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# Settlement Analysis of Single Granular Pile with Stiffened Top

K.S. Grover, J.K. Sharma, M.R. Madhav

**Abstract**— As per the various theoretical and experimental investigations, an axial load applied at the top of long granular pile is limited by bulging within a depth of 2 to 3 diameters from the top. The radial displacements of granular pile (GP) are prevented by the confining stresses generated by the surrounding soil. The capacity and thus the overall response of GP can be enhanced by restraining or strengthening the top 2 to 3 diameter length of the granular material where the lateral confining stresses are the least. The response of single GP with consideration of radial and vertical displacements compatibility along its interface is obtained with the provision of stiffer upper part of GP vulnerable to bulging based on elastic continuum approach. The overall responses of a single GP with stiffened top in terms of settlement influence factors namely for top and tip settlement influence factors and radial displacement influence factor are evaluated in the present paper.

**Index Terms**— **Granular pile, Stiffened Top**, Continuum approach, Compatibility of displacements, Vertical and Radial displacements, Settlement Influence Factor, Radial Influence Factor

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## 1 Introduction

Mattes & Poulos (1968) and Poulos & Mattes (1969) have presented solutions for settlement of floating and bearing compressible piles based on the continuum approach which can be applied to a granular pile. They presented their results considering only vertical displacement compatibility between a compressible pile and soil. Sharma and Madhav (1999) presented response of a single granular pile with consideration of compatibility of radial and vertical displacements along its interface within elastic continuum. The study by Sharma and Madhav (1999) also showed that when an axial load is applied at the top of granular pile (GP), it produces the bulge within a depth of 2 to 3 diameters of GP. The same result was established by the various experimental investigations earlier. The present work deals with the analysis of a single granular pile with stiffened top portion of granular pile using the elastic continuum approach as used by Sharma and Madhav (1999). The radial and vertical displacements of granular pile, with its top is stiffened, is evaluated based on Mindlin's equations for horizontal and vertical displacements due to both horizontal and vertical point forces within the elastic continuum. The overall responses of a single GP with stiffened top in terms of settlement influence factors for top and tip settlement influence factors and radial displacement influence factor are evaluated.

## 2 Problem Definition & Method of Analysis

Fig 1 shows a granular pile of length, L, and diameter, d=2a, under an axial load, P. The granular pile is compressible with a deformation modulus $E_{gp}$. The soft soil is characterized by its deformation modulus $E_s$, and Poisson ratio $\nu_s$. The relative stiffness of granular pile is defined as the ratio of modulus of deformation of GP to that of the soil, $K_{gp} = E_{gp}/E_s$. In the present analysis it is assumed that top portion of length $\eta \times L/d$, has relative stiffness, $E_{stgp}$, of granular pile $\chi$ times higher than that of the lower portion, i.e., $E_{gp}$. Therefore, the top portion has been stiffened with relative stiffness, $K_{stgp}$ of granular pile, $= \chi\, K_{gp}$ as compared to $K_{gp}$ for the lower portion. It is assumed that Poisson ratio $\nu_{gp}$ is constant throughout the GP. The pile base is assumed as smooth and rigid. The soil is assumed to be homogeneous, isotropic and linearly elastic. The pile is discretized in, 'nl', number of elements and each element is subdivided into sub-elements axially and radially for the purpose of numerical integration. The values of shear stress, $\tau$, and radial stress, $\sigma_r$ are evaluated.

The vertical displacements of soil at the midpoint of the periphery of each element and at the center of base due to the influence of shear, base and radial stresses in matrix form following Sharma and Madhav (1999) are

(1)

$$\left\{\rho^{nsv}\right\} = \left\{\frac{\rho^{sv}}{d}\right\} = \left[IC^{svv}\right]\left\{\frac{\tau}{E_s}\right\} + \left[IC^{srv}\right]\left\{\frac{\sigma_r}{E_s}\right\}$$

where $\{\rho^{sv}\}$ & $\{\rho^{nsv}\}$ are the vertical & normalized vertical soil displacement column vectors of size (nl+1) each respectively. $(\tau)$ & $(\sigma_r)$ are column vectors of size '(nl+1)' and 'nl' for interfacial shear stresses including base pressure & radial stresses respectively. Other terms are as follows: $[IC^{svv}]$ = vertical settlement influence coefficient matrix of size (nl+1)×(nl+1) for the influence of interface shear stresses and base pressure. $[IC^{srv}]$ = vertical soil settlement influence coefficient matrix of size (nl+1)×nl for the influence of radial stresses.

The radial displacements at mid-point of the periphery of each elements, are

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$$\left\{\rho^{nsr}\right\} = \left\{\frac{\rho^{sr}}{d}\right\} = \left[IC^{svr}\right]\left\{\frac{\tau}{E_s}\right\} + \left[IC^{srr}\right]\left\{\frac{\sigma_r}{E_s}\right\} \qquad (2)$$

where $\{\rho^{sr}\}$ & $\{\rho^{nsr}\}$ are radial & normalized radial soil displacement column vectors of size (nl+1) each respectively. The matrices in the above equation are as follows: $[IC^{svr}]$ = radial soil displacement influence coefficient matrix of size nl×(nl+1) for the influence of interface shear stresses and base pressure. $[IC^{srr}]$ = radial soil displacement influence coefficient matrix of size nl×nl for the influence of interface radial stresses.

## 2.2 Granular Pile Displacements

Vertical and radial displacements of elements of GP are evaluated based on a generalized stress-strain relationship as

$$\varepsilon_v = \frac{[\sigma_v - 2\nu_{gp}\sigma_{rr}]}{E_{gp}}$$

$$\text{and,}\quad \varepsilon_\theta = \varepsilon_r = \frac{[\sigma_r - \nu_{gp}(\sigma_r + \sigma_v)]}{E_{gp}} \qquad (3)$$

where $\varepsilon_v$, $\varepsilon_\theta$ and $\varepsilon_r$ are respectively the axial, tangential and radial strains of an element. $\sigma_v$ and $\sigma_r$ are the axial and radial stresses on the element respectively. When the pile is stiffened at top, the value of deformation modulus $E_{gp}$ is increased $\chi$ times, where $\chi$ may vary in the range 1 to 10 in the stiffened portion. Thus, deformation modulus $\chi E_{gp}$ and $E_{gp}$ are considered in top stiffened portion and in the remaining portion of the granular pile respectively. The value of Poisson's ratio $\nu_{gp}$ is taken constant throughout the length of granular pile. The depth of stiffened portion $\eta$ times of L/d up to which value of $E_{gp}$ is increased to $\chi E_{gp}$, may be changed is varied from 0.1 to 0.4. The above expression in terms of shear stresses and base pressures of the GP is established in the next section.

From the consideration of equilibrium, the total load, P, on GP is related to the shear stresses, $\tau$, and base pressure, $p_b$, as

$$P = \sum_{j=1}^{j=nl} \frac{\tau_j \pi d L}{nl} + p_b \frac{\pi d^2}{4} \qquad (4)$$

Where, 'nl' is the total number of elements of GP. Considering the element 'i', The axial forces on the top and bottom faces of an element are

$$P_{it} = P - \sum_{j=1}^{j=(i-1)} \frac{\tau_j \pi d L}{nl}$$

$$P_{ib} = P - \sum_{j=1}^{j=i} \frac{\tau_j \pi d L}{nl} \qquad (5)$$

Combining Eq.s (4) & (5)

$$P_{it} = \sum_{j=i}^{j=nl} \frac{\tau_j \pi d L}{nl} + p_b \frac{\pi d^2}{4}$$

$$P_{ib} = \sum_{j=(i+1)}^{j=nl} \frac{\tau_j \pi d L}{nl} + p_b \frac{\pi d^2}{4} \qquad (6)$$

The axial stresses on the top and bottom faces of the element 'i', are (Fig. 1 (c))

$$\sigma_{it} = p_b + \sum_{j=i}^{j=nl} \frac{4(L/d)\tau_j}{nl}$$

$$\sigma_{ib} = p_b + \sum_{j=(i+1)}^{j=nl} \frac{4(L/d)\tau_j}{nl} \qquad (7)$$

The average axial stress on the element, 'i', is equal to

$$\sigma_{vi} = \frac{\sigma_{it} + \sigma_{ib}}{2} = p_b + \sum_{j=(i+1)}^{j=nl} \frac{4(L/d)\tau_j}{nl} + \frac{2(L/d)\tau_i}{nl} \qquad (8)$$

