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Solving Blasius Problem by Adomian Decomposition Method

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Abstract - Using the Adomian decomposition method we solved the Blasius problem for boundary-layer flows of pure fluids (non-porous domains) over a flat plate. We obtained the velocity components as sums of convergent series. Furthermore we constructed the interval of admissible values of the shear-stress on the plate surface.

Index Terms - Convergent series, Decomposition technique, Fluid flow, Shear-stress.

Nomenclature

1. u velocity in the x-direction
2. u_0 velocity of the free stream
3. v velocity in the y-direction
4. x horizontal coordinate
5. y vertical coordinate
6. μ viscosity coefficient
7. ϱ density
8. $\nu = \frac{\mu}{\rho}$ kinematic viscosity of the fluid

1 INTRODUCTION

The problem of flow past a flat plate is one of interesting problems in fluid mechanics which was first solved by Blasius [5] by assuming a series solutions . Later, numerical methods were used in [7] to obtain the solution of the boundary layer equation. In [2] the first derivative with respect to y of the velocity component in the x direction at the point $y=0$ for the Blasius problem is computed numerically for the estimation of the shear-stress on the plate surface. Later in [9] one solved the problem above by assuming a finite power series where the objective is to determine the power series coefficients.

The purpose of this study is to obtain the solutions for the Blasius problem for two dimensional boundary layer using the Adomian decomposition technique and to compute the admissible values of the shear-stress on the wall, imposing the constraint on the first derivative with respect to y of the velocity component in the x direction at the point $y=0$.

2 MATHEMATICAL MODEL

The physical model considered here consists of a flat plate parallel to the x- axis with its leading edge at $x=0$ and infinitely long down stream with constant component u_0 of the velocity. For the mathematical analysis we assume the properties of the fluid such as viscosity and conductivity, to a first approximation, are constant. Under these assumptions the basic equations required for the analysis of the physical phenomenon are the equations of continuity and motion. According to the Boussinesq approximation these equations get the following expressions [2]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} \quad (2)$$

with the boundary conditions imposed on the flow in [2]

$$\psi=\frac{\partial \psi}{\partial y}=0,\ y=0\ ,\ \lim_{y\to\infty}\frac{\partial\psi}{\partial y}=u_0 \quad (3)$$

Where ψ is a stream function related to the velocity components as:

$$u=\frac{\partial\psi}{\partial y}, v=\frac{-\partial\psi}{\partial x} \quad (4)$$

3 ANALYTICAL SOLUTION and CONVERGENCE RESULTS

In this section we provide the analytical solutions ,i.e. the fluid velocity components as sums of convergent series using the Adomian decomposition technique and compute the admissible values of the shear-stress on the plate surface.

Consider the stream function ψ

$$\psi(x,y)=\sqrt{\nu u_0 x}f(\eta), \qquad \eta=y\sqrt{\frac{u_0}{\nu x}} \quad (5)$$

Where f is a function three times continuously differentiable on the interval $[0,\eta_0]$ and η_0 a constant positive real. Then (1) and (2) with (3) are transformed as

$$f'''+\frac{1}{2}ff''=0\ ,\ \mathrm{f}\ (0)=f'(0)=0\ ,\ f'(+\infty)=1 \quad (6)$$

where $(\ .\ )'$ stands for $\frac{d(.)}{d\eta}$

Definition 3.1

The problem (6) is called the Blasius problem for boundary-layer flows of pure fluids (non-porous domains) over a flat plate.

Let us transform (6) into the nonlinear integral equation. For this purpose, setting $g'(\eta)=f(\eta)$ we can write the equation in (6) as

$$g''''+\frac{1}{2}g'g'''=0 \quad (7)$$

Multiplying by $e^{\frac{1}{2}g}$ and integrating the result from 0 to η we reduce (7) to

$$g'''=Ke^{-\frac{1}{2}g}\ ,\ \text{where } K=g'''(0)e^{\frac{1}{2}g(0)} \quad (8)$$

