Properties of elements produced in a product of two wings & Analysis of Generalized Fermat Number by N-equation

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ABSTRACT

Natural Equation or simply N-equation is nothing but the systematic arrangement of all Pythagorean triplets, details of which was first published in August edition 2013 of this journal. Further developments of this N-equation took place intermittently with respect to its different properties and proof of different conjectures in Number Theory and were published in this journal in several bouts. Now this paper mainly contains the properties of four elements in an equality of two prime wings produced by the product of two prime wings and the analysis of Generalized Fermat Number to prove its composite nature with the help of N-equation. It also includes some special functions of composite nature and divisibility property of a prime number.

Keywords

Positive prime wing & negative prime wing; Positive composite wing & negative composite wing; In-symmetric, Ex-symmetric and Non-symmetric product or factor, conjugate wing, Power-wing's N-equation, Product wings & Produced wings.

1. Introduction

Once again we can recollect the definition and basic properties of a Natural equation or simply N-equation.
$a^2 + b^2 = c^2$ is said to be a N-equation when its comparable equation i.e. $(\alpha^2 - \beta^2)^2 + (2\alpha\beta)^2 = (\alpha^2 + \beta^2)^2$ has the property that α, β are the combination of odd & even positive integers. It is nothing but the systematic arrangement of all Pythagorean triplets a, b, c known as elements of the N-equation. According to this arrangement the N-equation has been divided into two kinds i.e. 1st kind includes where $k = c - b$ is in the form of $1^2, 3^2, 5^2$,…….and 2nd kind includes where $k = c - b$ is in the form of $2.1^2, 2.2^2, 2.3^2$, …….. assuming $a < b < c$ in both the cases. $(\alpha^2 \pm \beta^2)$ are said to be conjugate to each other and for $\gcd(\alpha, \beta) = 1$, they are known as positive or negative prime wings. If $\gcd(\alpha, \beta) \neq 1$ they are composite wings.
Any number, may be prime or composite if it fails to be expressed as $(\alpha^2 + \beta^2)$ is of 1st kind and if expressible it is of purely 2nd kind.
For a N-equation nature of RH odd element ± LH odd element = 2(integer)^2 and RH odd element ± LH even element = (odd integer)^2
The product & division rules of N_s operation are as follows.
$(e_1^2 + o_1^2).(e_2^2 + o_2^2) = (|e_1e_2 \pm o_1o_2|)^2 + (|e_1o_2 -/+ o_1e_2|)^2$ &
$(e_1^2 + o_1^2)/(e_2^2 + o_2^2) = \{|e_1e_2 \pm o_1o_2|/(e_2^2 + o_2^2)\}^2 + \{|e_1o_2 -/+ o_1e_2|/(e_2^2 + o_2^2)\}^2$ consider only one wing which has integer elements.
The product & division rules of N_d operation are as follows.

$(e_1^2 - o_1^2).(e_2^2 - o_2^2) = (|e_1e_2 \pm o_1o_2|)^2 + (|e_1o_2 \pm o_1e_2|)^2$ &
$(e_1^2 - o_1^2)/(e_2^2 - o_2^2) = \{|e_1e_2 \pm o_1o_2|/(e_2^2 - o_2^2)\}^2 + \{|e_1o_2 \pm o_1e_2|/(e_2^2 - o_2^2)\}^2$ consider only one wing which has integer elements. Here, e & o denotes even & odd integers respectively.
For $(a^2 + b^2)(c^2 + d^2) = v_1^2 + d_1^2 = v_2^2 + d_2^2$, $(a^2 + b^2)$ & $(c^2 + d^2)$ can be said as Product wings whereas $(v_1^2 + d_1^2)$ & $(v_2^2 + d_2^2)$ can be said as Produced wings. a, b, c, d, v, d all are known to be elements.
There exists following three kinds of N-equation in power form.
$a^n + b^2 = c^2$, $a^2 + b^{2n} = c^2$ & $a^2 + b^2 = c^n$ where n is any positive integer.
When two elements are found to be in power form it is not due to N-equation. It is for Nir-equation where one of the zygote elements must be irrational.
Based on the above theoretical background we can further discuss some other properties of N-equation and prove the generalized Fermat Number to be composite in nature for $n \ge m$.

2. A number of purely 2nd kind having n nos. of prime factors has exactly 2^{n-1} nos. of positive prime wings & a number of 1st kind or 2nd kind having n nos. of prime factors has exactly 2^{n-1} nos. of negative prime wings.

If P is a prime of 2nd kind it has exactly one positive prime wing.
Hence, P^m will also have exactly one positive prime wing as received by the formula in power form of c for a N-equation $a^2 + b^2 = c^2$ [Ref: Aug edition 2013 of IJSER]
Hence, $P_1^{m1}.P_2^{m2}$ will produce 2^1 prime wings & $P_1^{m1}.P_2^{m2}.P_3^{m3}$. will produce 2^{3-1} prime wings & so on.
Hence, a number having n nos. of prime factors has 2^{n-1} nos. of positive prime wings.
Similarly, for 1st kind or 2nd kind P^m has exactly one negative prime wing and on the same reason a number having n nos. of prime factors has 2^{n-1} nos. of negative prime wings.
If a number is a product of prime numbers only without any exponent of any prime then it has no composite wings. But if any prime is repeated twice or more then the number must have some composite wings [* in my paper published in Sept. edition 2015 it was written 'is repeated thrice or more' which is not correct. It may be ignored with example thereafter.]
The number can be expressed as product of two prime wings in 2^{n-2} ways. Because if we make two groups of r & n – r nos. of primes, the 1st group will produce 2^{r-1} prime wings & 2nd group will produce 2^{n-r-1} prime wings & total product of two prime wings = $2^{r-1}.2^{n-r-1} = 2^{n-2}$ nos.