The above equation relates the shaft shear stresses to axial stresses of the elements and is expressed in matrix form as

$$\left\{\sigma_v\right\} = \left[PA\right]\left\{\tau\right\} \qquad (9)$$

where $\{\tau\}$ and $\{\sigma_v\}$ are respectively the columns vectors of shear stresses on shaft including normalized stress on base and axial stresses of the elements, both of size '(nl+1)'. Matrix [PA] is an upper triangular square matrix of size '(nl+1)' which relates the axial and shear stresses as

$$\left[PA\right] = \begin{bmatrix} \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & \frac{4(L/d)}{n} & \frac{(L/d)}{n} & - & - & - & - & 1 \\ 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & \frac{4(L/d)}{n} & - & - & - & - & 1 \\ 0 & 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & - & - & - & - & 1 \\ & & & & & & & & \\ - & - & - & - & - & 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & 1 \\ - & - & - & - & - & - & 0 & \frac{2(L/d)}{n} & 1 \\ - & - & - & - & - & - & 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

## (ii) Vertical GP Displacements

The vertical displacements of granular pile are evaluated from displacement of the top of pile ρt downwards. The settlement of the first element of GP is-

$$\rho_1^{npv} = \frac{\rho_1^{pv}}{d} = \rho_t - \varepsilon_{v1}\frac{\Delta z}{2d} \qquad (11)$$

where $\varepsilon_{v1}$ is the axial strain of the first element of GP and $\Delta z$ =(L/nl), is element length. $\rho_1^{pv}$ and $\rho_1^{npv}$ are the displacement and normalized displacements of the first node respectively. Thus the displacement of any element 'i' is obtained as

$$\rho_i^{npv} = \rho_t - \sum_{j=1}^{j=(i-1)} \varepsilon_{vj}\frac{\Delta z}{d} - \varepsilon_{vi}\frac{\Delta z}{2d} \qquad (12)$$

where $\varepsilon_{vi}$ and $\varepsilon_{vj}$ are the axial strains of $i^{th}$ and

$$\varepsilon_b = -\frac{d\rho^{pv}}{dz} = \frac{p_b}{E_{gp}}$$

j[th] elements respectively. To evaluate the settlement of the base of granular pile, the strain at the base is

(13)

A finite difference scheme with unequal intervals of spacing, the above equation is

$$\frac{4\rho_{nl-1}^{pv} - 36\rho_{nl}^{pv} + 32\rho_{nl+1}^{pv}}{12(\Delta z/d)} = -\frac{p_b}{E_{gp}} \tag{14}$$

where $\rho_{nl-1}^{pv}$, $\rho_{nl}^{pv}$ and $\rho_{nl+1}^{pv}$ are the displacements of elements n-1, n and n+1 respectively. Writing the above equation in the normalized form, one gets

$$4\rho_{nl-1}^{npv} - 36\rho_{nl}^{npv} + 32\rho_{nl+1}^{npv} = -\frac{p_b}{E_{gp}}\frac{12(L/d)}{nl} \tag{15}$$

Substituting for the values of $\rho_{nl-1}^{npv}$ and $\rho_{nl}^{npv}$ from Eq. (12), and rearranging the terms, one gets

$$\rho_{nl+1}^{npv} = \rho_t - \sum_{j=1}^{j=(nl-2)} \varepsilon_{vj}\frac{\Delta z}{d} - \frac{34}{32}\varepsilon_{v(nl-1)}\frac{\Delta z}{d} - \frac{18}{32}\varepsilon_{vnl}\frac{\Delta z}{d} - \frac{6}{32}\frac{(L/d)}{nlK_{gp}}\frac{p_b}{E_s} \tag{16}$$

Combining Eqns (12) & (16), the vertical displacements of granular pile are

(17)

where, [PB] and [PC] are respectively lower triangular matrices of sizes (nl+1)×(nl+1) and (nl+1)×nl and are

$$[PB] = \frac{(L/d)}{nK_{gp}}\begin{bmatrix} \frac{-0.5}{\chi} & 0 & 0 & 0 & - & - & - & - & 0 \\ \frac{-1}{\chi} & \frac{-0.5}{\chi} & 0 & 0 & - & - & - & - & 0 \\ \frac{-1}{\chi} & \frac{-1}{\chi} & \frac{-0.5}{\chi} & 0 & - & - & - & - & - \\ & & & & & & & & \\ -1 & -1 & - & - & - & - & - & -0.5 & 0 \\ -1 & -1 & - & - & - & - & -\frac{34}{32} & -\frac{18}{32} & -\frac{6}{32} \end{bmatrix}\begin{pmatrix} Note: Kgp & will & be \\ replaced & by & \chi Kgp \\ upto & stiffened & depth \\ \eta x L/d. & Remaining & elements \\ will & have & Kgp \end{pmatrix}$$

$$\left\{\rho^{npv}\right\} = \rho_t\{1\} + [PB]\left\{\frac{\sigma_v}{E_s}\right\} + [PC]\left\{\frac{\sigma_r}{E_s}\right\} \tag{18}$$

$$[PC] = \frac{v_{gp}(L/d)}{nK_{gp}}\begin{bmatrix} \frac{1}{\chi} & 0 & 0 & 0 & - & - & - & - \\ \frac{2}{\chi} & \frac{1}{\chi} & 0 & 0 & - & - & - & - \\ \frac{2}{\chi} & \frac{2}{\chi} & \frac{1}{\chi} & 0 & - & - & - & - \\ & & & & & & & \\ 2 & 2 & - & - & - & - & - & 1 \\ 2 & 2 & - & - & - & \frac{68}{32} & \frac{36}{32} & 0 \end{bmatrix}\begin{pmatrix} Note: Kgp & will & be \\ replaced & by & \chi Kgp \\ upto & stiffened & depth \\ \eta x L/d. & Remaining & elements \\ will & have & Kgp \end{pmatrix}$$

It is very important to note that relative stiffness of granular pile, Kgp will be replaced by χKgp in the elements up to a depth of η x L/d where stiffened portion is provided in GP. Other elements will continue to have relative stiffness of granular pile Kgp. Using the elemental shear and axial stresses (Eq. 9), the vertical displacements of GP nodes in terms shaft shear stresses are

$$\left\{\rho^{pvv}\right\} = \rho_t\{1\} + [PD]\left\{\frac{\tau}{E_s}\right\} + [PC]\left\{\frac{\sigma_r}{E_s}\right\} \tag{19}$$

where [PD] is a square matrix of size, (nl+1) = [PB] [PA].

**(iii) Radial GP Displacements**

The radial displacements of nodes are evaluated based on stress-strain relationship for the axi-symmetric case as

$$\varepsilon_\theta = \frac{S^{prr}}{a} = -\frac{\left[\left(1-\nu_{gp}\right)\sigma_r - \nu_{gp}\sigma_v\right]}{E_{gp}} \quad (20)$$

where εθ, $S^{prr}$ and a are the tangential strain, radial displacement and radius of granular pile respectively. The equation in normalized and matrix form is

$$\left\{\rho^{prr}\right\} = \left\{\frac{S^{prr}}{d}\right\} = \left[PE\right]\left\{\frac{\sigma_v}{E_s}\right\} + \left[PF\right]\left\{\frac{\sigma_r}{E_s}\right\} \quad (21)$$

where [PE] and [PF] are the matrices of sizes nl×(nl+1) and nl×nl respectively and are

$$\left[PE\right] = \frac{\nu_{gp}}{2K_{gp}}\begin{bmatrix} \frac{1}{\chi} & 0 & - & - & 0 \\ 0 & \frac{1}{\chi} & - & - & - \\ & & & & \\ - & - & - & - & - \\ - & - & - & 1 & 0 \end{bmatrix}\begin{pmatrix} Note: Kgp & will & be \\ replaced & by & \chi Kgp \\ upto & depth & \eta xL/d \end{pmatrix} \quad (22)$$

$$\left[PF\right] = -\frac{\left(1-\nu_{gp}\right)}{2K_{gp}}\begin{bmatrix} \frac{1}{\chi} & 0 & - & - & 0 \\ 0 & \frac{1}{\chi} & - & - & - \\ & & & & \\ - & - & - & - & - \\ - & - & - & 0 & 1 \end{bmatrix}\begin{pmatrix} Note: Kgp & will & be \\ replaced & by & \chi Kgp \\ upto & depth & \eta xL/d \end{pmatrix}$$

The radial displacements of GP in terms of interfacial shear stresses of GP & soil (Eq.s (9) & (21)) are

$$\left\{\rho^{prr}\right\} = \left[PG\right]\left\{\frac{\sigma_v}{E_s}\right\} + \left[PF\right]\left\{\frac{\sigma_r}{E_s}\right\} \quad (23)$$

where [PG] is a matrix of size nl×(nl+1) = [PE][PA].

