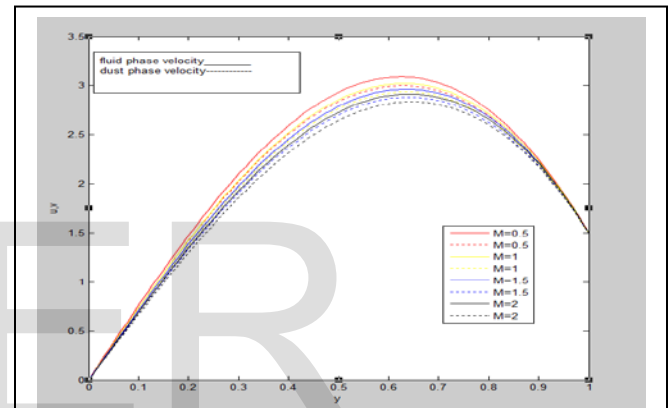


Fig.2. Variation of velocity for different values of  $M$   
 When  $f = 0.8$ ,  $t = 0.001$ ,  $Pr = 0.71$ ,  $S = 4$ ,  $\epsilon_p = 1$ ,  $Gr = 5$ ,  
 $Gc = 4$ ,  $Sc = 0.22$ ,  $Sr = 3$

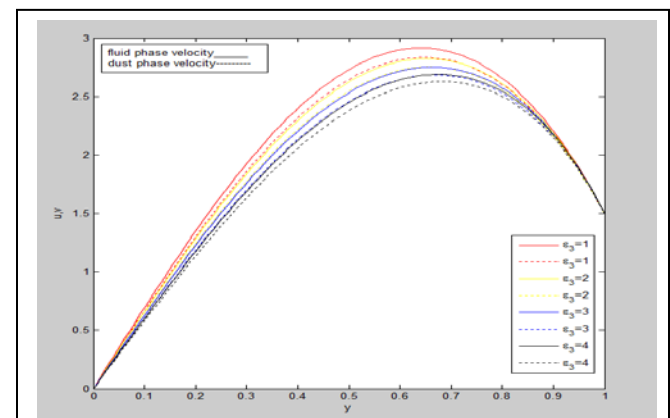


Fig.3. Variation of velocity for different values of  $\epsilon_p$   
 When  $f = 0.8$ ,  $t = 0.001$ ,  $Pr = 0.71$ ,  $S = 4$ ,  $M = 2$ ,  $S = 4$ ,  
 $Gr = 5$ ,  $Gc = 4$ ,  $Sc = 0.22$ ,  $Sr = 3$

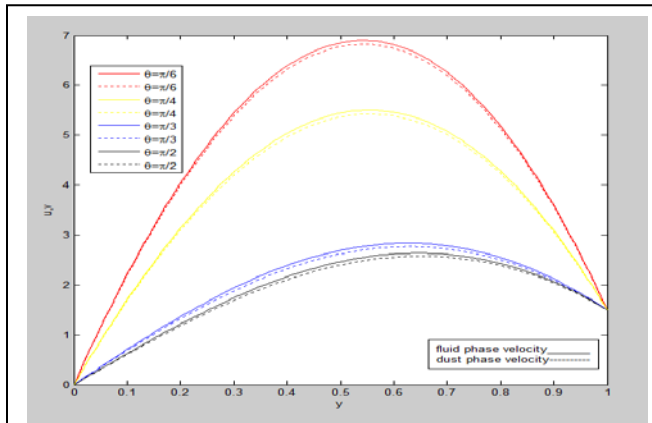


Fig.4. Variation of velocity for different values of  $\theta$   
 When  $Gr = 5, f = 0.2, t = 0.001, Pr = 0.71, S = 4, M = 4,$   
 $\epsilon_d = 1$

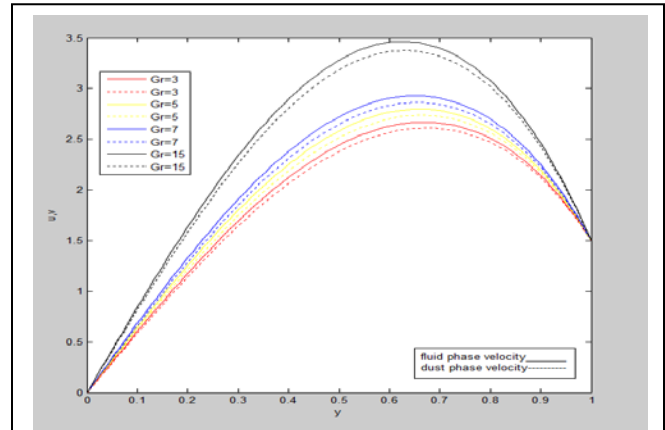


Fig.7. Variation of velocity for different values of  $Gr$   
 When  $f = 0.2, Pr = 0.71, S = 3, t = 0.001, M = 4, \epsilon_d = 1,$   
 $Gc = 4, Sc = 0.22, Sr = 3$