3.1 If gcd(p, q) = 1 then (p + q), (p – q), pq, ($p^2 + q^2$) all are prime to each other.

We know, if gcd(a, b) = 1 then gcd(a^m, b^n) = 1 & gcd(a + bq, b) = 1 for any integer value of q & for q = 1,
gcd(a + b, b) = 1 $\Rightarrow$ for N-equation $a^2 + b^2 = c^2$ if gcd(a, b) = 1 then gcd(a^2, b^2) = 1 i.e. gcd($a^2 + b^2$, b^2) = 1
i.e. gcd(c^2, b^2) = 1 i.e. gcd(c, b) = 1 and similarly, gcd(c, a) = 1. Hence, a, b, c are prime to each other.
Now the comparable equation of $a^2 + b^2 = c^2$ is $(p^2 - q^2)^2 + (2pq)^2 = (p^2 + q^2)^2$ where gcd(p, q) = 1
Hence, (p + q), (p – q), pq, ($p^2 + q^2$) all are prime to each other.

3.2 In a product of $\pi\,(e_i^2 + o_i^2) = (e_1^2 + o_1^2)(e_2^2 + o_2^2)(e_3^2 + o_3^2)\ldots\ldots(e_n^2 + o_n^2) = E(v_j^2 + d_j^2) = (v_1^2 + d_1^2) = (v_2^2 + d_2^2) = (v_3^2 + d_3^2) = \ldots\ldots 2^{n-1}$ wings where the symbol π & E stand for continued product & equalities, e, v for even integers & o, d for odd integers if all ($e_i^2 + o_i^2$) are prime numbers i.e. gcd(e_i, o_i) =1 then gcd(v_j, d_j) = 1

We have the product by N_S operation $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = (e_1e_2 \pm o_1o_2)^2 + (e_1o_2 \quad e_2o_1)^2 = v_1^2 + d_1^2 = v_2^2 + d_2^2$
Given, gcd(e_1, o_1) = gcd(e_2, o_2) = 1 $\Rightarrow$ gcd(e_1, o_2) $\neq$ gcd(e_2, o_1) for > 1 & gcd(e_1, e_2) $\neq$ gcd(o_1, o_2) for > 1
Now, if gcd(e_1, o_2) = k_1 & gcd(e_2, o_1) = k_2 where gcd(k_1, k_2) = 1 then obviously gcd(v_1, d_1) = gcd(v_2, d_2) = 1
Again, if gcd(e_1, e_2) = k_1 & gcd(d_1, d_2) = k_2 where gcd(k_1, k_2) = 1 then also gcd(v_1, d_1) = gcd(v_2, d_2) = 1
So, in general in a product of several primes all the produced wings will have the property gcd(v_j, d_j) = 1 & gcd(two or more or all d) = 1 and gcd(odd part of two or more or all v) = 1
In a product of two wings i.e. $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = v_1^2 + d_1^2 = v_2^2 + d_2^2$ it can be easily understood that if

$\gcd(o_1, o_2) = \alpha$ then $\gcd(v_1, v_2) = \alpha$ & if $\gcd(e_1, e_2) = \beta$ then $\gcd(d_1, d_2) = \beta$

3.3 Both the odd elements of produced wings are of opposite kind.

The numbers which are of the form $4x - 1$ & $4x + 1$ can be defined as 1st kind & 2nd kind respectively. If the 2nd kind number is capable of producing a positive wing it is called 'Purely 2nd kind'. The 1st kind numbers cannot produce any positive wing.

If P_i & Q_i be the respective prime numbers of 1st kind & 2nd kind then all the numbers $\pi(Q_i^{mi})$ are of purely 2nd kind and $\pi(P_i^{mi}Q_i^{ni})$ are of 2nd kind nature only for $\sum m_i$ is even & of 1st kind nature if $\sum m_i$ is odd.

Now, in a product of two wings $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = (e_1e_2 \pm o_1o_2)^2 + (e_1o_2 \quad e_2o_1)^2$ the produced odd elements are $(e_1e_2 \pm o_1o_2)$. Say o_1o_2 is of the form $4x + 1 \Rightarrow e_1e_2 + o_1o_2$ is of the form $4x + 1$ but $e_1e_2 - o_1o_2$ is of the form $4x - 1$

Similarly, for o_1o_2 is of the form $4x - 1$, $e_1e_2 + o_1o_2$ is of the form $4x - 1$ but $e_1e_2 - o_1o_2$ is of the form $4x + 1$.

Hence, they are of opposite kind.

Note:

- All 1st kind numbers produce negative wings like $(\text{even})^2 - (\text{odd})^2$ & all 2nd kind numbers produce negative wings like $(\text{odd})^2 - (\text{even})^2$.
- Among all the produced wings, the odd elements of 1st kind & 2nd kind occur in equal nos. of cases.
- Sum of two same kind numbers or difference of two opposite kind numbers is always in the form of 2(odd nos.)
- Sum of two opposite kind or difference of two same kind is always in the form of 2^p(odd nos.) where $p > 1$.