**(a) Compatibility of Displacements**

Satisfying the compatibility of displacements of the granular pile and the soil, solutions are obtained in terms of interface shear and radial stresses. Applying the compatibility condition for vertical displacements (Eq.s 1 & 19)

$$\left\{\rho^{sv}\right\} = \left\{\rho^{pvv}\right\} \quad \text{or}$$

$$\left[PAA\right]\left\{\frac{\tau}{E_s}\right\} + \left[PBB\right]\left\{\frac{\sigma_r}{E_s}\right\} = \rho_t\{1\} \quad (24)$$

For radial displacements, through compatibility (Eqs. 2 & 23)

$$\left\{\rho^{sr}\right\} = \left\{\rho^{prr}\right\} \quad \text{or}$$

$$\left[PCC\right]\left\{\frac{\tau}{E_s}\right\} + \left[PDD\right]\left\{\frac{\sigma_r}{E_s}\right\} = \{0\} \quad (25)$$

Simultaneous equations (24) & (25) are solved for interface shear and radial stresses. The matrices in the above equations are

$\left[PAA\right] = \left[I^{svv}\right] - \left[PD\right]$, of size (nl+1)×(nl+1).

$\left[PBB\right] = \left[I^{srv}\right] - \left[PC\right]$, of size (nl+1)×nl. (26)

$\left[PCC\right] = \left[I^{svr}\right] - \left[PG\right]$, of size nl×(nl+1).

$\left[PDD\right] = \left[I^{srr}\right] - \left[PF\right]$, of size nl×nl.

The top vertical displacement, ρtop, of a single granular pile is

$$\rho_{top} = \frac{P}{E_s d} I_{top} \quad (27)$$

Similarly, displacement, ρtip, of tip of a single granular pile is expressed as

$$\rho_{tip} = \frac{P}{E_s d} I_{tip} \quad (28) \quad (28$$

(28)

The radial displacements, ρr, of granular pile at any depth are

$$\rho_r = \frac{P}{E_s d} I_r \quad (29)$$

where Itop and Ir are the vertical and radial displacements influence factors respectively which depend on the various

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geometric & material characteristics related to soil and granular pile. The response of the granular pile is evaluated in terms of displacement influence factors, Itop & Ir, normalized shear and radial stress distributions along GP-soil interface and the percentage of load transferred to the base. Parameters affecting the response are (i) length to diameter ratio, L/d, of GP, (ii) relative stiffness, Kgp, of granular pile with respect to soil and (iii) the Poisson's ratio of soil, $\nu s$, and granular pile, $\nu gp$.

### 3.0 Validation of Results

The results of the present analysis are verified with the available results of the previous researchers, Sharma and Madhav (1999) who have results in good agreement with Mattes and Poulos (1969). The values obtained are given in the Table1 for comparison.

Table 1 Comparison of Results of Present work with previous

| | L/d | $K_{gp}$ | Results of Sharma and Madhav (1999) | Present Analysis |
|---|---|---|---|---|
| Top Settlement Influence Factor | 10 | 10 | 0.376 | 0.378 |
| Top Settlement Influence Factor | 10 | 100 | 0.186 | 0.187 |
| Top Settlement Influence Factor | 20 | 10 | 0.375 | 0.376 |
| Top Settlement Influence Factor | 20 | 100 | 0.158 | 0.159 |
| Tip Settlement Influence Factor | 10 | 10 | 0.077 | 0.0772 |
| Tip Settlement Influence Factor | 10 | 100 | 0.125 | 0.1257 |
| Tip Settlement Influence Factor | 20 | 10 | 0.031 | 0.0314 |
| Tip Settlement Influence Factor | 20 | 100 | 0.061 | 0.0617 |
| Radial displacement influence Factor at 0.95 depth | 10 | 10 | 0.0013 | 0.0014 |
| Radial displacement influence Factor at 0.95 depth | 10 | 200 | 0.0002 | 0.00022 |
| Radial displacement influence Factor at 0.95 depth | 20 | 50 | 0.0003 | 0.00033 |

The comparisons of various values shown in Table 1 indicate that results obtained match with the previous results and are in close agreement

### 4.1 Effect of Stiffened Pile Top on Vertical Settlement Influence Factor $I_{top}$

The results in Table 2 along with Figure 2 show the vertical settlement influence factor, Itop, with relative GP-soil stiffness for L/d=10 with 10% stiffened length ($\eta$= 0.1). The values of the vertical settlement influence factor, Itop, for same Kgp ($\chi$=1) throughout the length of pile are matching with results given by Sharma and Madhav (1999). When GP is assumed stiff by taking 2 times Kgp in top portion i.e.$\chi$=2 (with $\eta$=0.1), Itop, decrease for the lower Kgp values of 10 and 20. The values of Itop are lower by about 9.58% at Kgp=10 and 7.25% at Kgp=20. The decrease in values of Itop for higher Kgp such as Kgp=200 and Kgp=500 GP for $\chi$=2 is only 2.65% and 1.18% respectively. For more stiff top i.e. $\chi$=5 and $\chi$=10 in top 10 % GP ($\eta$=0.1), the values of vertical settlement influence factor, Itop further decrease. The values are lesser by 15.88% and 18.09% at Kgp=10, 11.84% and 13.40% less at Kgp=20, 1.18% and 1.33% at Kgp=500 as compared to un-stiffened piles. When the Kgp values of the pile are in the range of 200-500, the vertical settlement influence factor, Itop, do not decrease at the same rate and values are lower by 1-3% only as compared to un-stiffened GP.

Refer results tabulated in Table 3 and Fig 3. When top 20% ($\eta$=0.2) of the portion of the pile having L/d=20 is made stiff by taking $\chi$=2, the variation in values of Itop is about 18.95% at Kgp=10 where as for Kgp=500 the change is 3.76%. For stiffer top taking $\chi$=5, the values of vertical settlement influence factor, Itop further decrease at lower Kgp especially in the range of 10-100. The variation is about 34.30% at Kgp=10 and about 29.77% at Kgp=20. As the Kgp values of the pile are 10 times stiffer in top 20%, the vertical settlement influence factor, Itop, decrease by 40.48% and 34.48% for Kgp=10 and Kgp=20 respectively. The change in values of vertical settlement influence factor Itop is less than 7% for Kgp=500.

Results are presented in Table 4 and Fig 4 to show the variation of stiffened length of GP i.e. $\eta$ on vertical settlement influence factor Itop. It is very clear that with increase in stiffened length of GP, there is reduction in the values of Itop as compared to un-stiffened GP. With the increase in stiffened portion in a GP, the percentage reduction in values of Itop increase. The values for Itop show maximum reduction at $\eta$=0.4 for all Kgp values. The reduction in Itop values is more in case of lower Kgp values of 10 and 20 as compared to 200 or 500.

It is evident from results shown that stiff piles have less vertical settlement influence factor Itop. It may also be observed that due to stiffened top values of vertical settlement influence factor Itop decrease. The reduction in values of Itop is more for less stiff GP. Also the GP having more stiffened top will have lesser vertical settlement influence factor Itop i.e. for higher $\chi$ values, reduction in vertical settlement influence factor will be more. It may also be seen that with increase in val-

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ue of η, there is decrease in values of vertical settlement influence factor Itop. The percentage change in the vertical settlement influence factor is more in the GP with less Kgp. The percentage change in the vertical settlement influence factor is more in the GP with more η for same Kgp.