Integrating three times (8) from 0 to η,τ,σ and taking into account the boundary conditions in (6) we reduce (8) to the nonlinear integral equation

$$g(\eta)-K\int_0^\eta\int_0^\tau\int_0^\sigma e^{-\frac{1}{2}g(S)}\,ds\,d\sigma\,d\tau=a_0 \quad (9)$$

$$a_0=g(0)=const$$

which is a functional equation

$$g-N(g)=a_0\ ,\ \text{where} \quad (10)$$

$$N(g)=K\int_0^\eta\int_0^\tau\int_0^\sigma e^{-\frac{1}{2}g(S)}\,ds\,d\sigma\,d\tau\ , \quad (11)$$

$$K=\left[\int_0^{+\infty}e^{-\frac{1}{2}g(S)}ds\right]^{-1} \quad (12)$$

Here $N(g)$ is a nonlinear operator from a Hilbert space H into H. In [4] G. Adomian has developed a decomposition technique for solving nonlinear functional equation such as (10). We assume that (10) has a unique solution. The Adomian technique allows us to find the solution of (10) as an infinite series $g = \sum_{n\geq 0} g_n$ using the following scheme:

$$g_0 = a_0$$

$$g_1 = A_0$$

$$\vdots$$

$$g_{n+1} = A_n(g_0, g_{1,\dots,}g_n)$$

$N(g) = \sum_{n\geq 0} A_n$, where

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i\geq 0} \lambda^i g_i\right)\right]_{\lambda=0}$$

$$n = 0,1,2, \dots.$$

The proofs of convergence of the series $\sum_{n\geq 0} g_n$ and

$\sum_{n\geq 0} A_n$ are given below. Without loss of generality we set $a_0 = 0$ and we have the following scheme:

$$\frac{d^n}{dg^n} N(g_0) = \left(-\frac{1}{2}\right)^n N(g_0) \qquad (13)$$

$$g_0 = 0 \qquad (14)$$

$$g_1 = \frac{1}{6} K\eta^3 \equiv b_1 K\eta^3 \qquad (15)$$

$$g_2 = -\frac{1}{72}(K\eta^3)^2 \equiv b_2(K\eta^3)^2 \qquad (16)$$

$$g_3 = \frac{1}{576}(K\eta^3)^3 \equiv b_3(K\eta^3)^3 \qquad (17)$$

$$g_4 = -\frac{1}{3888}(K\eta^3)^4 \equiv b_4(K\eta^3)^4 \qquad (18)$$

$$g_5 = \frac{125}{2985984}(K\eta^3)^5 \equiv b_5(K\eta^3)^5 \qquad (19)$$

$$g_6 = -\frac{569}{79626240}(K\eta^3)^6 \equiv b_6(K\eta^3)^6. \qquad (20)$$

By induction, we have

$$g_{n+1} = \frac{1}{n!}\sum_{k=0}^{n-1} C_{n-1}^k (n-k)!\, g_{n-k}\left[\frac{d^k}{d\lambda^k}\frac{dN}{dh}(h)\right]_{\lambda=0}$$

i.e $g_{n+1} \equiv b_{n+1}(K\eta^3)^{n+1},\ n \geq 1$ (21)

$$h = \sum_{i\geq 0} \lambda^i g_i$$

where the b_n are real numbers. Then we obtain

$$|K - 0.332|\sqrt{\frac{\mu\rho u_0^3}{x}} < 10^{-n} \Leftrightarrow$$

$$0.332 - \sqrt{\frac{x}{\mu\rho u_0^3}} \times 10^{-n} < K < 0.332 + \sqrt{\frac{x}{\mu\rho u_0^3}} \times 10^{-n}$$

We arrive at the following result

Lemma 4.1

The admissible values of the shear-stress

$$\Gamma = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = K\sqrt{\frac{\mu\rho u_0^3}{x}}$$

on the plate surface belong to the open interval

$$\left]0.332\sqrt{\frac{\mu\rho u_0^3}{x}} - 10^{-n}; 0.332\sqrt{\frac{\mu\rho u_0^3}{x}} + 10^{-n}\right[\qquad (22)$$

for each given value of $x > 0$ and for the given approximation precision depending on $n \in \mathbb{N}^*$

5 CONCLUSION

In this paper, we have investigated the analytical solutions for the Blasius problem which are the sums of convergent series, using the Adomian decomposition technique. Then we estimated the error by approximating the exact values of the shear-stress on the plate surface obtained in this paper by the approximate values of the shear-stress obtained in [2]. Doing so, we constructed the interval of admissible values of the shear-stress on the plate surface.

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