Reliable Control of PMDC Motor Speed Using Matlab

M. O. Charles, R. C. Okoro, I. A. Ikposhi, D. E. Oku

Abstract: This research investigated several controllers designed to control the speed of a 380-volt permanent magnet DC motor set to rotate at a speed of 45 rads/sec. The aim is to design a controller which is robust and reliable, with a good disturbance rejection suitable for use in any industrial equipment or domestic appliance which rotates at a speed of 45 rads/sec. The system modeling was carried out using simulink for proper analysis of the time responses of the various controllers. The reliability of the system model was tested with a torque of 10Nm and the responses show that the cascade-PID controller amongst all controller designs tested can produce a stable, reliable, and robust but not very sensitive system, while the ordinary PID controller can produce a highly sensitive, but less stable system.

Keywords: Speed control, Cascade PID, Cascade PI, DC Motor Modeling, Reliable and Sensitive Systems, Simulink, PMDC

1 Introduction

In the modern day, almost all electric power generation, transformation, transmission, and distribution systems are AC due to the fact that generation of large amounts of electric power is easier with AC. However, in most electrical appliances in homes, and in special applications such as in trains, electric vehicles, and process control, it is advantageous to convert AC into DC in order to use DC motors, which have become the most important machine in control systems [1]. The reason is that speed/torque characteristics of DC motors are superior to that of AC motors [2]. DC motors are in particular popular in high power and precise servo applications due to their reasonable cost and ease of control [3].

A cascade structure consisting of a Generic Model Control (GMC) in the inner loop and a PID in the outer loop was used to control the speed of a DC motor [4]. Results obtained from [4] though satisfactory, show chattering in output speed when a load of torque 30Nm was applied. It is known however, that this chattering in shaft position is dangerous for the life span of the motor and consequently the system. Similar works in [5] compared the performances of a conventional PI cascade controller to that of an intelligent self-tuning PI cascade controller using fuzzy algorithm and results proved that the auto-tuning PI controller offered a better settling time when a load is applied. A self-tuning PI cascade was used in [5] due to the difficulty involved in properly tuning the PI parameters to perfection without a deep knowledge of the behavior of the system to be controlled and subsequently a good mathematical model of the system.

Many DC motor models exists which can accurately represent the machine behavior, however, the model parameters must be set correctly for the mathematical model to provide a correct behaviour [6],[7]. After obtaining a good mathematical model, it was necessary then to painstakingly tune the conventional PI controller amongst other controllers, to obtain satisfactory results which will include insensitivity to load disturbances and a good settling time. This is because it is more cost effective using the conventional PI cascade controller compared to the auto-tuning PI controller.

2 MATERIALS AND METHOD

2.1 System Modeling

A PMDC motor with parameters given in Table 1 is chosen for its excellent electrical and mechanical performances and a pictorial model is given in Figure 1.

Table 1: DC motor parameters [8]

Motor Parameters	*Value*
Armature Resistance (R_a)	0.5Ω
Armature Inductance (L_a)	0.01H
Torque constant (K_t)	1.22
Back e.m.f. constant (K_b)	1.22
Frictional damping (B)	0.01Nm/rads/s
Inertia (J)	0.037kgm^2
Motor voltage (Vm)	380V
Power of DC motor (kW)	7.5 kW

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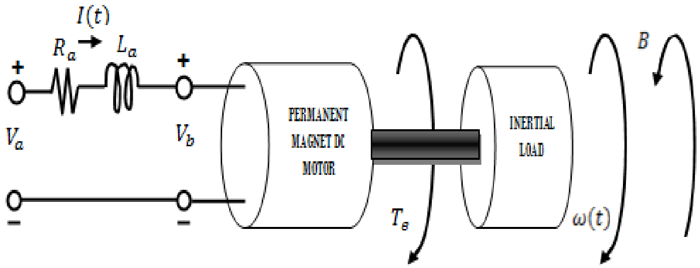

Fig.1: PMDC motor pictorial model [8]

2.2 Electrical Part

Consider the DC motor driving an inertial load in Fig. 1. Assuming that the field due to the magnet is constant, it can be shown from Kirchhoff's voltage law that,

$$V_m - V_R - V_L - V_b = 0\,, \tag{1}$$

Where $V_m = motor\ voltage,\ V_R = R_a i_a(t)$,

$V_L = L_a \frac{di_a(t)}{dt}$, $i_a(t) = armature\ current.$

$V_b = back\ e.m.f\ and\ is\ given\ as\ \ K_b\omega(t)$

where $\omega(t)$ = Angular or rotational velocity and K_b is the velocity constant.