**Table2 Values of $I_{top}$ with Kgp and x for L/d=10 along with Percent change in $I_{top}$GP Stiffened in Top h=0.1**

| Kgp | $I_{top}$ Values | | | | % Change in $I_{top}$ values | | |
|---|---|---|---|---|---|---|---|
| | χ=1 | χ=2 | χ=5 | χ=10 | χ=2 | χ=5 | χ=10 |
| 10 | 0.378287 | 0.342028 | 0.318187 | 0.309846 | -9.5852 | -15.8875 | -18.0925 |
| 20 | 0.296936 | 0.275382 | 0.261776 | 0.25712 | -7.25887 | -11.841 | -13.4088 |
| 50 | 0.221659 | 0.211695 | 0.205584 | 0.203524 | -4.49503 | -7.25196 | -8.1812 |
| 100 | 0.187018 | 0.18171 | 0.178489 | 0.177409 | -2.83818 | -4.56052 | -5.13792 |
| 200 | 0.166523 | 0.163768 | 0.162106 | 0.16155 | -1.65431 | -2.65265 | -2.98639 |
| 500 | 0.152788 | 0.151658 | 0.150979 | 0.150752 | -0.73962 | -1.18444 | -1.33288 |

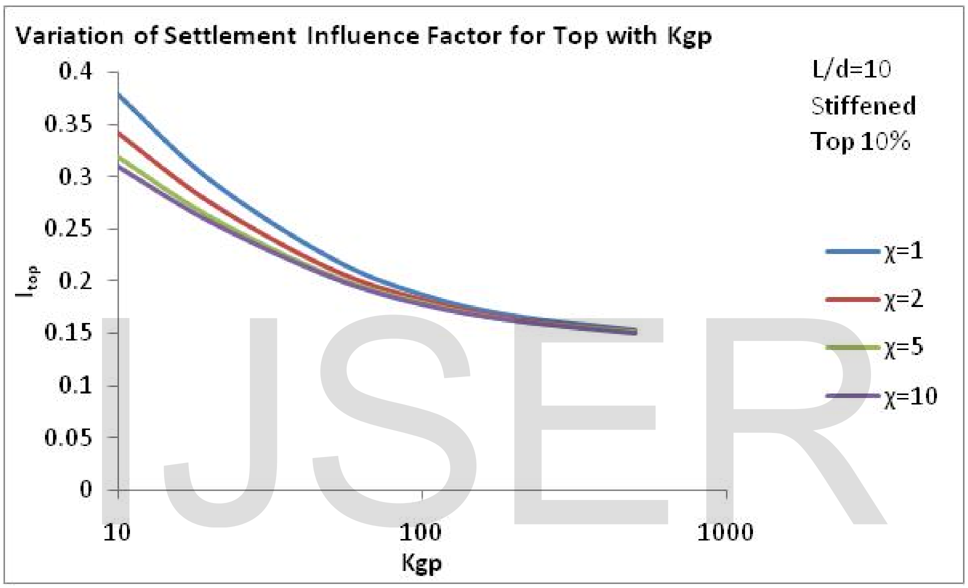



**Fig 2 Values of $I_{top}$ with Kgp and χ for L/d=10 GP Stiffened in Top η=0.1**

**Table 3 Values of $I_{top}$ with Kgp and x with Percent change in $I_{top}$ for L/d=20 GP Stiffened in Top h=0.2 Length**

| Kgp | Values of $I_{top}$ | | | | % Change in $I_{top}$ | | |
|---|---|---|---|---|---|---|---|
| | χ=1 | χ=2 | χ=5 | χ=10 | χ=2 | χ=5 | χ=10 |
| 10 | 0.376182 | 0.304894 | 0.247135 | 0.223877 | -18.9504 | -34.3044 | -40.4872 |
| 20 | 2.91E-01 | 2.41E-01 | 2.04E-01 | 1.91E-01 | -17.1843 | -29.7716 | -34.4885 |
| 50 | 0.205187 | 0.177446 | 0.158934 | 0.152394 | -13.5202 | -22.5417 | -25.7295 |
| 100 | 0.15971 | 0.143286 | 0.132799 | 0.129186 | -10.2834 | -16.8497 | -19.1119 |
| 200 | 1.29E-01 | 1.20E-01 | 1.14E-01 | 1.12E-01 | -7.11877 | -11.5413 | -13.0422 |
| 500 | 0.106302 | 1.02E-01 | 0.099856 | 0.099037 | -3.76811 | -6.06359 | -6.83468 |

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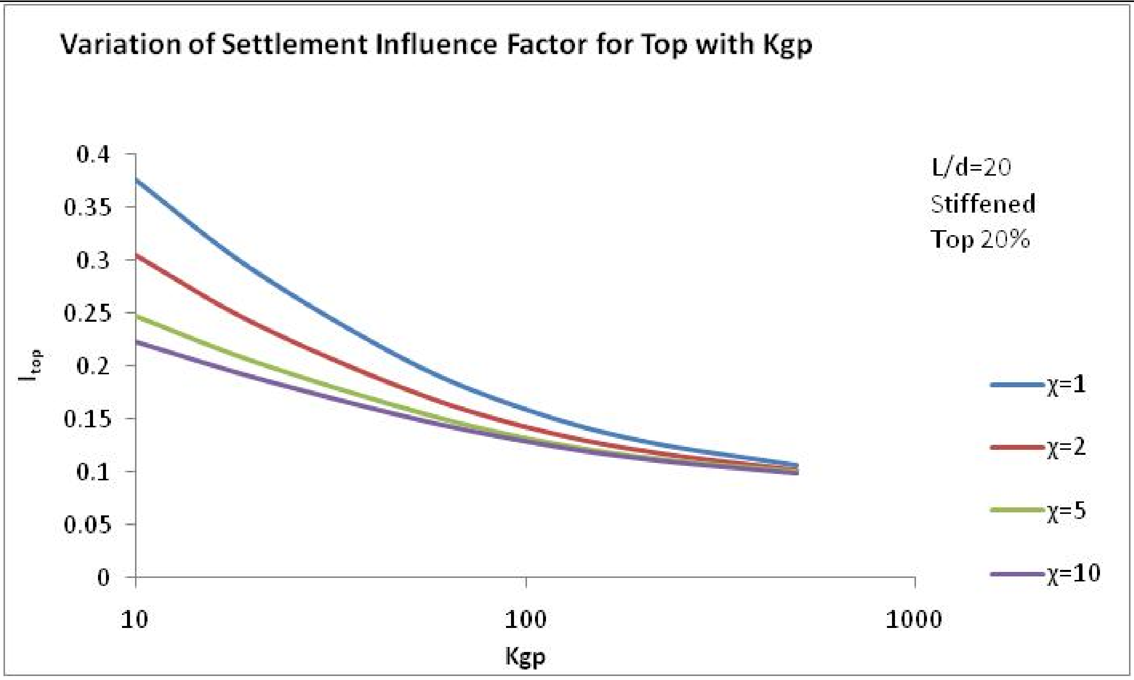


**FIG 3 VALUES OF $I_{TOP}$ WITH KGP AND X FOR L/D=20 GP STIFFENED IN TOP H=0.2**



**TABLE4 VALUES OF $I_{TOP}$ WITH KGP AND H FOR L/D=10 ALONG WITH PERCENT CHANGE IN $I_{TOP}$ GP STIFFENED IN TOP BY X=2**

| Kgp | Values of $I_{top}$ | | | | | % Change in $I_{top}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | η =0.1 | η =0.2 | η =0.3 | η =0.4 | η =0.1 | η =0.2 | η =0.3 | η =0.4 |
| 10 | 0.3783 | 0.3182 | 0.2850 | 0.2641 | 0.2499 | -15.8875 | -24.6674 | -30.1972 | -33.9315 |
| 20 | 0.2969 | 0.2618 | 0.2390 | 0.2230 | 0.2120 | -11.8410 | -19.5146 | -24.8996 | -28.6041 |
| 50 | 0.2217 | 0.2056 | 0.1936 | 0.1846 | 0.1776 | -7.2520 | -12.6458 | -16.7369 | -19.8782 |
| 100 | 0.1870 | 0.1785 | 0.1717 | 0.1664 | 0.1622 | -4.5605 | -8.1731 | -11.0265 | -13.2724 |
| 200 | 0.1665 | 0.1621 | 0.1585 | 0.1560 | 0.1530 | -2.6526 | -4.8360 | -6.3194 | -8.1209 |
| 500 | 0.1528 | 0.1510 | 0.1494 | 0.1482 | 0.1472 | -1.1844 | -2.1858 | -3.0107 | -3.6758 |