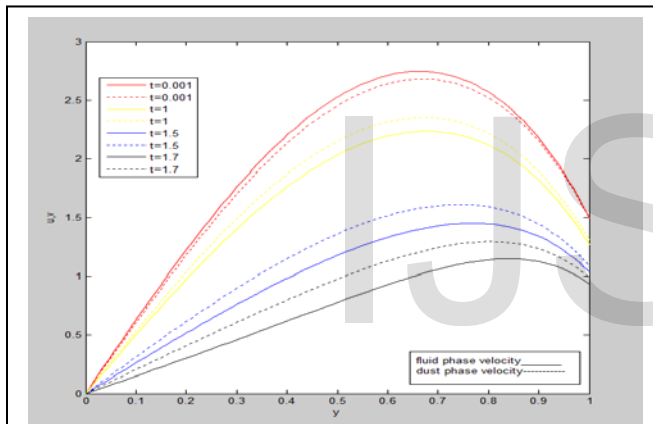


Fig.5. Variation of velocity for different values of  $t$   
 When  $Gr = 5, f = 0.2, Pr = 0.71, S = 4, M = 4, Gr = 5,$   
 $Gc = 4, Sc = 0.22, Sr = 3, \epsilon_d = 1$

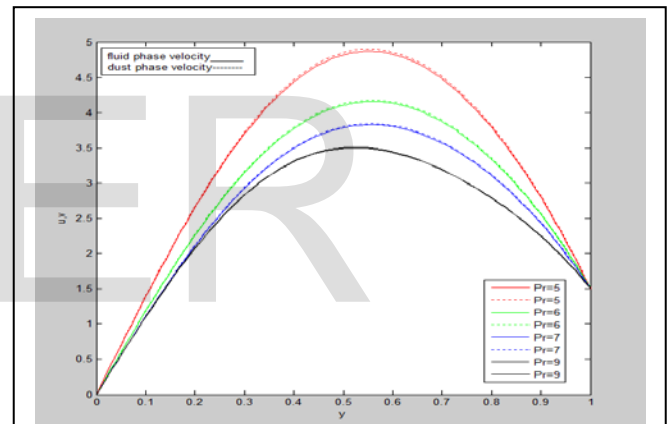


Fig.8. Variation of velocity for different values of  $Pr$   
 When  $f = 0.2, Gr = 5, S = 3, t = 0.001, M = 4, \epsilon_d = 1,$   
 $Gc = 4, Sc = 0.22, Sr = 3$

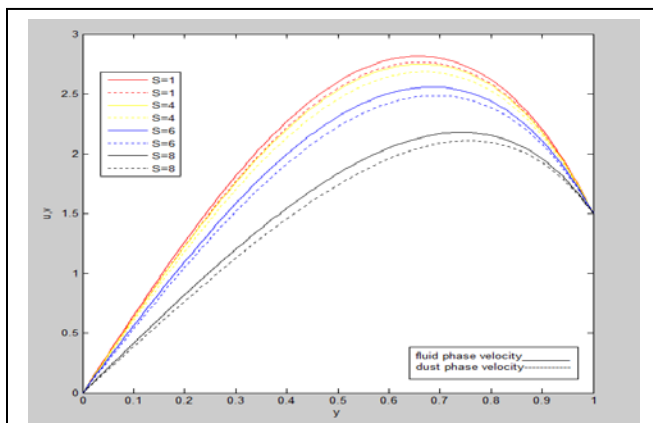


Fig.6. Variation of velocity for different values of  $S$   
 When  $f = 0.2, Pr = 0.71, Gr = 5, t = 0.001, M = 1, \epsilon_d = 1,$   
 $Gr = 5, Gc = 4, Sc = 0.22, Sr = 3$

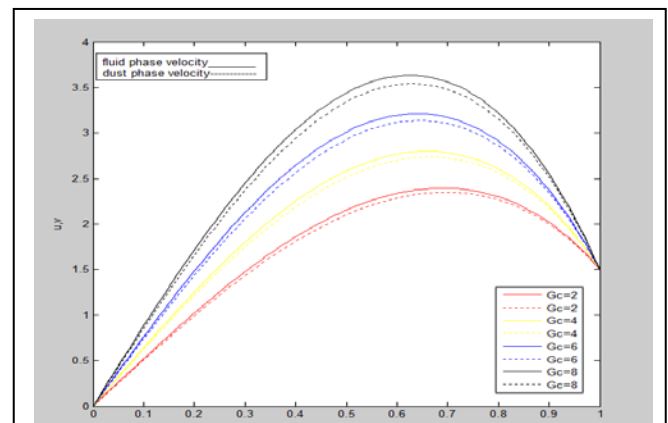


Fig.9. Variation of velocity for different values of  $Gc$   
 When  $f = 0.2, Gr = 5, S = 3, t = 0.001, M = 4, \epsilon_d = 1,$   
 $Pr = 0.71, Sc = 0.22, Sr = 3$