4.1 In a product of $(e_1^2 + o_1^2)(e_2^2 + o_2^2)$ if $e_1e_2 > o_1o_2$ both the odd elements of the produced wings cannot be in the power form.

We have $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = (e_1e_2 \pm o_1o_2)^2 + (e_1o_2 \quad e_2o_1)^2 = v_1^2 + d_1^2 = v_2^2 + d_2^2$

For $e_1e_2 > o_1o_2$ say $e_1e_2 + o_1o_2 = u^m$ & $e_1e_2 - o_1o_2 = \omega^n \Rightarrow e_1e_2 = (u^m + \omega^n)/2$. But for any two odd integers $(u^m + \omega^n)$ is in the form of $2(2k + 1)$. Hence, $e_1e_2 =$ in odd integer is an absurd result.

Hence, both the odd elements cannot be in power form.

4.2 Ina prime wing of odd-even combination if $e^2 + o^2$ is in power form (power > 2) then $e^2 \sim o^2$ cannot be in power form or vice-versa.

As per earlier theorem for $e > o$, obviously $e^2 \sim o^2$ cannot be in power form.

For $o > e$ if we consider $o^2 + e^2 = u^m$ & $o^2 - e^2 = v^n$ then $o^2 = (u^m + v^n)/2$ & $e^2 = (u^m - v^n)/2$ which seems to be accepted. But in view of the fact that N_s operation & N_d operation both cannot run simultaneously to produce power of LH odd element $e^2 \sim o^2$ and that of RH odd element $e^2 + o^2$ simultaneously in a N-equation, we can conclude that $(e^2 + o^2)$ & $(e^2 \sim o^2)$ cannot produce power (> 2) simultaneously.

4.3 For a N-equation like $a^2 + (b_1b_2)^2 = (c_1c_2)^2$ where b is even, $(c_1^2 + b_1^2)(c_2^2 + b_2^2)$ will always produce a pair of prime wings where both the odd elements are in power form.

As $\gcd(b_1b_2, c_1c_2) = 1$, $\gcd(b_1, c_1) = \gcd(b_2, c_2) = 1$. Hence, produced wings are prime wings.

Now, $(c_1^2 + b_1^2)(c_2^2 + b_2^2) = (c_1c_2 \pm b_1b_2)^2 + (c_1b_2 \quad c_2b_1)^2$ where c_1c_2 obviously $> b_1b_2$ & $(c_1c_2 \pm b_1b_2)$ are always in the form of $(2k + 1)^2$, say I_1^{2m} & I_2^{2n}. Hence, $(c_1^2 + b_1^2)(c_2^2 + b_2^2) = I_1^{4m} + (c_1b_2 \quad c_2b_1)^2 = I_2^{4n} + (c_1b_2 \quad c_2b_1)^2$

Example: we have $33^2 + 56^2 = 65^2$ i.e. $33^2 + (4.14)^2 = (5.13)^2$. Hence, $(5^2 + 14^2)(13^2 + 4^2) = 11^4 + 162^2 = 3^4 + 202^2$.

We have $3^2 + 4^2 = 5^2$. Hence, $(1^2 + 2^2)(5^2 + 2^2) = 1^2 + 12^2 = 3^4 + 8^2$.

4.4 In a product of two positive prime wings like $\{(2^{2p-1}o_1)^2 + (c_2)^2\}\{(2^{2p-1}c_1)^2 + (o_2)^2\}$ both the even elements will be in power form with respect to N-equation $(o_1o_2)^2 + b^2 = (c_1c_2)^2$ where b is even.

Obviously, by properties of N-equation $(c_1c_2 \pm o_1o_2)$ are in the form of $2x^{2q}$ & $2y^{2r}$.
Hence, by N_s operation the even elements of the product are $\{2^{2p-1}(c_1c_2 \pm o_1o_2)\}$ i.e. $2^{2p-1}.2x^{2q}$ & $2^{2p-1}.2y^{2r}$
i.e. $(2^px^q)^2$ & $(2^py^r)^2$ i.e. e_1^{2m} & e_2^{2n} where m = gcd(p, q) & n = gcd(p, r)
Example: say, $33^2 + 56^2 = 65^2$ i.e. $(3.11)^2 + 56^2 = (5.13)^2$
Hence the required product = $\{(2^{2p-1}.3)^2 + 13^2\}\{(2^{2p-1}.5)^2 + 11^2\}$ i.e. $\{2^{2p-1}(13.5 \pm 3.11)\}^2 + (2^{4p-2}.15 \quad 13.11)^2$
i.e. $(2^{2p-1}.2.7^2)^2 + (15.2^{4p-2} - 143)^2$ & $(2^{2p-1}.2.4^2)^2 + (15.2^{4p-2} + 143)^2$
i.e. $(7.2^p)^4 + (15.2^{4p-2} - 143)^2$ & $2^{4(p+2)} + (15.2^{4p-2} + 143)^2$ where p = 1, 2, 3, …….
Say, p = 1 i.e. $(6^2 + 13^2)(10^2 + 11^2) = 14^4 + 83^2 = 2^{12} + 203^2$. For p = 2, $(24^2 + 13^2)(40^2 \; 11^2) = 28^4 + 817^2 = 2^{16} + 1103^2$
& so on.
Corollary1. When both the odd elements are in power, even elements cannot be in power form or vice-versa
Corollary2. All the four elements cannot be in power form i.e. $a^{2p} + b^{2q} = c^{2r} + d^{2s}$ has no existence for p, q, r, s > 1
Collary3. No element can be zero. Hence, $a^{2n} + b^{2n} = c^{2n}$ has no existence for n > 1