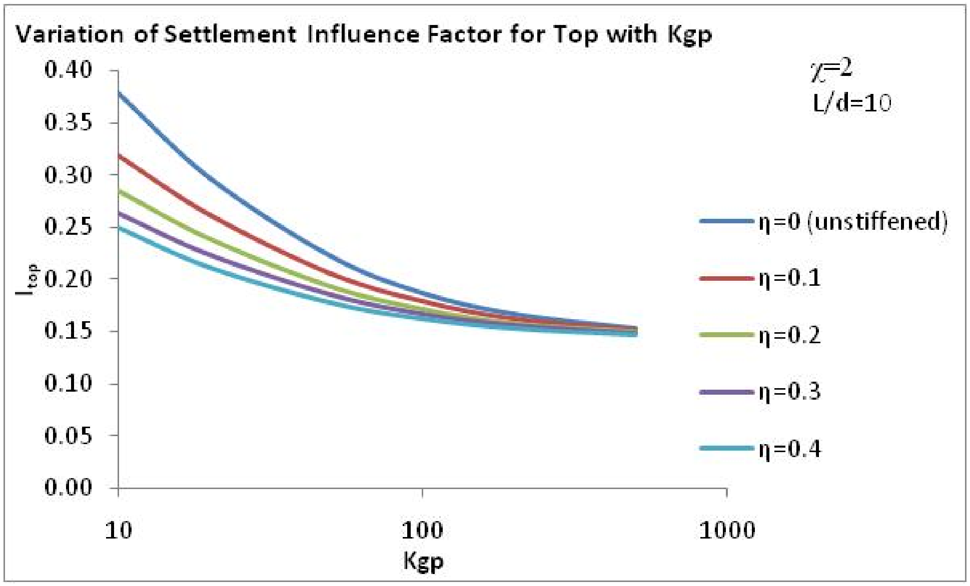


Fig 4 Values of $I_{top}$ with Kgp and η for L/d=10 GP Stiffened in Top χ=2

**4.2 Effect of Stiffened Pile Top on Tip Settlement Influence Factor $I_{tip}$**

The results in Table 5 and Figure 5 show the tip settlement influence factor, Itip, with relative GP-soil stiffness for L/d=20 with 10% stiffened length i.e. η=0.1. It may be seen that Itip, increase for the all Kgp values at all values of χ. Thus, the values are higher by about 1.098% at Kgp=10 and 0.957% at Kgp=20 for χ=2. For more stiff top i.e. χ=5 or χ=10 in top 10 %, the values of tip settlement influence factor, Itip further increase. The values are higher by 1.916% and 2.223% at Kgp=10 and 1.606% and 1.837% less at Kgp=20 as compared to unstiffened piles. When the Kgp values of the pile are in the range of 200-500, the tip settlement influence factor, Itip, do not increase at the same rate and values are higher by 0.2 to 0.45% only.

Similar results are obtained when η=0.3 is considered. At the same value of χ, there is reduction in the value of Itip with increasing Kgp. There is increase in tip displacement influence factor values for same Kgp with increasing χ. The increase in value of Itip is more for lower Kgp values as compared to higher Kgp values. The tip settlement influence factor values and %change in increase in values are shown in Table 6 along with Fig 6.

Another result may be obtained by comparing the effect of length of stiffened portion. It is evident from the Table 7 along with Fig 7 that when length of stiffened portion is increased, higher tip settlement influence factor values are obtained and thus values for η=0.4 are having the maximum tip displacement values at any Kgp. For a constant η, it may be observed that Itip values are more for higher vaue of Kgp.

Thus, with the increase in stiffness of GP or with increase in stiffness of top i.e. for higher χ or with increase in length of stiff portion i.e. for higher η, the tip displacement influence factor values increase.

Table 5 Values of $I_{tip}$ with Kgp and χ for L/d=20 along with Percent change in $I_{tip}$ GP Stiffened in Top η =0.1

| Kgp | $I_{tip}$ Values | | | | % Change in $I_{tip}$ values | | |
|---|---|---|---|---|---|---|---|
| | χ=1 | χ=2 | χ=5 | χ=10 | χ=2 | χ=5 | χ=10 |
| 10 | 0.031468 | 0.031814 | 0.032071 | 0.032168 | 1.098515 | 1.916809 | 2.223189 |
| 20 | 0.037509 | 0.037868 | 0.038111 | 0.038198 | 0.957335 | 1.606976 | 1.837853 |
| 50 | 0.050406 | 0.050741 | 0.050953 | 0.051026 | 0.66463 | 1.086125 | 1.230647 |
| 100 | 0.061775 | 0.06204 | 0.062205 | 0.06226 | 0.430257 | 0.696165 | 0.786138 |
| 200 | 0.07163 | 0.07181 | 0.07192 | 0.071957 | 0.251534 | 0.404823 | 0.456321 |
| 500 | 0.080286 | 0.080376 | 0.08043 | 0.080448 | 0.11224 | 0.180024 | 0.202693 |

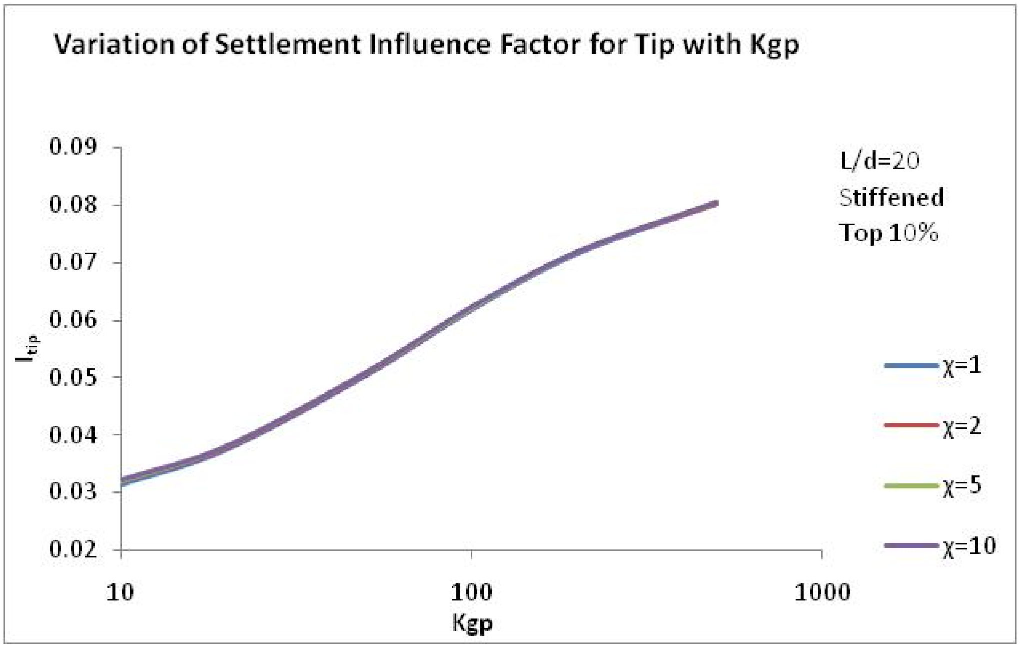


Fig 5 Values of $I_{tip}$ with Kgp and $\chi$ for L/d=20 (GP Stiffened in Top $\eta$=0.1)