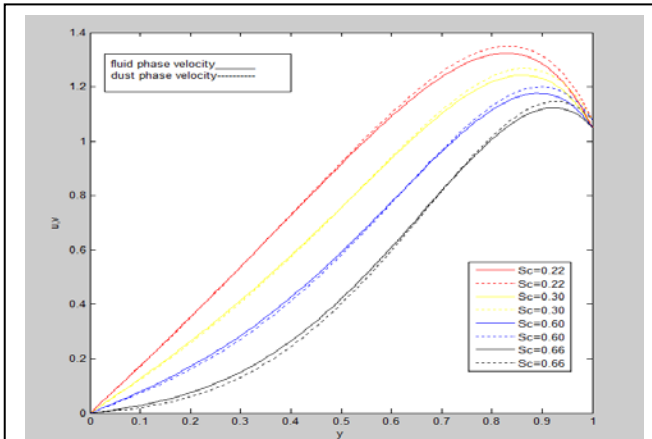


Fig.10. Variation of velocity for different values of Sc  
 When  $f = 0.2, Gr = 4, S = 3, t = 0.001, M = 4, \epsilon_2 = 1,$   
 $Pr = 0.71, Gc = 5, Sr = 3$

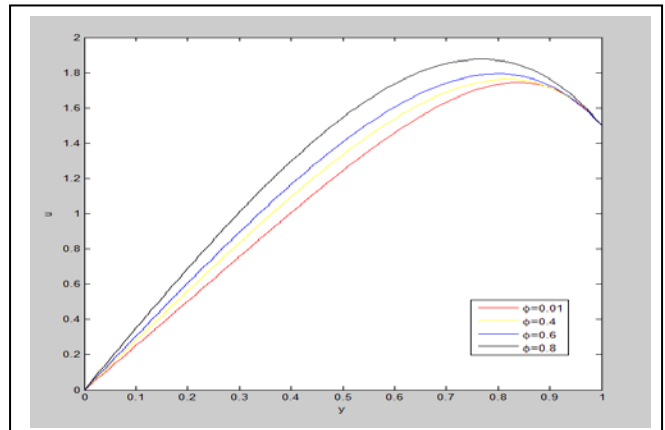


Fig.13. Variation of fluid velocity for different values  
 of  $\phi$  When  $f = 0.2, Pr = 0.71, Gr = 5, S = 3, t = 0.001,$   
 $M = 1, \epsilon_2 = 4, S = 3, Gc = 3, Sc = 0.22, Sr = 1$

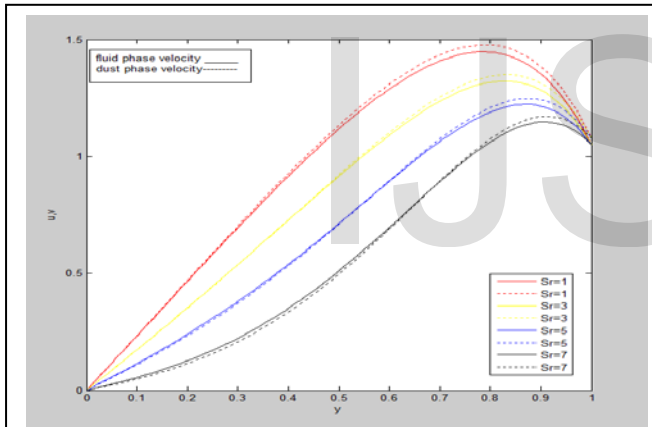


Fig.11. Variation of velocity for different values of Sr  
 When  $f = 0.2, Gr = 4, S = 3, t = 0.001, M = 4, \epsilon_2 = 1,$   
 $Pr = 0.71, Gc = 5, Sc = 0.22$

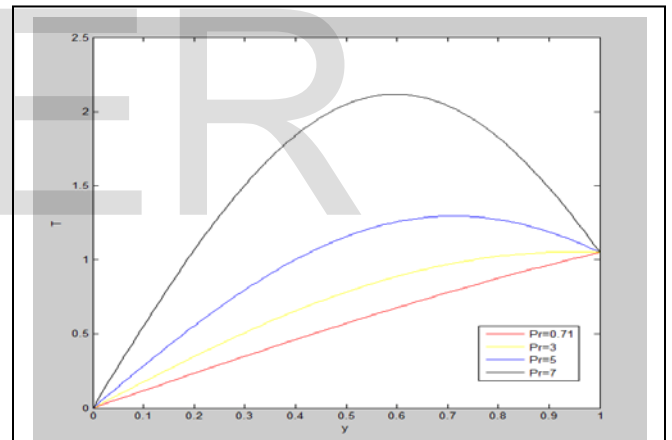


Fig.14. Variation of temperature field for different  
 values of Pr When  $S=1, t = 0.001, \epsilon = 0.5$