4.5 If there exists a N-equation like $(a_1a_2)^2 + (2^{2p}d_1d_2)^2 = (c_1c_2)^n$ where $(a_1d_1)^2 + b^{2r} = (a_2d_2)^2$ then there must exist a Product wings having all the four elements in power form where both the elements of a particular Produced wing are also in power form. n = 2, 3, 4, ……..

By N_s operation we have $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = (e_1e_2 + o_1o_2)^2 + (e_1o_2 \sim e_2o_1)^2 = (e_1e_2 \sim o_1o_2)^2 + (e_1o_2 + e_2o_1)^2$
Say, $e_1e_2 + o_1o_2 = (2^{2p}d_1d_2)^2 + (a_1a_2)^2$ where $e_1 = (2^pd_2)^2$ & $e_2 = (2^pd_1)^2$, $o_1 = a_1^2$ & $o_2 = a_2^2$
$\Rightarrow e_1e_2 + o_1o_2 = (c_1c_2)^n$ & n = 2, 3, 4, …… & $e_1o_2 - e_2o_1 = (2^p)^2\{(a_2d_2)^2 - (a_1d_1)^2\} = (2^p.b^r)^2 = v^{2m}$ where gcd(p, r) = m
Hence, both the elements are in power form & both powers can be equal also for which n must be even.
Elements of other wing i.e. $(o_1o_2 - e_1e_2)$ & $(e_1o_2 + e_2o_1)$ fail to produce power simultaneously even when $(o_1o_2 + e_1e_2)$ & $(e_1o_2 - e_2o_1)$ remain silent as it cannot be equated with any N-equation.
The existence of such type of N-equations cannot be ignored because very often we get the result of $(a^{2m} + b^{2n})$ composite nature.
Note: any one of the elements $(o_1o_2 \sim e_1e_2)$ & $(e_1o_2 + e_2o_1)$ must be silent to produce power.
The N-equation $(a_1a_2)^2 + (2^{2p}d_1d_2)^2 = (c_1c_2)^n$ can be said as Power-wing's N-equation or simply Np-equation when it can produce another N-equation like $(a_1d_1)^2 + b^{2r} = (a_2d_2)^2$.

4.7 If all the four elements of a Product wing have equal power greater than two then all the four elements of its Produced wing must be power free.

Let us consider the equality of Product wing & Produced wing:
$\{(e_1^n)^2 + (o_1^n)^2\}\{(e_2^n)^2 + (o_2^n)^2\} = \{(o_1o_2)^n \pm (e_1e_2)^n\}^2 + \{(e_1o_2)^n \quad (e_2o_1)^n\}^2$
Obviously, all the four elements of Produced wings are powerless for n > 2 as per FLT.
In more general way we can say that for GCD of all powers of 4 product elements > 2, all 4 produced elements are power free.
It confirms that if NP-equation exists it will exist only for n = 2 as shown below.
Say, $e_1 = 2^pd_1$, $e_2 = 2^qd_2$ & n = 2 to have a produced wings = $\{(o_1o_2)^2 \pm (2^pd_12^qd_2)^2\}^2 + \{(2^pd_1o_2)^2 \quad (2^qd_2o_1)^2\}^2$
Now, if we consider the existence of a N-equation $(o_1o_2)^2 + (2^{p+q}d_1d_2)^2 = c_1^m$ so that $(d_1o_2)^2 - (2^{q-p}d_2o_1)^2 = c_2^t$
i.e. called NP-equation, we can have a produced wing $c_1^{2m} + c_2^{2t} = \alpha^2 + \beta^2$ where α, β obviously powerless as $g^2 \pm h^2$ both cannot be in power form.

5. Proof of Fermat's Last Theorem (FLT)

Refer my paper published in May edition 2014 of this journal regarding Ramanujam Number of higher exponents. There the algebraic polynomial equation contains a free constant and produces pair of roots (a, b), (c, d), …… so as to form a relation $a^{2n+1} + b^{2n+1} = c^{2n+1} + d^{2n+1} =$ …….. Due to presence of free constant we cannot have a root zero so that a relation likes $a^{2n+1} + b^{2n+1} = c^{2n+1}$ will exist.

Now, we have proved $a^{2n} + b^{2n} = c^{2n}$ has also no existence.
Hence, follows the proof of FLT that $a^n + b^n = c^n$ has no existence for $n > 2$.

- Of course, N-equation clearly confirms that $a^x + b^y = c^z$ has no existence for x, y, z > 2 & where a, b, c are prime to each other. It also covers the proof of FLT.