TABLE 6 VALUES OF $I_{TIP}$ WITH KGP AND X FOR L/D=20 ALONG WITH PERCENT CHANGE IN $I_{TIP}$

GP STIFFENED IN TOP H=0.3

| | $I_{tip}$ Values | | | | % Change in $I_{tip}$ values | | |
|---|---|---|---|---|---|---|---|
| Kgp | $\chi$=1 | $\chi$=2 | $\chi$=5 | $\chi$=10 | $\chi$=2 | $\chi$=5 | $\chi$=10 |
| 10 | 0.0314679 | 0.0326454 | 0.0338642 | 0.0344401 | 3.741838 | 7.614845 | 9.445034 |
| 20 | 0.0375086 | 0.0391129 | 0.0405076 | 0.0410882 | 4.277009 | 7.995468 | 9.543397 |
| 50 | 0.0504057 | 0.0523084 | 0.0537059 | 0.0542278 | 3.774695 | 6.547353 | 7.58275 |
| 100 | 0.0617746 | 0.0634529 | 0.0645894 | 0.0649938 | 2.716744 | 4.556468 | 5.211141 |
| 200 | 0.0716300 | 0.0728444 | 0.0736252 | 0.0738951 | 1.695433 | 2.785486 | 3.162246 |
| 500 | 0.0802857 | 0.0809225 | 0.0813165 | 0.0814500 | 0.793209 | 1.284029 | 1.450233 |

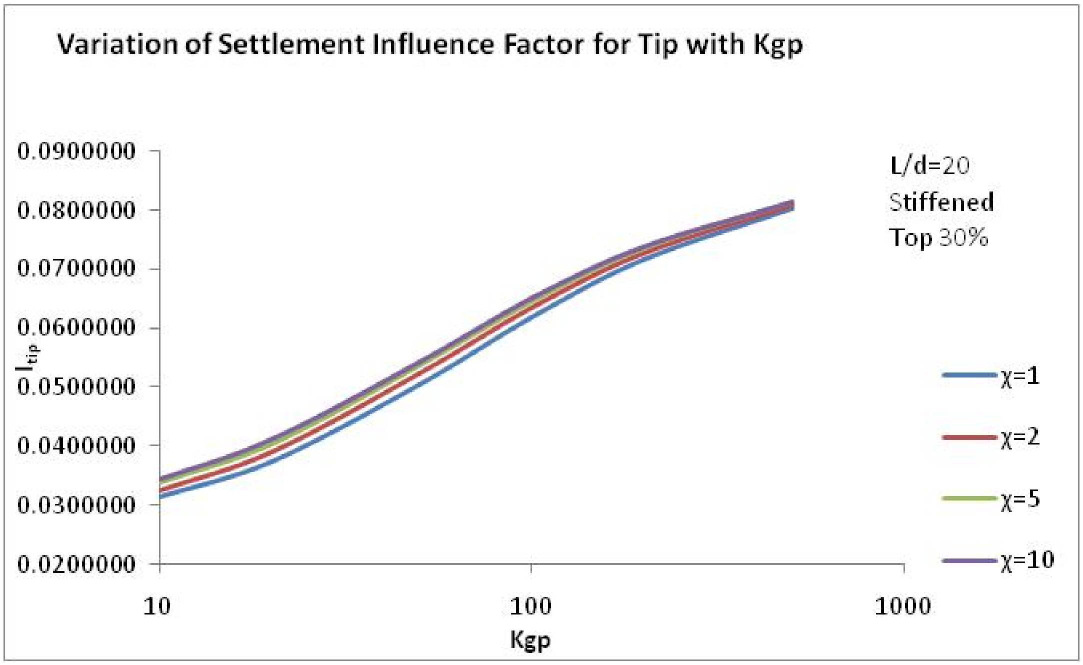


Fig 6 Values of $I_{tip}$ with Kgp and $\chi$ for L/d=20 (GP Stiffened in Top $\eta$=0.3)

Table 7 Values of $I_{tip}$ with Kgp and $\chi$=2 for L/d=10 GP Stiffened in Top

| Kgp | Values of $I_{tip}$ | | | | | % Change in $I_{tip}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Not Stiffened | $\eta$= 0.1 | $\eta$= 0.2 | $\eta$= 0.3 | $\eta$= 0.4 | $\eta$= 0.1 | $\eta$= 0.2 | $\eta$= 0.3 | $\eta$= 0.4 |
| 10 | 0.077103 | 0.077766 | 0.078735 | 0.079921 | 0.081301 | 0.859625 | 2.116152 | 3.654244 | 5.444964 |
| 20 | 0.093064 | 0.093628 | 0.0946 | 0.095855 | 0.097402 | 0.605206 | 1.649984 | 2.998082 | 4.66108 |
| 50 | 0.113992 | 0.114354 | 0.115039 | 0.116018 | 0.117238 | 0.317878 | 0.91856 | 1.777486 | 2.848073 |
| 100 | 0.125715 | 0.125939 | 0.126385 | 0.127039 | 0.127862 | 0.178728 | 0.533226 | 1.053431 | 1.708096 |
| 200 | 0.13331 | 0.133438 | 0.134 | 0.13409 | 0.134584 | 0.096121 | 0.51764 | 0.585452 | 0.955941 |
| 500 | 0.138725 | 0.138781 | 0.139 | 0.139076 | 0.139301 | 0.040572 | 0.198529 | 0.253242 | 0.415486 |

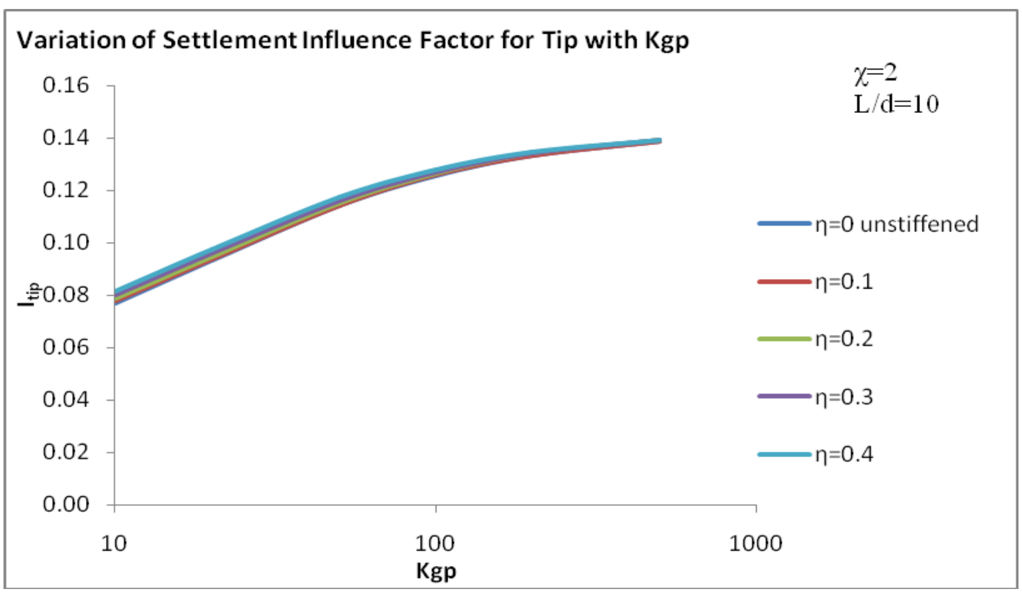

Fig 7 Values of $I_{tip}$ with Kgp and $\chi$=2 for L/d=10 GP Stiffened in Top with different $\eta$

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**4.3 Effect of Stiffened Pile Top on Radial Displacement influence factor $I_r$**

The results in Table 8 and Figure 8 show the radial displacement influence factor, Ir, with relative GP-soil stiffness for Kgp=20, L/d=20 with 10% stiffened length i.e. η=0.1. A sharp decrease in the values of Ir may be seen up to η=0.1. It may be seen that Ir, decrease for the all χ values. Thus, the values are lower by about 47.24% at χ=2, 78.19% at χ=5 and 88.97% for χ=10 at a normalized depth of 0.025. At depths up to 0.075 Ir is decreasing with stiff top i.e. χ=2 to 10 in top 10 %. After normalized depth of 0.075, the values of radial displacement influence factor, Ir increase with χ as compared to un-stiffened piles at any depth. However, for a constant χ value, the values of Ir reduce with depth as shown in Table 7 as compared to un-stiffened piles values.