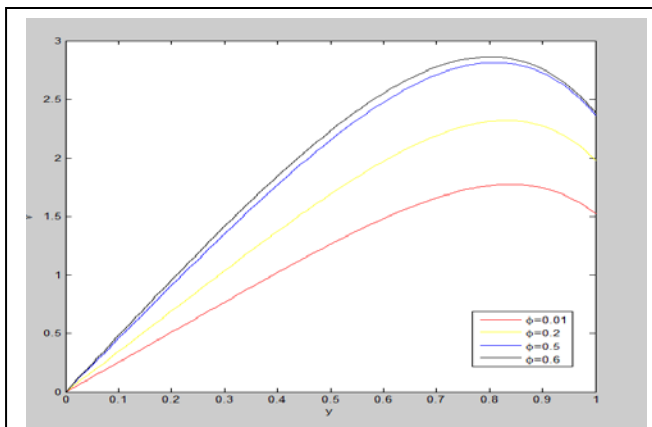


Fig.12. Variation of particle velocity for different values  
 of  $\phi$  When  $f = 0.2, Pr = 0.71, Gr = 5, S = 3,$   
 $t = 0.001, M = 1, \epsilon_2 = 4, S = 3, Gc = 3, Sc = 0.22, Sr = 1$

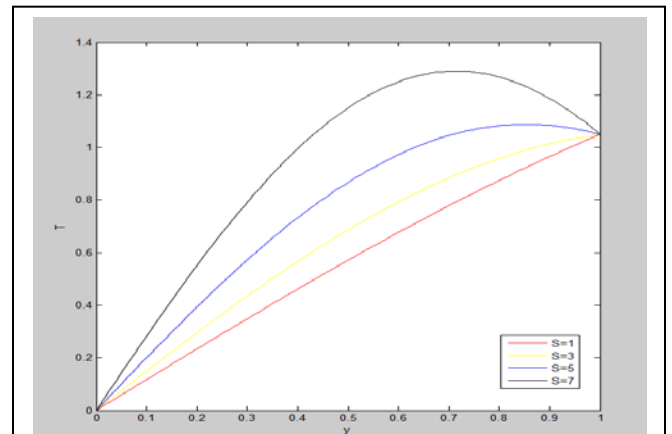


Fig.15. Variation of temperature field for different values  
 of S When  $Pr = 0.71, t = 0.001, \epsilon = 0.5$



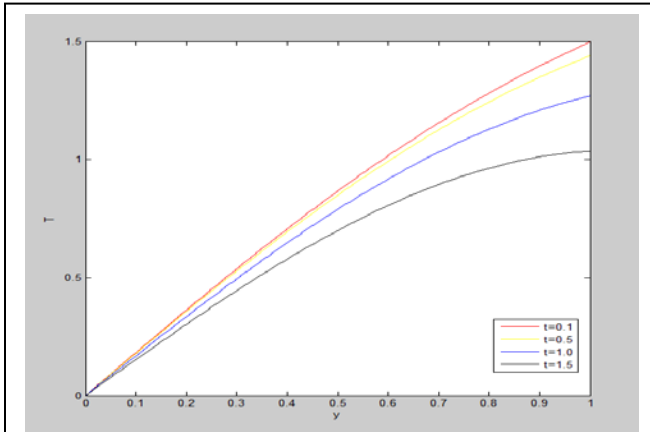


Fig.16. Variation of temperature field for different values of  $t$  When  $Pr = 0.71$ ,  $S = 5$ ,  $\epsilon = 0.5$

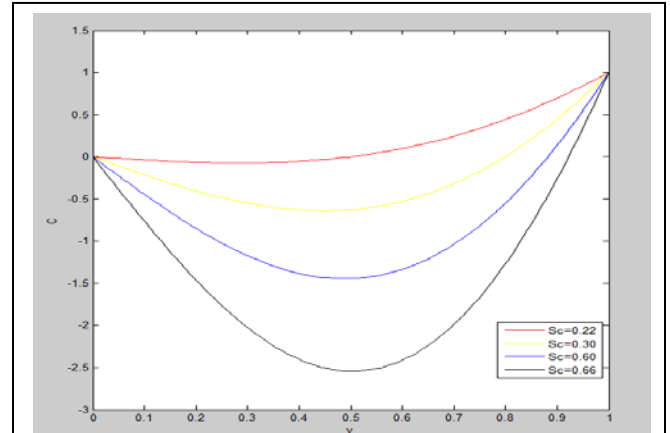


Fig.19. Variation of concentration field for different values of  $Sc$  When  $Pr = 0.71$ ,  $Sr = 3$ ,  $M = 2$ ,  $\epsilon = 0.05$ ,  $S = 3$