6.1 $(2x^2)^{2(2r-1)} + d^{4q(2r-1)}$ where x, q, r all are of positive integers, always represents a composite number and divisible by the factors $(2x^2 \pm 2d^q x + d^{2q})$

By N_s operation we have the product of two symmetric factors with respect to x like
$\{(x + d)^2 + x^2\}\{x^2 + (x - d)^2\} = (2x^2)^2 + (d^2)^2 = (2dx)^2 + (2x^2 - d^2)^2$
$\Rightarrow (2x^2)^2 + (d^2)^2$ is always composite & $(2x^2)^2 \equiv - d^4 \pmod{2x^2 \pm 2dx + d^2}$
Now replacing d by d^q & then raising power $(2r - 1)$ on both sides $(2x^2)^{2(2r-1)} \equiv - d^{4q((2r-1)} \pmod{2x^2 \pm 2d^q x + d^{2q}}$
These two factors with respect to a common element x can be said as 'Product of symmetric factors' (PSF)
Two numbers are said to be symmetric to each other when their product produces two prime wings in the form of $(2x^2)^2 + (d^2)^2 = (2dx)^2 + (2x^2 - d^2)^2$
Example: PSF for x = 4 & d = 1, $= (3^2 + 4^2)(4^2 + 5^2) = (2.4^2)^2 + 1^4 = (2.1.4) + (2.4^2 - 1^2)^2$
i.e. $(3^2 + 4^2)(4^2 + 5^2) = 32^2 + 1^2 = 8^2 + 31^2$
On the contrary, the wing $32^2 + 1^2$ has the property $32 = 2.4^2$ form & $1 = 1^4$ form. So it must have PSF
i.e. $\{(4 + 1)^2 + 4^2\}\{4^2 + (4 - 1)^2\} = (5^2 + 4^2)(4^2 + 3^2)$
Find the symmetric numbers of 65^2:
65 is the product of two primes i.e. $5.13 \Rightarrow 65^2$ has two prime wings
i.e. $65^2 = (3^2 + 4^2)(5^2 + 12^2) = 33^2 + 56^2 = 16^2 + 63^2$.
Hence, Sym(65^2) with respect to $33 = 33^2 + \{33 - (56 - 33)\}^2 = 33^2 + 10^2 = 1189$
$\Rightarrow 65^2.1189 = (2.33^2)^2 + 23^4 = (2.23.33)^2 + (2.33^2 - 23^2)^2$ i.e. $65^2.1189 = 2178^2 + 23^4 = 1518^2 + 1649^2$
Again, Sym(65^2) with respect to $56 = 56^2 + \{56 + (56 - 33)\}^2 = 56^2 + 79^2 = 9377$
$\Rightarrow 65^2.9377 = (2.56^2)^2 + 23^4 = (2.23.56)^2 + (2.56^2 - 23^2)^2$ i.e. $65^2.9377 = 6272^2 + 23^4 = 2576^2 + 5743^2$
Similarly, Sym(65^2) with respect to $16 = 16^2 + 31^2 = 1217$ & with respect to $63 = 63^2 + 110^2 = 16069$
So, if a number N of 2nd kind has n nos. of prime factors then the number of Sym(N) = 2^n.
Number of 1st kind has no symmetric numbers.

6.2 With respect to N-equation $a^2 + b^2 = c^n$, $(c^n \sim 4x^2)^2 + \{4.(a \text{ or } b).x\}^2$ always represents a composite number for x, n are of any positive integer.

The above product can be rightly said as 'Product of symmetric factors internally' (Posfi).
'Product of symmetric factors externally' (Posfe) is with respect to y ($y \neq x$) i.e. $\{(x + d)^2 + y^2\}\{(x - d)^2 + y^2\} =$
$(x^2 + y^2 - d^2)^2 + (2yd)^2 = (x^2 - y^2 - d^2)^2 + (2xy)^2$.
Replacing x, y by a, b & d by any even number e_i, we have $(c^n \sim e_i^{\,2})^2 + \{2.(a \text{ or } b).\, e_i\}^2$
i.e. $(c^n \sim 4x^2)^2 + \{4.(a \text{ or } b).x\}^2$ that always represents a composite number for x, n of any positive integer.
Or, in general $(c^n \sim 4x^2)^{2(2r-1)} + \{4.(a \text{ or } b).x\}^{2(2r-1)}$ represents a composite number for x, n, r = 1, 2, 3, ……..
* c can be any integer of 2nd kind which is always expressible in the form of $a^2 + b^2$ and corresponding factors are $c^n + 4x^2 \pm 4(a \text{ or } b)x$ (can be said as ex-symmetrical factors)
There can be three designated terms: In-symmetric product, Ex-symmetric product, Non-symmetric product.
Non-symmetric product is the most general case.

6.3 In an equality of two prime wings i.e. $a_1^2 + b_1^2 = a_2^2 + b_2^2$ maximum three elements can be in power form

For double wings equality we can have maximum three elements in power form for a particular case of a product in between two In-symmetric factors.
We have the In-symmetric pair of wings with respect to x, $(2x^2)^2 + (d^2)^2 = (2xd)^2 + (2x^2 - d^2)^2$.

