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Singularities in five dimensional Vaidya space time with exponential mass function

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Abstract: This paper deals with nature of singularities for imploding radiations in Five dimensional Vaidya space times. We will examine the nature of singularities with exponential mass function. We observe strong curvature naked singularity is the outcome.

Keywords: Gravitational collapse, Black holes, Naked singularities.

1. INTRODUCTION

Cosmic Censorship Hypothesis (CCH) is one of the famous unsolved problem in General Relativity and Astro Physics. R. Penrose;[1] proposed CCH that the space time singularity must not be visible to any observer. Even after three decades of efforts of formulating properly and proving CCH, it becomes apparent[2] that any generally acceptable formulation for it has not emerged. Number of attempts [2] have been made[3] to construct examples that counter the spirit of this hypothesis- examples of naked singularities. Occurrence of naked singularities, in some examples of spherically symmetric gravitational collapse[4] is a well-established phenomenon.

The development of locally naked singularities in a variety of cases such as collapse of radiation shell, spherically symmetric self-similar collapse of perfect fluid. Collapse of spherical inhomogeneous dust cloud[5] and other physically relevant situation is well known. In all these families of non-space like geodesic emerge from the naked singularity, therefore these cases can be considered as serious examples of locally naked singularity of strong curvature type.

along a non-spacelike curve meeting the singularity in the past in the limit of approach to the singularity.

Kaluza and Kein [15] introduced the idea that space time shoud be extended to higher dimension so as to unify gravity and electromagnetism.Five dimentional space time is particularly more important because ten and eleven dimensional super gravity theories yield solutions where a five dimensional space time results after dimensional reduction [16].

2. Five dimensional Vaidya Space-times

Five dimensional Vaidya space-time describing imploding radiation is [13]

$$ds^2 = -\left(1 - \frac{m(v)}{r^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2 \quad (1)$$

Where $d\Omega^2 = d{\theta_1}^2 + \sin^2\theta_1 \left(d{\theta_2}^2 + \sin{\theta_2}^2 d{\theta_3}^2\right)$ is the metric of the 3- sphere, v is a null coordinate with $-\infty < v < \infty$, $m(v)$ is an arbitrary but non negative increasing mass function.

The stress- Energy tensor for radial flux of radiation is

$$T_{ab} = \frac{3}{2r^3}\frac{dm}{dv} U_a U_b \quad (2)$$

Where

$U_a = -\delta_a^v,\ U_a U^b = 0$ *and* a, b take values $v, r, \theta_1, \theta_2, \theta_3$ Further, We are using units in which $8\pi G = c = 1$. We note that v= 0, r = 0 is the singularity of the metric (1). We note that, for the weak energy condition to be satisfied, we require that $\frac{dm}{dv}$ be non negative. The Kreetschmann scalar $K = R_{abcd}R^{abcd}$, *where* R_{abcd} is the Riemann tensor for the metric (1) takes the form

$$K = \frac{72m^2(v)}{r^8} \quad (3)$$

Which diverges along r = 0 establishing a scalar polynomial singularity. Initially, higher dimensional space-time is flat and empty for v < 0. Now for v> T, $\frac{dm}{dv} = o$, $m(v) > 0$. The metric for v = 0 to v = T is higher dimensional Vaidya metric and for v $> T$ we have the higher dimensional SchwarZchild solution. The first shell arrives at r =0 at time v = 0 and the final at v = T. the growing centeral singularity is developed at r = 0.

3. Nature of singularities in higher dimensional Vaidya Space-time.

To discuss the nature of singularity it is sufficient to consider the family of null geodesics. Let $K^a = \frac{dx^a}{dk}$ be the tangent vector to the null geodesic, where k is an affine parameter. The geodesic equation, on using the null condition $K^a K_a = 0$ take the form

$$\frac{dK^v}{dk} + \frac{m(v)}{r^3}(K^v)^2 = 0 \quad (4)$$

$$\frac{dK^r}{dk} + \frac{1}{2r^2}\frac{dm}{dv}(K^v)^2 = 0 \quad (5)$$

Folowing [8,9], we introduce

$$K^v = \frac{P}{r} \quad (6)$$

And from the null condition we obtain

$$K^r = \left(1 - \frac{m(v)}{r^2}\right)\frac{P}{2r} \quad (7)$$

Where $P(v, r)$ obeys the differential equation

$$\frac{dP}{dk} - \left(1 - \frac{3m(v)}{r^2}\right)\frac{P^2}{2r^2} = 0 \quad (8)$$