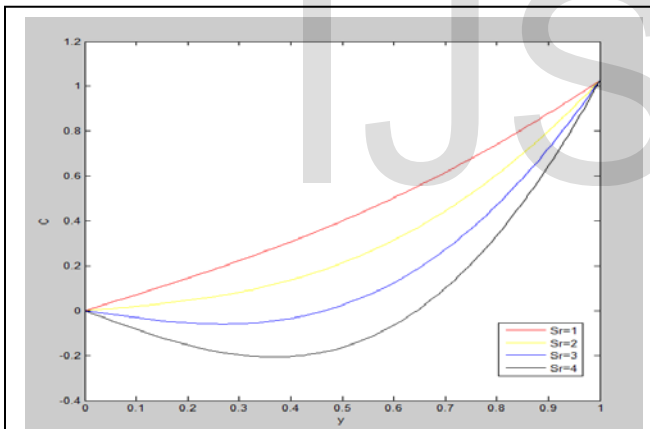


Fig.17. Variation of concentration field for different values of  $Sr$  When  $Pr = 0.71$ ,  $S = 3$ ,  $\epsilon = 0.05$ ,  $Sc = 0.22$

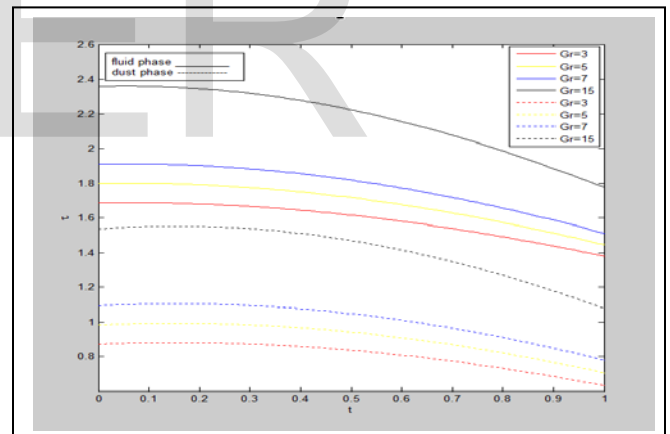


Fig.20. Skin-friction for different values of  $Gr$  when  $Gr = 5$ ,  $Gc = 1$ ,  $f = 0.2$ ,  $Pr = 0.71$ ,  $Sr = 2$ ,  $M = 4$ ,  $\epsilon = 0.5$ ,  $S = 3$ ,  $\epsilon_2 = 1$ ,  $Sc = 3$

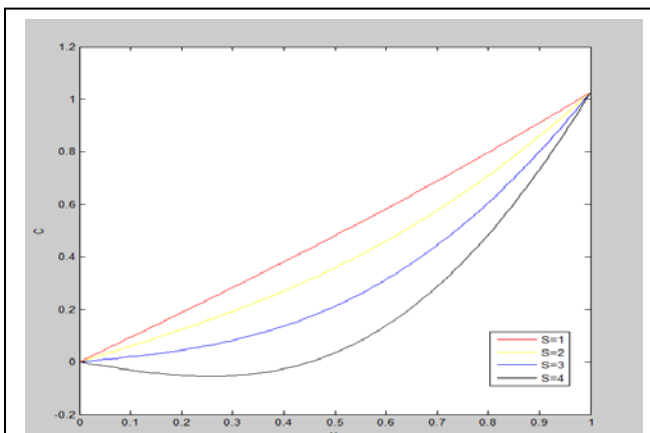


Fig.18. Variation of concentration field for different values of  $S$  When  $Pr = 0.71$ ,  $Sr = 2$ ,  $\epsilon = 0.05$ ,  $Sc = 0.22$

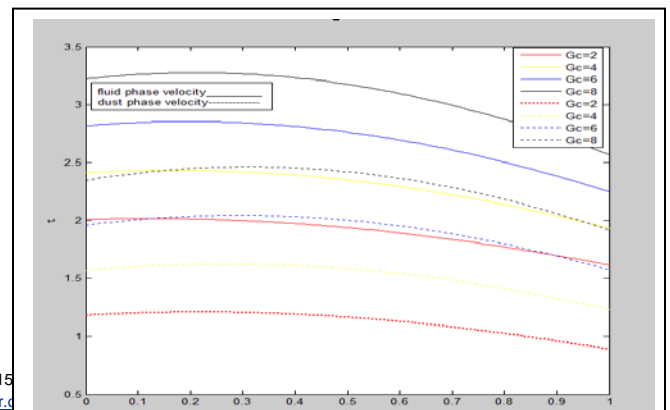


Fig.21. Skin-friction for different values of  $Gc$  When  $Gr = 5$ ,  $f = 0.2$ ,  $Pr = 0.71$ ,  $Sr = 2$ ,  $M = 4$ ,  $\epsilon = 0.5$ ,  $S = 3$ ,  $\epsilon_2 = 1$ ,  $Sc = 3$