Put $x = 2$ & replace d by d^{2m} i.e. $2^6 + d^{8m} = (2d^m)^4 + (2^3 \sim d^{4m})^2$ where obviously $2^3 \sim d^{4m}$ is a powerless number.
$d^{4m} \pm 4d^{2m} + 8$ i.e. $(d^{2m} \pm 2)^2 + 2^2$ are the two in-symmetric factors with respect to 2, product of which (Posfi) produces two prime wings having three elements in power form where two exponents are fixed and one is variable i.e. 8m. All these numbers can be denoted by $(Posfi)_{max}$ & $= 2^6 + 1^8, 2^6 + 3^8, 2^6 + 5^8, \ldots$
Or $= (2.1)^4 + (2^3 - 1^4), (2.3)^4 + (2^3 - 3^4), (2.5)^4 + (2^3 - 5^4), \ldots$ for $m = 1$ i.e. $(Posfi)_{max} = 65, 6625, 390689, \ldots$
$(Posfi)_{max}$ may have several wings. Because the in-symmetric factors of it may have again sub-factors. But out of all wings only one pair can show power form of maximum elements. If $(Posfi)_{max}$is multiplied by any factor of 2nd kind its nature of 'Maximum elements in power form' is destroyed.
Example: say $d = 3$ & $m= 2$. Hence, $(Posfi)_{max} = \{(3^4 + 2)^2 + 2^2\}\{(3^4 - 2)^2 + 2^2\}$
i.e. $(83^2 + 2^2)(79^2 + 2^2) = 3^{16} + 2^6 = 6553^2 + 18^4$ by N_S operation.
In more generalized way we can put $x = 2^n$ & replace d by d^m to receive $(2^{2n+1})^2 + (d^{2m})^2 = (v^p)^2 + d_0^2$ where $p = \gcd(n + 1, m)$ & > 1 & d_0 is powerless.
Corollary: if $(2^p.d)^{2m} + o^{2n}$ is composite for odd factor $d \neq 1$, then elements of all the produced wings are powerless. It once again confirms that all th elements of produced wings of a Generalized Fermat Number are powerless except the number-form itself.

7.1 In a pair of produced positive prime wings $(OE)_{max\ or\ min}$ & $(EE)_{max\ or\ min}$ lie on opposite sides of equality & for produced negative prime wings they lie on the same side. If none of the product wings contains consecutive elements then in pair of produced wings $(OE)_{max}$ or min $\pm$ $(EE)_{max\ 0r\ min}$ are composite. [OE & EE denotes odd & even elements respectively]

We have by N_s operation $(e_1^2 + o_1^2)(e_2^2 + o_2^2) = (e_1e_2 + o_1o_2)^2 + (e_1o_2 \sim e_2o_1)^2 = (e_1e_2 \sim o_1o_2)^2 + (e_1o_2 + e_2o_1)^2$
$\Rightarrow$ Max OE & Max EE or Min OE & Min EE lie opposite sides of equality.
Now, $(OE)_{max} + (EE)_{max} = (e_1 + o_1)(e_2 + o_2) =$ composite
$(OE)_{min} + (EE)_{min} = (e_1 - o_1)(e_2 + o_2) =$ composite if e_1 & o_1 are not consecutive.
$(OE)_{max} \sim (EE)_{max} = (e_1 - o_1)(e_2 - o_2) =$ composite if e_1, o_1 & e_2, o_2 are not consecutive.
$(OE)_{min} \sim (EE)_{min} = (e_1 + o_1)(e_2 - o_2) =$ composite if e_2 & o_2 are not consecutive.
Similarly, for N_d operation i.e. $(e_1^2 \sim o_1^2)(e_2^2 \sim o_2^2) = (e_1e_2 \pm o_1o_2)^2 - (e_1o_2 \pm e_2o_1)^2$ where both $(OE)_{max\ or\ min}$ & $(EE)_{max\ or\ min}$ lie on same side we can observe the same phenomenon.

Note: If all the elements of Product wings are in equal power form then its Produced wings must have all these composite properties as no two elements of Product wings are possible to be consecutive. It is true for different powers also excepting the case 2^3 & 3^2.
In all In-symmetric pair of Produced wings all these composite properties exist. This implies that in a pair of Produced wings where maximum three elements are in power form all these composite properties exist.

7.2 In an equality of two positive prime wings $d_1^2 + v_1^2 = d_2^2 + v_2^2$ where d, v are not consecutive & denote odd & even integers respectively we have

7.2.1 $|d_1^2 \pm v_1^2| \pm 2d_2v_2$ is composite (4 cases) & $|d_2^2 \pm v_2^2| \pm 2d_1v_1$ is composite (4 cases)
7.2.2 $|d_1^4 + v_1^4 - 6d_1^2v_1^2| + 4d_2v_2(d_2^2 - v_2^2) \in$ composite & $|d_2^4 + v_2^4 - 6d_2^2v_2^2| + 4d_1v_1(d_1^2 - v_1^2) \in$ composite