Similar results are obtained when η=0.3 is considered. At the same value of χ, there is reduction in the value of Ir with increase in depth. There is decrease in radial displacement influence factor values for same depth with increasing χ up to 0.275. From depth 0.325 there is increase in value of Ir with χ. The radial displacement influence factor values and %change in increase in values are shown in Table 9 along with Fig 9.

Another result is presented by comparing the effect of length of stiffened portion. It is evident from the Table 10 along with Fig 10 that when length of stiffened portion is increased, lower radial settlement influence factor values are obtained up to the stiffened length for a constant η. When the stiffened portion is over, for η=0.4 is having the maximum radial settlement influence factor values at any depth.

Table 8 Values of $I_r$ with Kgp=20 and χ for L/d=20 along with Percent change in $I_r$ GP Stiffened in Top η =0.1

| | $I_R$ Values | | | | % Change in $I_R$ values | | |
|---|---|---|---|---|---|---|---|
| Depth | χ=1 | χ=2 | χ=5 | χ=10 | χ=2 | χ=5 | χ=10 |
| -0.025 | 0.007696 | 0.00406 | 0.001678 | 0.000848 | -47.2416 | -78.1954 | -88.9782 |
| -0.075 | 0.00718 | 0.003807 | 0.001577 | 0.000797 | -46.9742 | -78.0436 | -88.8976 |
| -0.125 | 0.006191 | 0.006277 | 0.006321 | 0.006333 | 1.390267 | 2.091634 | 2.288592 |
| -0.175 | 0.005232 | 0.005339 | 0.005405 | 0.005426 | 2.053266 | 3.300773 | 3.712591 |
| -0.225 | 0.004386 | 0.004492 | 0.00456 | 0.004584 | 2.411424 | 3.963309 | 4.496494 |
| -0.275 | 0.00367 | 0.003765 | 0.003827 | 0.003848 | 2.568468 | 4.263574 | 4.85555 |
| -0.325 | 0.003075 | 0.003155 | 0.003209 | 0.003227 | 2.605144 | 4.347159 | 4.960557 |
| -0.375 | 0.002583 | 0.00265 | 0.002695 | 0.002711 | 2.574627 | 4.309271 | 4.922934 |
| -0.425 | 0.002178 | 0.002233 | 0.00227 | 0.002283 | 2.509351 | 4.207903 | 4.810522 |
| -0.475 | 0.001843 | 0.001888 | 0.001918 | 0.001929 | 2.428277 | 4.077007 | 4.663055 |
| -0.525 | 0.001564 | 0.001601 | 0.001626 | 0.001635 | 2.342167 | 3.935915 | 4.503183 |
| -0.575 | 0.001331 | 0.001361 | 0.001382 | 0.001389 | 2.256947 | 3.795332 | 4.343471 |
| -0.625 | 0.001134 | 0.001159 | 0.001176 | 0.001182 | 2.175773 | 3.661015 | 4.1907 |
| -0.675 | 0.000966 | 0.000986 | 0.001 | 0.001005 | 2.100289 | 3.536009 | 4.048473 |
| -0.725 | 0.00082 | 0.000837 | 0.000848 | 0.000852 | 2.031384 | 3.421995 | 3.918796 |
| -0.775 | 0.000691 | 0.000705 | 0.000714 | 0.000718 | 1.96967 | 3.320163 | 3.803098 |
| -0.825 | 0.000575 | 0.000586 | 0.000594 | 0.000596 | 1.915882 | 3.231915 | 3.703053 |
| -0.875 | 0.000467 | 0.000476 | 0.000482 | 0.000484 | 1.871411 | 3.159837 | 3.621731 |
| -0.925 | 0.000363 | 0.00037 | 0.000374 | 0.000376 | 1.839674 | 3.110144 | 3.566443 |
| -0.975 | 0.000259 | 0.000263 | 0.000267 | 0.000268 | 1.831304 | 3.101966 | 3.559616 |

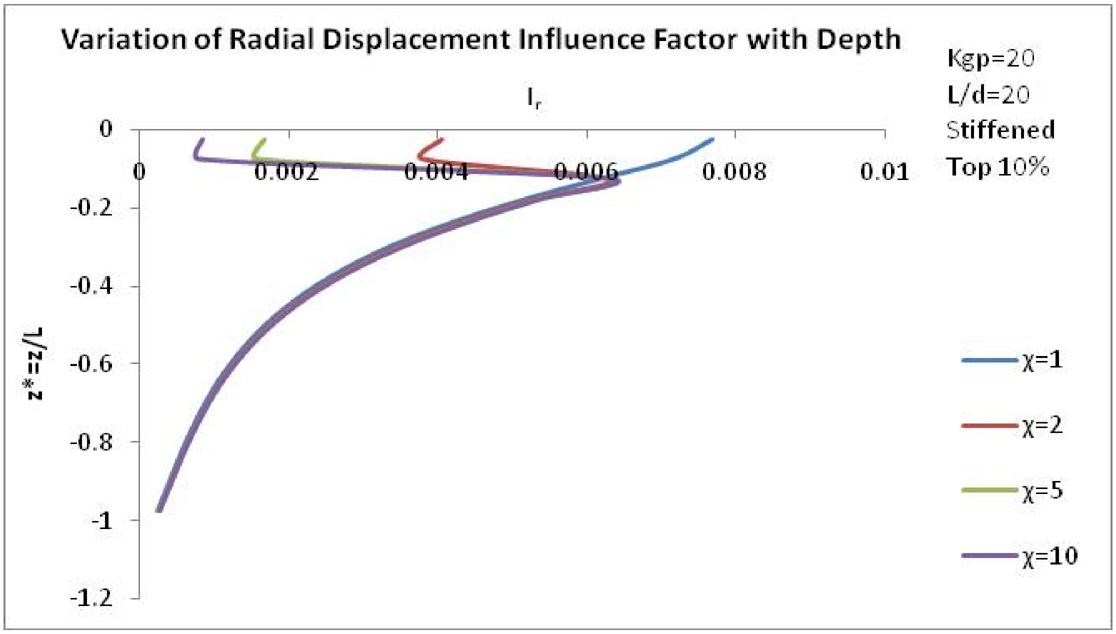


Fig. 8 Variation of $I_r$ along Normalized Depth for Kgp=20, L/d=20 GP Stiffened in Top η =0.1

Table 9 Values of $I_r$ with Kgp=20 and $\chi$ for L/d=20 along with Percent change in $I_r$ GP STIFFENED IN TOP H=0.3

| | $I_R$ VALUES | | | | % CHANGE IN $I_R$ VALUES | | |
|---|---|---|---|---|---|---|---|
| DEPTH | x=1 | x=2 | x=5 | x=10 | x=2 | x=5 | x=10 |
| | 0.00770 | 0.00412 | 0.00172 | 0.00087 | -46.404 | -77.6449 | -88.6778 |
| | 0.00718 | 0.00393 | 0.00165 | 0.00083 | -45.2435 | -77.045 | -88.4033 |
| | 0.00619 | 0.00353 | 0.00151 | 0.00077 | -43.0019 | -75.6036 | -87.6212 |
| | 0.00523 | 0.00310 | 0.00136 | 0.00069 | -40.7711 | -74.0553 | -86.7466 |
| | 0.00439 | 0.00268 | 0.00120 | 0.00062 | -38.9704 | -72.683 | -85.9329 |
| | 0.00367 | 0.00228 | 0.00104 | 0.00054 | -37.9866 | -71.7732 | -85.3431 |
| | 0.00307 | 0.00361 | 0.00401 | 0.00414 | 17.46206 | 30.34989 | 34.72035 |
| | 0.00258 | 0.00302 | 0.00336 | 0.00349 | 16.72222 | 29.99225 | 34.94519 |
| | 0.00218 | 0.00252 | 0.00281 | 0.00292 | 15.84723 | 28.98353 | 34.14209 |
| | 0.00184 | 0.00212 | 0.00235 | 0.00245 | 14.92964 | 27.64017 | 32.77841 |
| | 0.00156 | 0.00178 | 0.00197 | 0.00205 | 14.02626 | 26.17205 | 31.16993 |
| | 0.00133 | 0.00151 | 0.00166 | 0.00172 | 13.17201 | 24.70749 | 29.50955 |
| | 0.00113 | 0.00127 | 0.00140 | 0.00145 | 12.38372 | 23.31585 | 27.90389 |
| | 0.00097 | 0.00108 | 0.00118 | 0.00122 | 11.66716 | 22.0305 | 26.40705 |
| | 0.00082 | 0.00091 | 0.00099 | 0.00103 | 11.02226 | 20.86504 | 25.04393 |
| | 0.00069 | 0.00076 | 0.00083 | 0.00086 | 10.44651 | 19.82354 | 23.82496 |
| | 0.00058 | 0.00063 | 0.00068 | 0.00071 | 9.937159 | 18.90735 | 22.75583 |
| | 0.00047 | 0.00051 | 0.00055 | 0.00057 | 9.493622 | 18.12185 | 21.84705 |
| | 0.00036 | 0.00040 | 0.00043 | 0.00044 | 9.123248 | 17.49052 | 21.13264 |
| | 0.00026 | 0.00028 | 0.00030 | 0.00031 | 8.861064 | 17.10289 | 20.73417 |