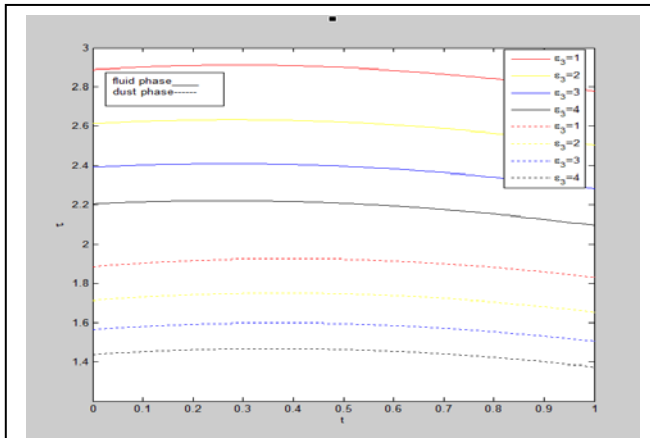


Fig.22. Skin-friction for different values of  $\epsilon_3$  When  $Gr = 5, Gc = 5, f = 0.8, Pr = 0.71, Sr = 2, M = 4, \epsilon = 0.05, S = 2, Sc = 2$

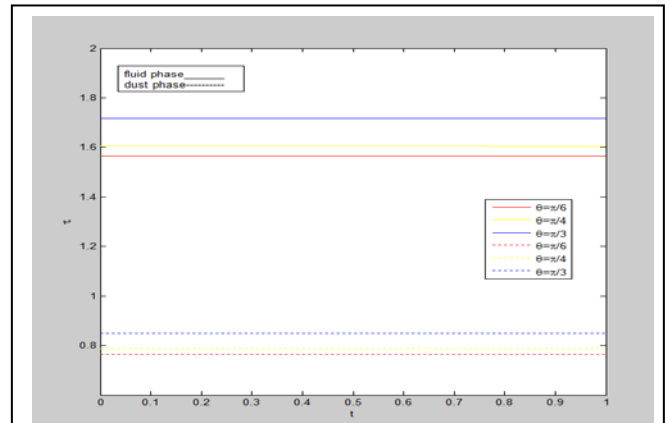


Fig.25. Skin-friction of fluid for different values of  $\theta$  When  $Gr = 5, Gc = 3, Pr = 0.71, f = 0.2, Sr = 1, \epsilon_3 = 1, \epsilon = 0.05, S = 1, Sc = 0.22$

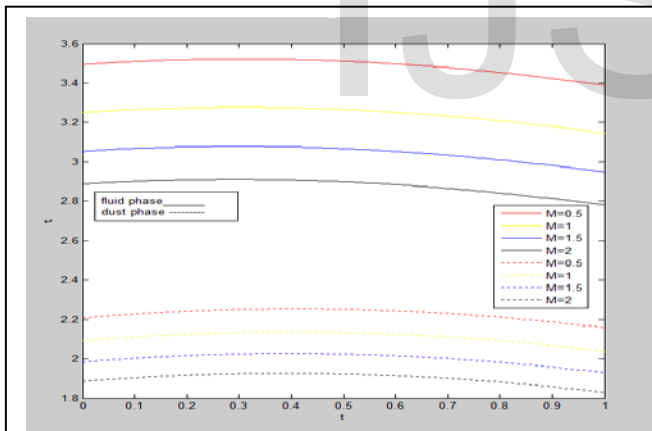


Fig.23. Skin-friction for different values of  $M$  When  $Gr = 5, Gc = 5, f = 0.8, Pr = 0.71, Sr = 1, \epsilon_3 = 1, \epsilon = 0.05, S = 2, Sc = 0.22$

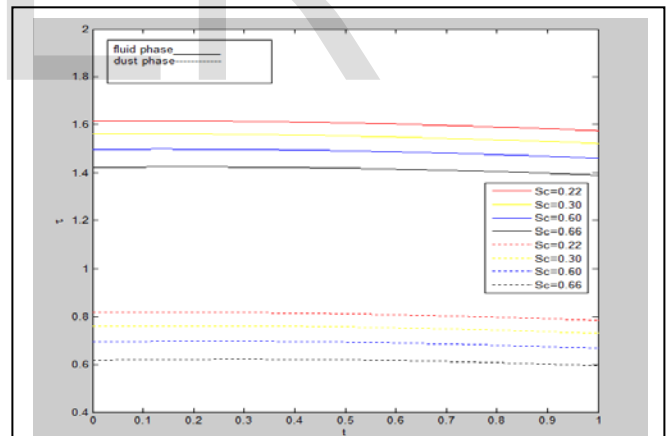


Fig.26. Skin-friction of fluid for different values of  $Sc$  When  $Gr = 5, Gc = 3, Pr = 0.71, f = 0.2, M = 1, Sr = 1, \epsilon_3 = 1, \epsilon = 0.05, S = 1$