$$V_m(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b\omega(t) = 0. \tag{2}$$

2.3 Mechanical Part

For energy balance in the system, the sum of torques must be equal to zero. Hence,

$$T_e - T_{\omega'} - T_\omega - T_L = 0\,, \tag{3}$$

where T_e = Electromagnetic torque, T_L = Load torque

$T_{\omega'}$ = Torque due to rotational acceleration of the rotor, and

T_ω = Damping torque due to velocity of motor,

Since the magnetic field strength is fixed, then

$$T_e = K_t i_a(t)\,, \tag{4}$$

where K_t is the torque constant, and

$$T_{\omega'} = J\frac{d\omega(t)}{dt}\,, \tag{5}$$

where J = inertia of the rotor and equivalent load.

Also,

$$T_\omega = B\omega(t)\,, \tag{6}$$

where B is the damping coefficient associated with rotation of motor.

Substituting (4), (5), and (6) into (3), we have that

$$J\frac{d\omega(t)}{dt} = K_t i_a(t) - T_L - B\omega(t)\,. \tag{7}$$

The differential equations for the armature current ($i_a(t)$) and the angular velocity ($\omega(t)$) can be written respectively as

$$\frac{di_a(t)}{dt} = -\frac{R_a i_a(t)}{L_a} - \frac{K_b\omega(t)}{L_a} + \frac{V_m}{L_a}\,, \tag{8}$$

and

$$\frac{d\omega(t)}{dt} = \frac{K_t i_a(t)}{J} - \frac{B\omega(t)}{J} - \frac{T_L}{J}\,. \tag{9}$$

Equations (8) and (9) conform to the characteristic equations of the DC motor in [9],[10].

Taking Laplace transforms of both sides of (8) and (9) respectively, and ignoring initial conditions, we get

$$sI_a(s) = -\frac{R_a}{L_a}I_a(s) - \frac{K_b}{L_a}W(s) + \frac{1}{L_a}V_m(s) \tag{10}$$

and

$$sW(s) = \frac{K_t}{J}I_a(s) - \frac{B}{J}W(s) - \frac{1}{J}T_L(s), \tag{11}$$

From (10),

$$I_a(s) = [-K_b W(s) + V_m(s)] * \frac{1}{L_a s + R_a}\,, \tag{12}$$

where the factor ($\frac{1}{L_a s + R_a}$), is the transfer function of the electrical part.

Comparing this with the general form of transfer function for a first order system,

$$\frac{1}{L_a s + R_a} \approx \frac{1}{\tau s + 1} \tag{13}$$

Therefore the *Electrical time constant*

$$(\boldsymbol{\tau_e}) = \frac{\boldsymbol{L_a}}{\boldsymbol{R_a}}\,, \tag{14}$$

provided R_a =1. From (11),

$$W(s) = [K_t I_a(s) - T_L(s)] * \frac{1}{Js + B}\,. \tag{15}$$

The factor ($\frac{1}{Js+B}$) is the *transfer function* of the *mechanical part.*

Also **c**omparing as in (13), the *Mechanical time constant*

$$(\boldsymbol{\tau_m}) = \frac{\boldsymbol{J}}{\boldsymbol{B}}\,, \tag{16}$$

provided B = 1

Equations (12) and (15) are merged to form the DC motor block diagram as shown in Fig. 2.

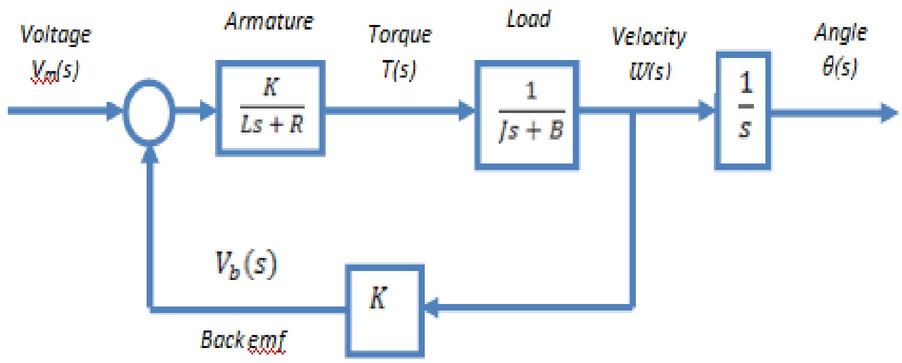

Fig. 2: Block diagram representation of the PMDC motor

2.4 Transfer Functions

From the block diagram, the transfer function of the PMDC motor is obtained as follows:

$$W(s) = T(s) * \frac{1}{Js+B} \ , \tag{17}$$

where

$$T(s) = \frac{K_t}{sL_a+R_a} e \tag{18}$$

and

$$e = V_m(s) - W(s)K_b \ . \tag{19}$$

Therefore

$$T(s) = \frac{K_t}{sL_a+R_a}[V_m(s) - W(s)K_b]$$

$$T(s) = \frac{K_t V_m(s)}{sL_a+R_a} - \frac{W(s)K_t K_b}{sL_a+R_a} \tag{20}$$

Substituting (20) into (17), we obtain

$$W(s) = \frac{1}{Js+B}\left[\frac{K_t V_m(s)}{sL_a+R_a} - \frac{W(s)K_t K_b}{sL_a+R_a}\right] \tag{21}$$

$$W(s)\left[1 + \frac{K_t K_b}{(Js+B)(sL_a+R_a)}\right] = \frac{K_t V_m(s)}{(Js+B)(sL_a+R_a)} \tag{22}$$

$$W(s)\left[\frac{(Js+B)(sL_a+R)+K_t K_b}{(Js+B)(sL_a+R_a)}\right] = \frac{K_t V_m(s)}{(Js+B)(sL_a+R_a)} \tag{23}$$

Hence, the open loop transfer function $G(s)$ from the input voltage $V_m(s)$ to the angular velocity $W(s)$ is given as,

$$G(s) = \frac{W(s)}{V_m(s)} = \frac{K_t}{(Js+B)(sL_a+R_a)+K_t K_b} \tag{24}$$

Opening up the denominator and dividing by $L_a J$,

$$G(s) = \frac{\frac{K_t}{L_a J}}{s^2 + s\frac{(JR_a+BL_a)}{L_a J} + \frac{R_a B}{L_a J} + \frac{K_t K_b}{L_a J}} \ . \tag{25}$$

Substituting the values of the motor parameters in Table 1 into (25), we have that

$$G(s) = \frac{3297}{s^2+50.27s+4036} \tag{26}$$

2.5 Design of Controllers

The main control objectives here are: stability, set-point tracking, and to make the system less-sensitive to disturbances. It is also desirable to obtain a fast transient response to the input signal, a short settling time, and a low overshoot. Finally, since the complex property of the poles is known to introduce oscillations to a system, it will also be necessary to design a controller which will attenuate oscillations to the barest minimum if complete elimination is not possible.

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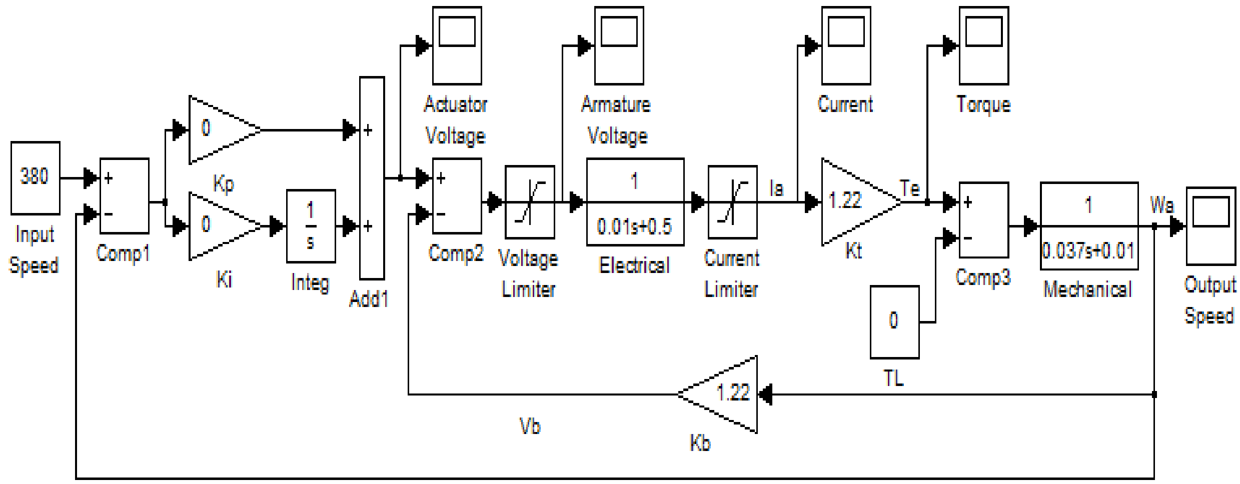