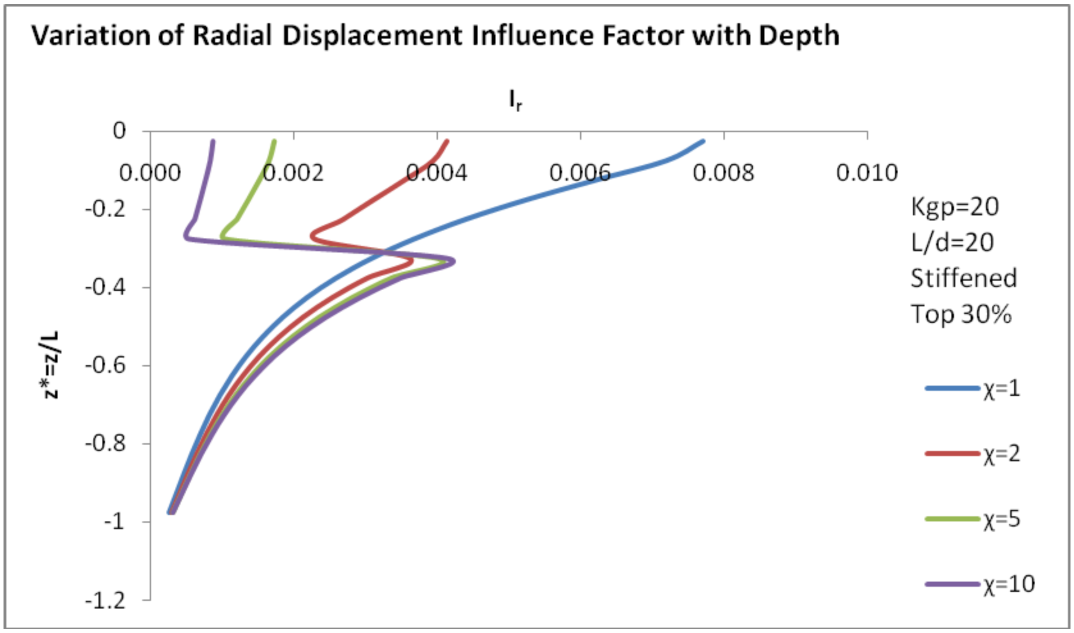


Fig. 9 Variation of $I_r$ along Normalized Depth for Kgp=20, L/d=20 GP Stiffened in Top η =0.3

Table 10 Values of $I_r$ with Kgp=50, χ=2and for L/d=10 along with Percent change in $I_r$ GP STIFFENED IN TOP

| DEPTH | η=0 | η=0.1 | η=0.2 | η=0.3 | η=0.4 | η=0.1 | η=0.2 | η=0.3 | η=0.4 |
|---|---|---|---|---|---|---|---|---|---|
| -0.05 | 0.003344 | 0.001708 | 0.001707 | 0.00171 | 0.001713 | -48.923 | -48.940 | -48.858 | -48.761 |
| -0.15 | 0.003196 | 0.00319 | 0.00163 | 0.001633 | 0.001637 | -0.187 | -48.992 | -48.912 | -48.779 |
| -0.25 | 0.002889 | 0.002888 | 0.00289 | 0.001485 | 0.001492 | -0.012 | 0.029 | -48.610 | -48.342 |
| -0.35 | 0.002555 | 0.002558 | 0.002565 | 0.00258 | 0.001332 | 0.131 | 0.397 | 0.991 | -47.886 |
| -0.45 | 0.00222 | 0.002225 | 0.002235 | 0.002253 | 0.002276 | 0.231 | 0.684 | 1.459 | 2.505 |
| -0.55 | 0.001896 | 0.001901 | 0.001912 | 0.00193 | 0.001952 | 0.298 | 0.883 | 1.808 | 2.989 |
| -0.65 | 0.001585 | 0.001591 | 0.001601 | 0.001618 | 0.001638 | 0.341 | 1.011 | 2.037 | 3.328 |
| -0.75 | 0.00129 | 0.001294 | 0.001304 | 0.001318 | 0.001335 | 0.367 | 1.087 | 2.168 | 3.520 |
| -0.85 | 0.001006 | 0.00101 | 0.001017 | 0.001028 | 0.001042 | 0.378 | 1.118 | 2.212 | 3.567 |
| -0.95 | 0.000724 | 0.000727 | 0.000732 | 0.00074 | 0.000749 | 0.372 | 1.095 | 2.145 | 3.422 |

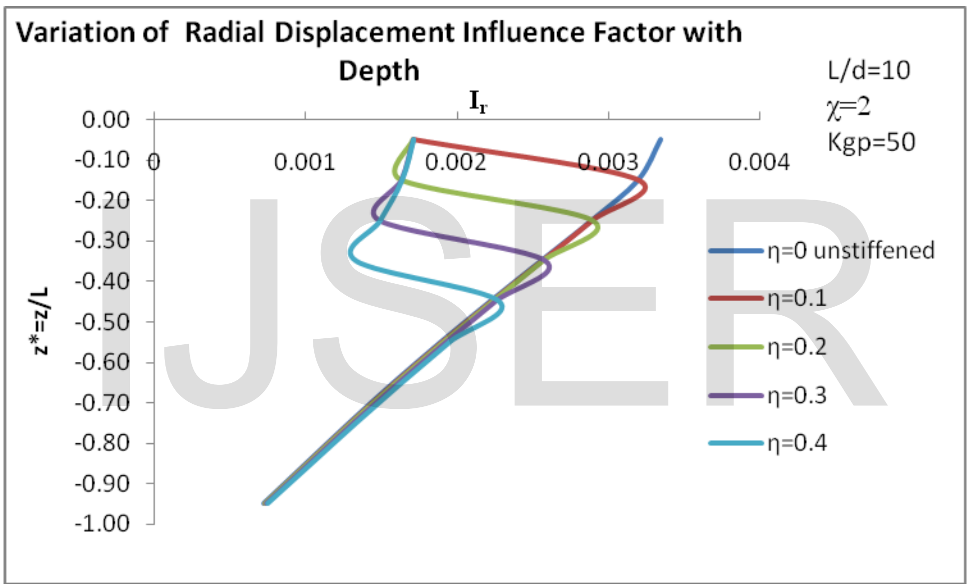


**Fig. 10 Variation of $I_r$ along Normalized Depth for Kgp=50, χ=2, L/d=10 GP Stiffened in Top**

5. Conclusions

1. It is evident from results that stiff piles have less settlement influence factor Itop and GP having less Kgp will have higher vertical settlement influence factor.

2. Stiffening the top reduces the settlement influence factor Itopof GP.

3. Settlement influence factor Itop decreases with increase in length of stiffened portion of GP.

4. Tip settlement influence factors are very small compared to top settlement influence factors.

5. For a GP stiffened the top or with higher length of stiffened portion, tip settlement influence factor Itip will be higher as compared to un-stiffened GP but the increment is very small.

6. For GP with stiffened top portion, due to increased stiffness at top, lower radial displacement influence factors Ir are obtained up to the stiffened length.

7. In a GP stiffened top, the values for radial displacement influence factors, Ir increase beyond the stiffened length but the increments are very small.

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