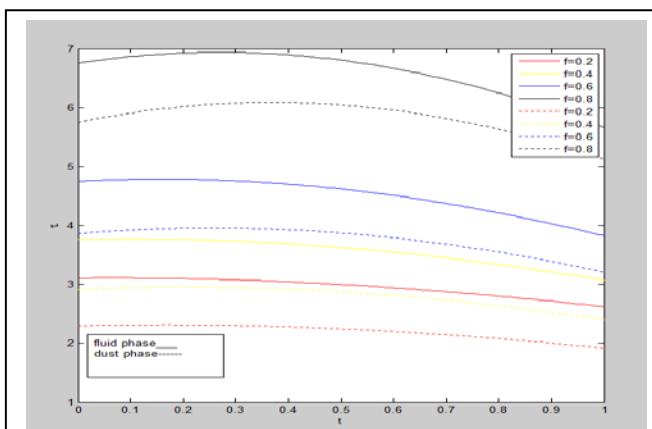


Fig.24. Skin-friction for different values of  $f$  When  $Gr =$

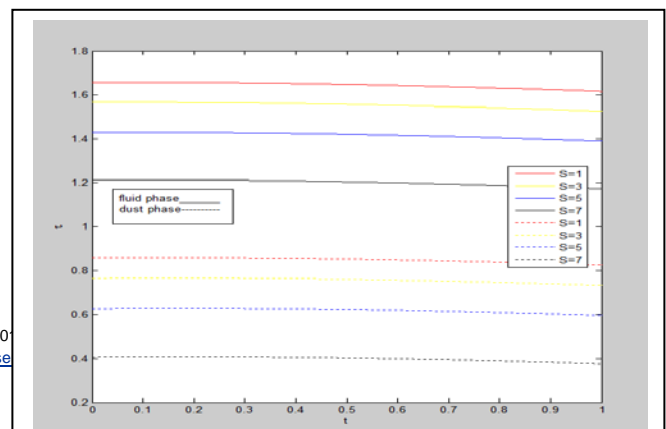


Fig.27 Skin-friction of fluid for different values of  $S$

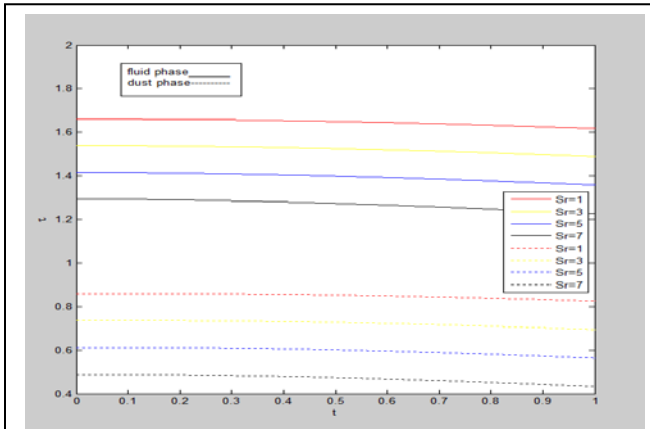


Fig.28. Skin-friction of fluid for different values of  $Sr$   
 When  $Gr = 5, Gc = 3, Pr = 0.71, f = 0.2, M = 1, Sr = 1,$   
 $\epsilon_3 = 4, \epsilon = 0.05, Sc = 0.22$

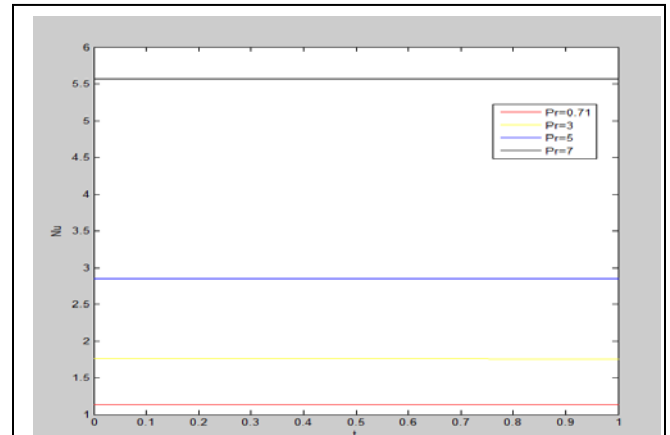


Fig.31. Nusslet number for different values of  $Pr$   
 When  $S = 1, \epsilon = 0.05$

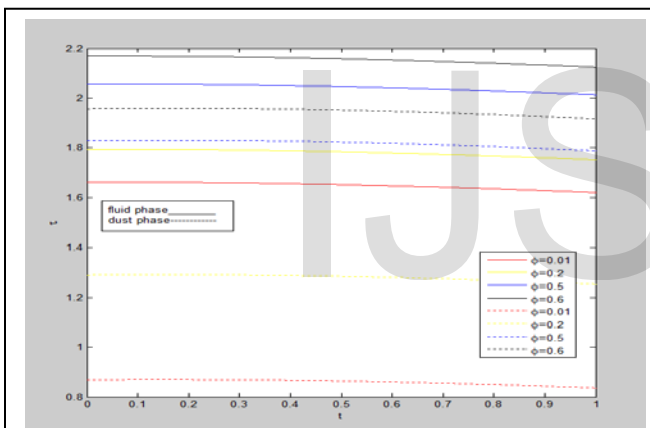


Fig.29. Skin-friction of fluid for different values of  $\phi$   
 When  $Gr = 5, Gc = 3, Pr = 0.71, f = 0.2, M = 1, Sr = 1,$   
 $\epsilon_3 = 4, \epsilon = 0.05, Sc = 0.22, Sr = 1$