We have $d_1^2 + v_1^2 = d_2^2 + v_2^2$ & squaring both sides $(d_1^2 \sim v_1^2)^2 + (2d_1v_1)^2 = (d_2^2 \sim v_2^2)^2 + (2d_2v_2)^2$
$\Rightarrow |d_1^2 - v_1^2| \pm 2d_2v_2$ is composite and $|d_2^2 - v_2^2| \pm 2d_1v_1$ is composite
Again, $(d_1^2 + v_1^2)^2 = (d_2^2 \sim v_2^2)^2 + (2d_2v_2)^2 = d_3^2 - v_3^2 + (2d_2v_2)^2$ [assuming at least 4 prime factors of $(d_2^2 \sim v_2^2)$, Ref 8]
i.e. $(d_1^2 + v_1^2)^2 + v_3^2 = d_3^2 + (2d_2v_2)^2 \Rightarrow (d_1^2 + v_1^2) \pm 2d_2v_2$ is composite & similarly, $(d_2^2 + v_2^2) \pm 2d_1v_1$ is composite.
Again, from $\{(d_1^2 \sim v_1^2)^2 + (2d_1v_1)^2\}^2 = \{(d_2^2 \sim v_2^2)^2 + (2d_2v_2)^2\}^2$ we get the relation
$\{(d_1^2 \sim v_1^2)^2 - (2d_1v_1)^2\}^2 + \{4d_1v_1(d_1^2 \sim v_1^2)\}^2 = \{(d_2^2 \sim v_2^2)^2 - (2d_2v_2)^2\}^2 + \{4d_2v_2(d_2^2 \sim v_2^2)\}^2$
$\Rightarrow |d_1^4 + v_1^4 - 6d_1^2v_1^2| + 4d_2v_2(d_2^2 - v_2^2) \in$ composite & $|d_2^4 + v_2^4 - 6d_2^2v_2^2| + 4d_1v_1(d_1^2 - v_1^2) \in$ composite

7.3 For α is odd & β is even where $\gcd(\alpha, \beta) = 1$ and $\alpha \sim \beta \neq 1$

7.3.1 $\left| \alpha^{2n+1} - {}^{2n+1}c_2\alpha^{2n-1}\beta^2 + {}^{2n+1}c_4\alpha^{2n-3}\beta^4 - \ldots\ldots \right| \pm (\alpha^2 + \beta^2)^n\beta \in$ **composite**

7.3.2 $\left| {}^{2n+1}c_1\alpha^{2n}\beta - {}^{2n+1}c_3\alpha^{2n-2}\beta^3 + \ldots\ldots \right| \pm (\alpha^2 + \beta^2)^n\alpha \in$ **composite**

7.3.3 $\left| \alpha^{2n} - {}^{2n}c_2\alpha^{2n-2}\beta^2 + {}^{2n}c_4\alpha^{2n-4}\beta^4 - \ldots\ldots \right| \pm (\alpha^2 + \beta^2)^{n-1}(2\alpha\beta) \in$ **composite**

7.3.4 $\left| {}^{2n}c_1\alpha^{2n-1}\beta - {}^{2n}c_3\alpha^{2n-3}\beta^3 + \ldots\ldots \right| \pm (\alpha^2 + \beta^2)^{n-1}(\alpha^2 - \beta^2) \in$ **composite**

We know when c element of a N-equation produces power.
$(\alpha^n - {}^nc_2\alpha^{n-2}\beta^2 + {}^nc_4\alpha^{n-4}\beta^4 - \ldots\ldots)^2 + ({}^nc_1\alpha^{n-1}\beta - {}^nc_3\alpha^{n-3}\beta^3 + \ldots\ldots)^2 = (\alpha^2 + \beta^2)^n$.
If we replace n by 2n + 1 RHS = $\{(\alpha^2 + \beta^2)^n\alpha\}^2 + \{(\alpha^2 + \beta^2)^n\beta\}^2$ hence follows the proof.
Similarly, If we replace n by 2n RHS = $\{(\alpha^2 + \beta^2)^{n-1}(\alpha^2 - \beta^2)\}^2 + \{(\alpha^2 + \beta^2)^{n-1}(2\alpha\beta\}^2$ hence follows the proof.
If $\alpha \sim \beta = 1$ any one of 7.3.1 or 7.3.2 and of 7.3.3 or 7.3.4 must be composite.

7.4 If $a^2 + b^2 = \alpha^2 + \beta^2$ where a, b & α, β are non-consecutive odd even integers

7.4.1 $\left| a^n - {}^nc_2a^{n-2}b^2 + {}^nc_4a^{n-4}b^4 - \ldots\ldots \right| \pm \left| {}^nc_1\alpha^{n-1}\beta - {}^nc_3\alpha^{n-3}\beta^3 + \ldots\ldots \right| \in$ **composite**

7.4.2 $\left| \alpha^n - {}^nc_2\alpha^{n-2}\beta^2 + {}^nc_4\alpha^{n-4}\beta^4 - \ldots\ldots \right| \pm \left| {}^nc_1a^{n-1}b - {}^nc_3a^{n-3}b^3 + \ldots\ldots \right| \in$ **composite**

Same as above and it is the general case of 7.2.1 & 7.2.2.
Here all the formulae under 7.3 are applicable if in the 2nd part or in the 1st part α, β are replaced by a, b.

8. How Generalized Fermat Number (GF_n) is formed and its proof of composite nature for $n \geq m$ where GF_m is composite.