In general Eq.(8) may not yield an analytical solution. We consider radial null geodesic for the metric(10, by virtue of Eqs. (6) and (7), satisfy

$$\frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{m(v)}{r^2}\right) \quad (9)$$

We observe that this differential equation has a singularity at r = 0, v = 0 further, the real tangent to the null geodesic establishes that a family of null geodesic going in to the central singularity is visible to an asymptotic observer. This will establish that the central singularity is locally naked. The character of the singularity depends on the exact form of $m(v)$. When $m(v) \sim \lambda v^2$ it is found that the space time is self-similar[18], further, it admits a homothetic Killing vector and nature of singularity can be decided at ease. However, self-similarity is a strong geometric condition on the space time. It is therefore, of interest to us to examine more general forms of the function $m(v)$. We construct specific example which satisfy weak energy condition but develop strong curvature naked singularities.

4. Naked Singularity in Five dimensional Vaidya Space-time for exponential mass Function.

Here we study the situation when mass function is an exponential function. Our specific choice of mass function is $m(v) = e^{\lambda v^2} - 1$ where $\lambda > 0$ is constant. Initially at $v = 0$ $m(v) = 0$ and

$\frac{dm}{dv} = 2v\lambda e^{\lambda v^2} \geq 0$ satisfy the weak energy condition.

Using(1) radial null geodesic equation is

$$\frac{dv}{dr} = \frac{2}{\left(1 - \frac{e^{\lambda v^2} - 1}{r^2}\right)} \quad (3)$$

$$\frac{dv}{dr} = \frac{2}{\left(1 - \frac{(e^{\lambda v^2} - 1)}{r^2}\right)\frac{\lambda v^2}{r^2}} \quad (4)$$

We define new parameter,

$X = \frac{v}{r}$, $and\ set\ X_0 = \lim_{\substack{v \to o \\ r \to 0}} \frac{v}{r} \quad \lim_{\substack{v \to o \\ r \to 0}} \frac{dv}{dr}$

Taking limit as $v \to o, r \to o$

$\lambda X_0^3 - X_0 + 2 = 0$ .It can be easily observed that X_0 is real if $0 < \lambda \leq \frac{1}{27}$. Thus future directed outgoing null geodesics from the central singularity are observable to an asymptotic observer making singularity naked.

We now examine the strength of this singularity by evaluating the behavior of scalar $\Psi = R_{ab}\ U^a U^b$

Where R_{ab} is the Ricci tensor and where the geodesic terminate at $\lambda = 0$. Now, we consider the following condition:

$\lim_{k \to 0} k^2 \Psi > 0$ This condition is equivalent to the termination of a geodesic in a strong sense of Tipler (cf[19])

$$\lim_{k \to 0} k^2 \Psi = \lim_{k \to 0} \frac{3}{2r}\frac{dm}{dv}\left(\frac{kP}{r^2}\right)^2$$

$$\lim_{k \to 0} k^2 \Psi = \lim_{k \to 0} \frac{3}{r}\lambda v e^{\lambda v^2}\left(\frac{kP}{r^2}\right)^2$$

Using the fact that as singularity is approached, $k \to 0\ and\ r \to 0$, and using L' Hospital Rule, $\lim_{k \to 0}\left(\frac{kP}{r^2}\right)^2 = \frac{2}{1+\lambda X_0^2}$

For $P_0 = \infty$ and , $\lim_{k \to 0}\left(\frac{kP}{r^2}\right)^2 = \frac{2}{1-\lambda X_0^2}$, For $P_0 \neq \infty$

Where $P_0 = \lim_{k \to 0} P$

Thus $\lim_{k \to 0} k^2 \Psi = \frac{12\lambda X_0}{(1+\lambda X_0^2)^\wedge 2} > 0\ for\ P_0 = \infty$

And $\lim_{k \to 0} k^2 \Psi = \frac{12\lambda X_0}{(1-\lambda X_0^2)^\wedge 2} > 0\ for\ P_0 \neq \infty$

Where $X_0 \neq \frac{1}{\lambda}$. Thus along radial null geodesic, the curvature condition is satisfied. Thus ,this singularity is a strong curvature singularity.

CONCLUSIONS

Thus, we have seen that strong naked singularity is the outcome of the Five dimensional Vaidya space-times, when mass function selected is exponential in nature. Further, investigation are necessary with other forms of mass function satisfying the required energy conditions.

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