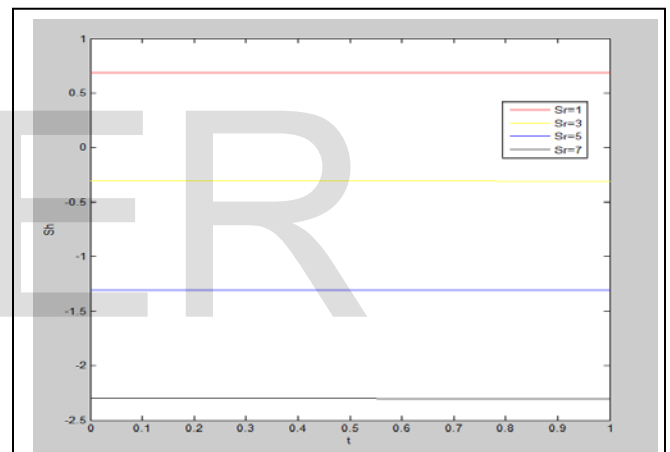


Fig.32. Sherwood number for different values of  $Sr$   
 when  $Pr = 0.71, Sc = 0.22, S = 2, \epsilon = 0.05$

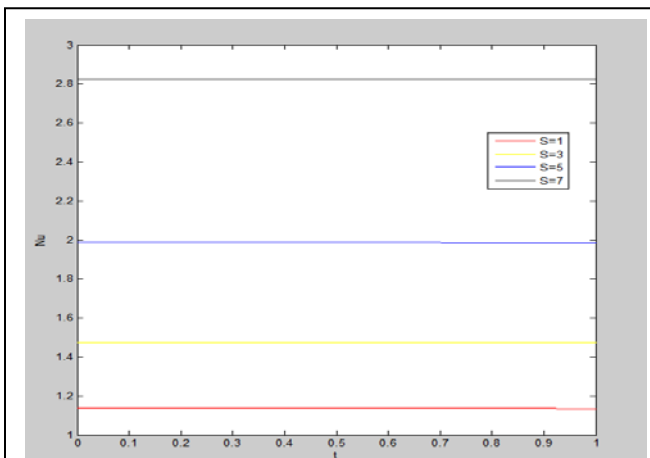


Fig.30. Nusslet number for different values of  $S$   
 when  $Pr = 0.71, \epsilon = 0.05$

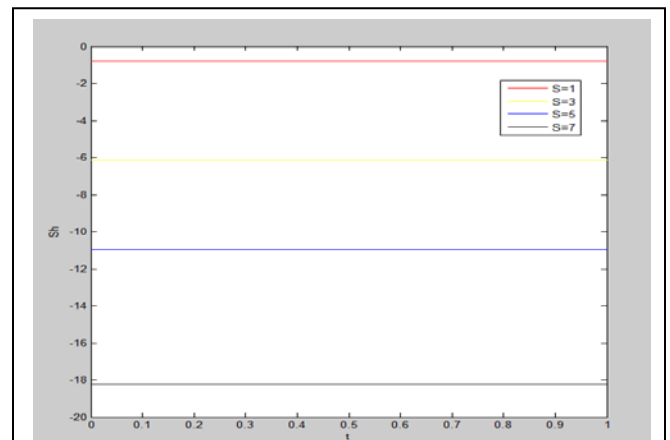


Fig.33. Sherwood number for different values of  $S$   
 when  $Pr = 0.71, Sc = 0.22, Sr = 4, \epsilon = 0.05$

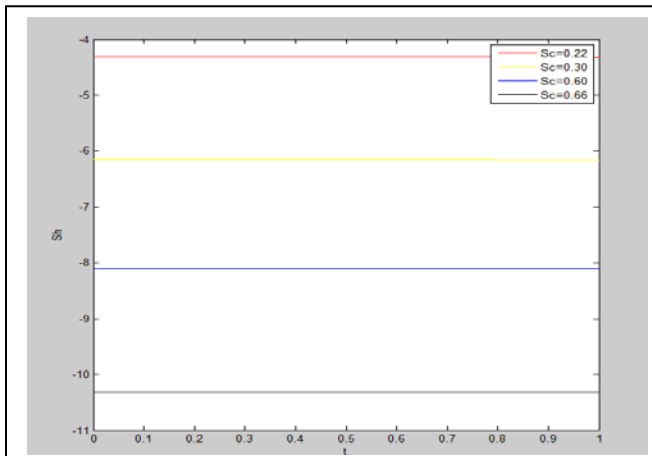


Fig.34. Sherwood number for different values of Sc when  $Pr = 0.71$ ,  $S = 3$ ,  $Sr = 3$ ,  $\varepsilon = 0.03$

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