Fig. 3: PI controller

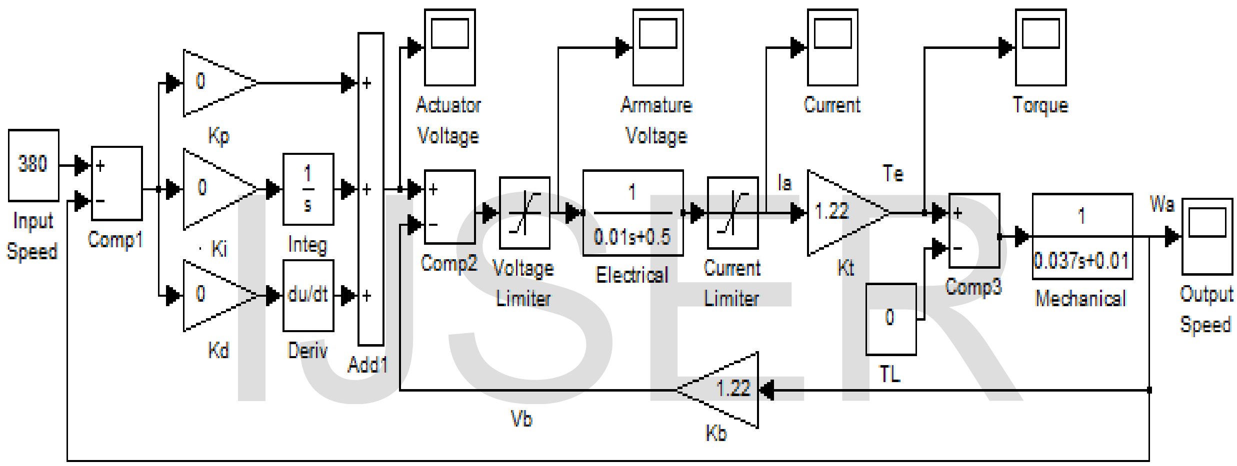

Fig. 4: PID controller

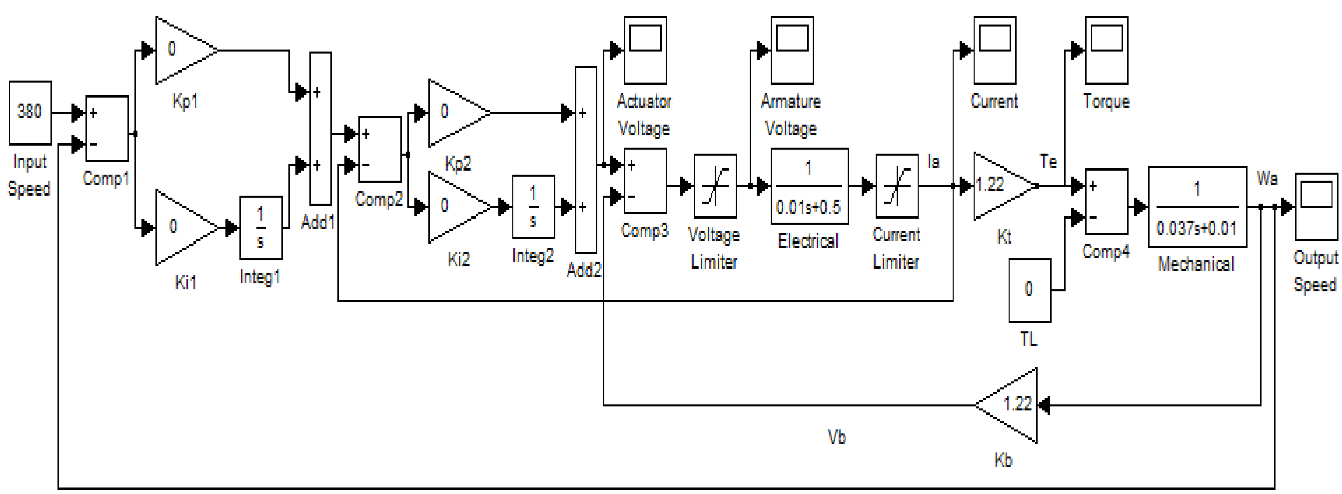

Fig. 5: Cascade PI controller

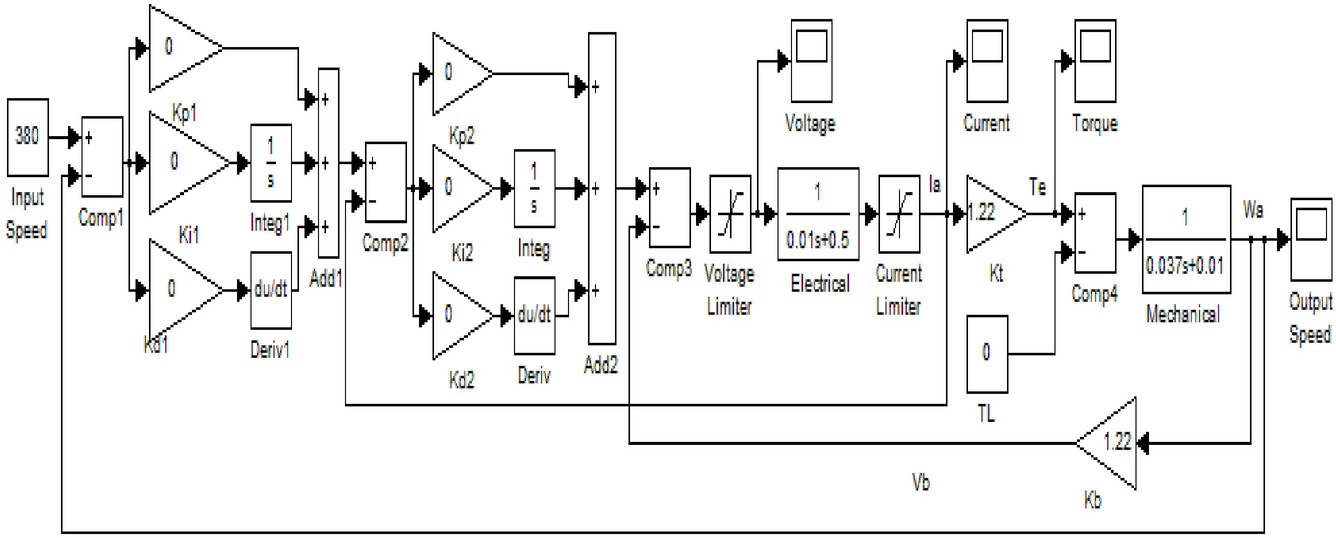

Fig. 6: Cascade PID controller

3 RESULTS AND DISCUSSION

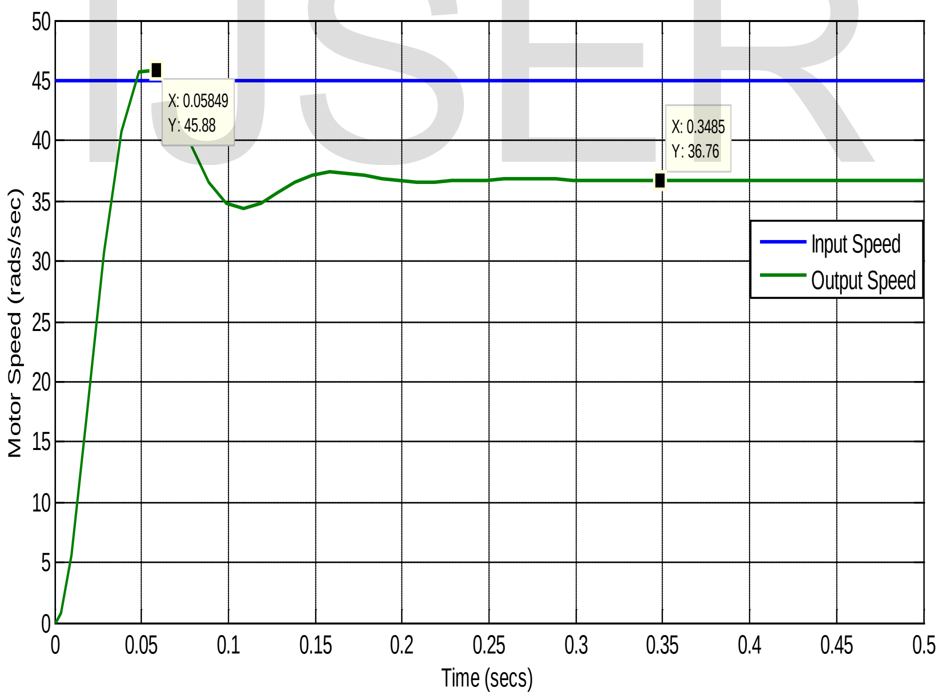

Fig. 7a: Open-loop response: $T_L = 0$Nm

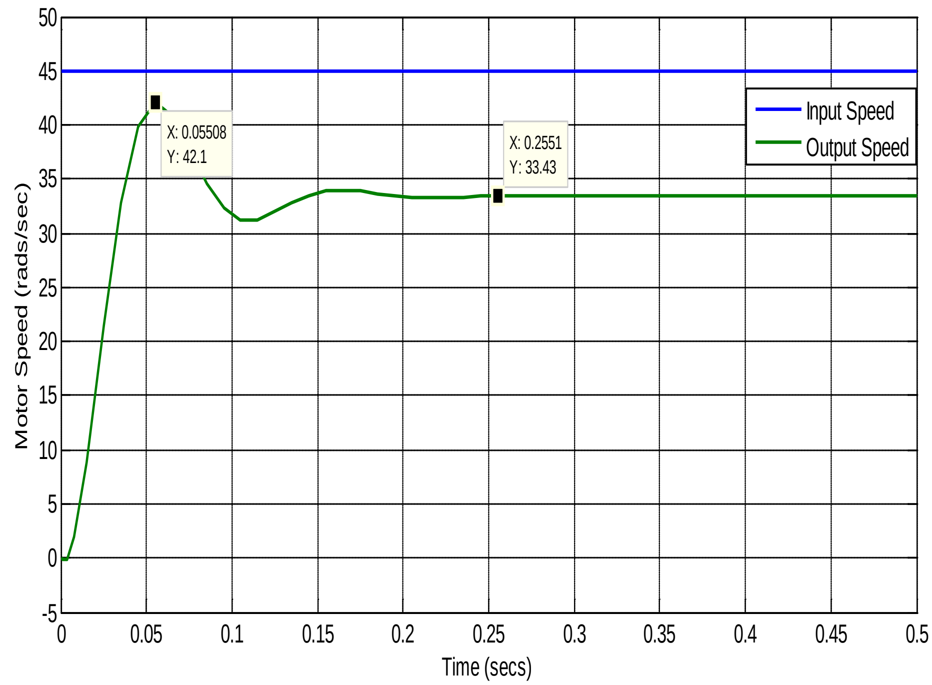

Fig. 7b: Open-loop response: T_L = 10Nm

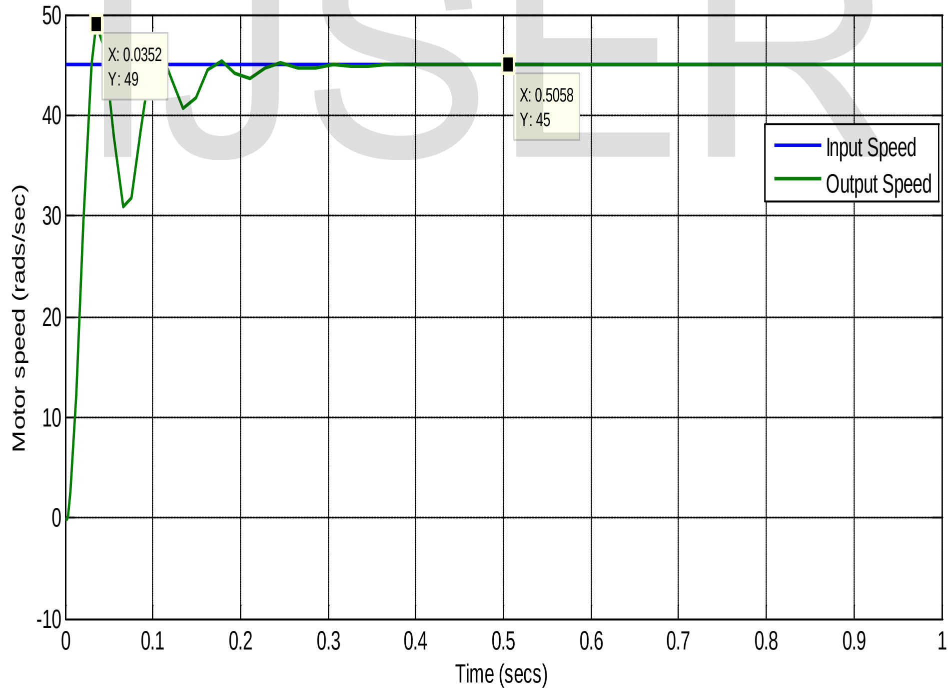

Fig. 8: Closed loop response for PI Controller with T_L = 10Nm (K_p = 1.6, K_i = 45)

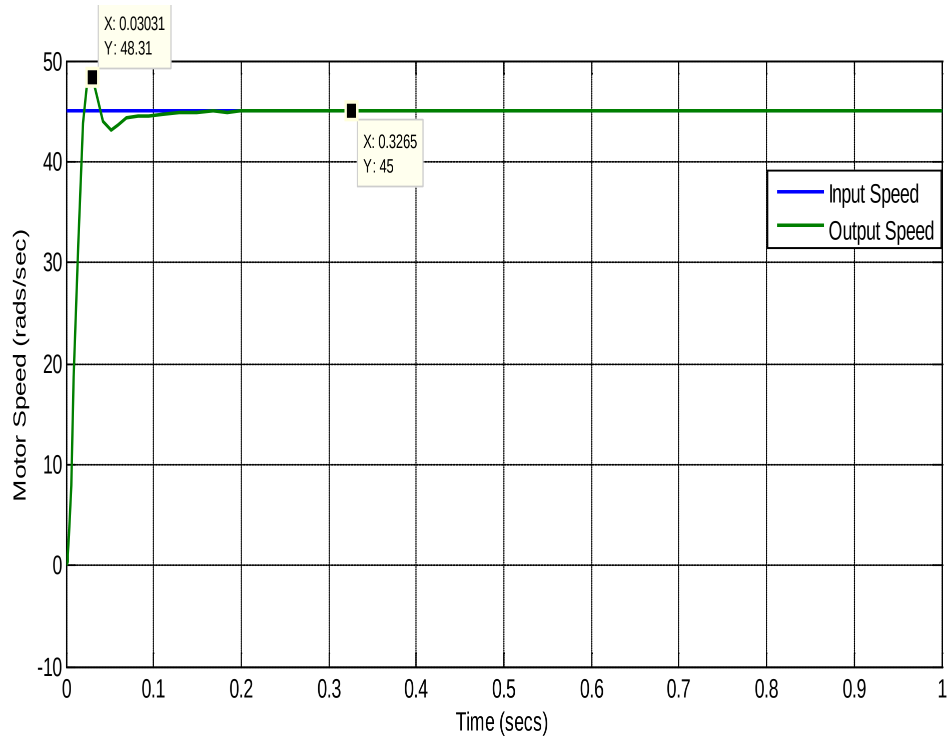

Fig. 9: Closed loop response for PID Controller with T_L = 10Nm (K_p = 5, K_i = 118, K_d = 0.03)

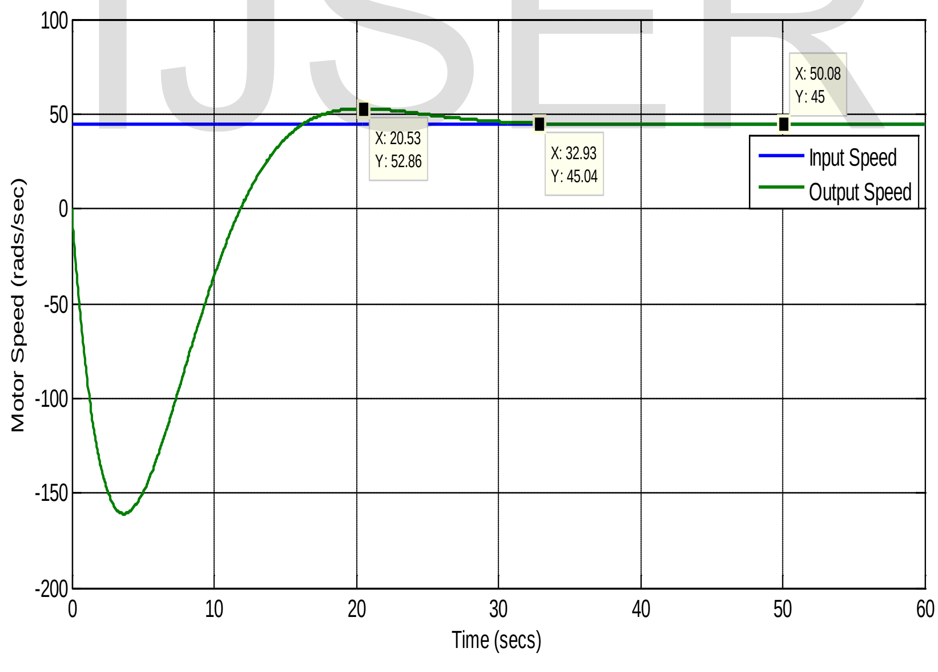

Fig. 10: Closed loop response for Cascade PI with T_L = 10Nm [(Inner Loop: K_p = 0.6, K_i = 31)

(Outer Loop: K_p = 0.019, K_i = 0.0051)]

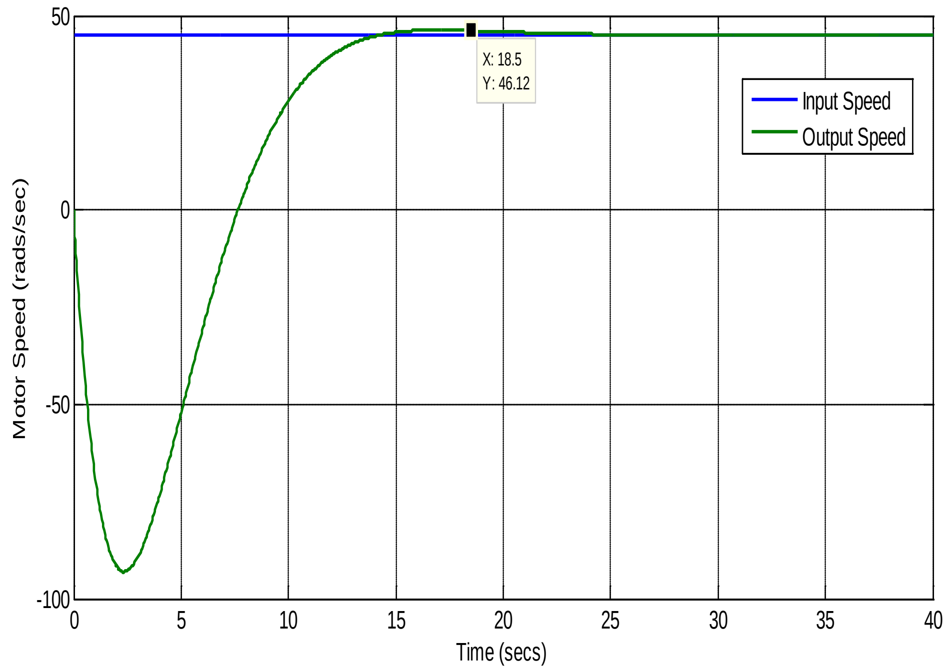

Fig. 11: Closed loop response for Cascade PI with T_L = 10Nm [(Inner Loop: *0.7*, $K_i = 33$, $K_d = 0.001$)

(Outer Loop: $K_p = 0.035$, $K_i = 0.01$, $K_d = 0.001$)]

The open-loop response of the plant as shown in Fig. 7b has a little overshoot but settles below set-point. Settling at a steady state value suggests a stable system but an inaccurate system since set-point is not tracked; hence, the need of a controller to eliminate any disturbances which introduced the error.

The PI controller as seen in Fig. 8 showed very high sensitivity, giving a small rise time and a zero steady state (S-S) error while also producing a good settling time (0.5 seconds, with and without a load). It however produced ripples in its output before maintaining S-S. This, as we know causes chattering in the shaft position of a PMDC motor and is not desired for smooth and accurate machinery operation. Although results are not shown, it was very insensitive to load disturbances as it still tracked set-point with a load of torque 100Nm. It showed high sensitivity, a high disturbance rejection but poor stability and robustness.

From Fig. 9, the PID controller showed a good disturbance rejection as it tracked set point and settled to S-S at about 0.33 seconds (with and without a load of 10 Nm). It also presents a smaller overshoot compared to the PI, and a very minimal ripple in its output. It offered a high sensitivity and showed stability and robustness

Cascade control was investigated to try to eliminate initial oscillations in output observed in the PI and PID controlled system, provide a smaller overshoot while maintaining a good settling time. The voltage and a current limiters where found to be redundant; nonetheless they were included for precaution. The cascade PI in Fig. 10 produced an acceptable overshoot but a slow rise time which still managed to track set-point and settle to S-S in about 32.9 seconds (approx. 9 mechanical time constants τ). The PI controlled system could handle a load of up to 20Nm but this was at the expense of its overshoot which peaked to 60 rads/sec. Its sensitivity was low, but this was compensated for in its high accuracy.

The Cascade PID as shown in Fig. 11 presents the best all-round performance, giving an insignificant level of overshoot,

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a zero S-S error and a reasonable settling time of about 24 seconds (approx. 6τ). Its sensitivity was low, but this was compensated for in its high accuracy and robustness.

4. CONCLUSION

The desire for any control design is to achieve three primary objectives namely:

1. Stability
2. Set-point tracking
3. Disturbance attenuation

Since system stability is not usually perfect, asymptotic stability is usually accepted. The output speed of the system as observed, tracked the set-point/reference input asymptotically for all four controllers. It can thus be concluded that the PID controller will be best suited for a system where sensitivity is the focus but if robustness is the desire, then the cascade-PID will be the best bet.

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