We have $a \pm b = c$ or $a^2 + b^2 = c^2 \pm 2ab = d$. or, $a^{2^2} + b^{2^2} = d^2 - 2a^2b^2 = d_1$
& similarly, $a^{2^3} + b^{2^3} = d_1^2 - 2a^{2^2}b^{2^2} = d_2$ and so on.
Here, $(e_1e_2 + o_1o_2)^2 + (e_1o_2 \sim e_2o_1)^2$ or $(o_1o_2 \sim e_1e_2)^2 + (e_1o_2 + e_2o_1)^2$ is being replaced element wise. In earlier theorem one element was replaced by a complete wing.
Hence, in all the product wings of GF_n only one wing can show full power form of elements. Elements of all the produced wings except $(a^{2^n} + b^{2^n})$ are power free. If a, b, c are prime to each other all the factors of GF_n are prime numbers and gcd(any two GF_n) = 1. For a common factor in between any two GF_ns there must be the same common factor in between b & c or a & c.
Let us assume GF_m is composite for n = m & $GF_m = a^{2^m} + b^{2^m} = \alpha^2 + \beta^2 = pq$.
Now, $(a^{2^m})^4 - (b^{2^m})^4 = GF_{m+1}.(GF_m).(a^{2^m} - b^{2^m})$ = product of at least four prime factors which is always expressible in the form of $(\alpha_1^2 - \beta_1^2)(\alpha_2^2 - \beta_2^2)$ where α_1, β_1 & α_2, β_2 are not consecutive.
$\Rightarrow (a^{2^{m+1}})^2 - (b^{2^{m+1}})^2 = \alpha^2 - \beta^2$. Hence, $(a^{2^{m+1}} + b^{2^{m+1}})$ i.e. GF_{m+1} is composite.
So, once a Generalized Fermat Number is formed as composite for n = m it will be always composite for $n \geq m$.
Corollary:

8.1 For a relation $\alpha_1^2 + \beta_1^2 = \alpha_2^2 + \beta_2^2 = \alpha_3^2 + \beta_3^2 = \ldots\ldots$, $\alpha_1^{2^n} + \beta_1^{2^n}$, $\alpha_2^{2^n} + \beta_2^{2^n}$, $\alpha_3^{2^n} + \beta_3^{2^n}$ all are composite.

8.2 For o-e combination where gcd(a, b) = 1 if $a^2 - b^2$ has at least 4 prime factors then $a \pm b \in$ composite.

8.3 GF_n is bound to be composite for $n \leq 5$.

9.1 If (2p + 1) is prime then $p^n \not\equiv x^n \pmod{2x + 1}$ and $(p + 1)^n \not\equiv (-x)^n \pmod{2x + 1}$

If any number $(2p + 1) = (p + 1)^2 - (p)^2$ is divisible by another number $(x + 1)^2 - (x)^2$ then by division rule of N_d operation we have the quotient = $[\{(p + 1)(x + 1) \pm px\}/\{(x + 1)^2 - (x)^2\}]^2 - [\{(p + 1)x \pm p(x + 1)\}/\{(x + 1)^2 - (x)^2\}]^2$
$= \{(2px + x + p + 1)/(2x + 1)\}^2 - \{(2px + x + p)/(2x + 1)\}^2$ or $\{(x + p + 1)/(2x + 1)\}^2 - \{(p - x)/(2x + 1)\}^2$
Obviously 1st one does not stand. It is true when x = 0 i.e. (2p + 1) is divisible by 1.
But from 2nd one we can say $p + 1 \equiv -x \pmod{2x + 1}$ i.e. $(p + 1)^n \equiv (-x)^n \pmod{2x + 1}$
& $p \equiv x \pmod{2x + 1}$ i.e. $p^n \equiv x^n \pmod{2x + 1}$ where there exists at least one value of x which will satisfy both.
Hence, it follows the proof when (2p + 1) is prime, there is no existence of x.

Note: the above two theorems are independently true. It will never happen that in one case it is integer & in other case it is fraction as because sum of the numerators is the number itself i.e. $(2p + 1)$. This implies that if $(2x + 1)$ divides $(p - x)$ it will also divide $(p + 1 + x)$ or vice-versa.

9.2 Conversely, if $p^n \not\equiv x^n \pmod{2x + 1}$ or $(p + 1)^n \not\equiv (-x)^n \pmod{2x + 1}$ for $x < (p - 1)/3$ then $(2p + 1)$ is a prime.

Here obviously $x + p + 1 > 2x + 1$ and $p - x > 2x + 1 \Rightarrow x < (p - 1)/3$

9.3 If all $(2p_i + 1)$ are primes then $(\sum p_i^n) \not\equiv rx^n \pmod{2x + 1}$ & $\sum(p_i + 1)^n \not\equiv r(-x)^n \pmod{2x + 1}$ where $i = 1, 2, 3, \ldots\ldots, r$

9.4 If all $(2p_i + 1)$ are primes then $(\prod p_i^n) \not\equiv x^{rn} \pmod{2x + 1}$ & $\prod(p_i + 1)^n \not\equiv (-x)^{rn} \pmod{2x + 1}$ where $i = 1, 2, 3, \ldots\ldots, r$

The proof of the above two theorems 9.3 & 9.4 can be given below.
If $a_1 \equiv b_1 \pmod m$, $a_2 \equiv b_2 \pmod m$, $a_3 \equiv b_3 \pmod m$, …….. then $a_1 = b_1 + mk_1$, $a_2 = b_2 + mk_2$, $a_3 = b_3 + mk_3$…….
Adding all $\sum a_i \equiv \sum b_i \pmod m$. Similarly multiply all $\prod a_i \equiv \prod b_i \pmod m$.
Now we can replace a by p^n or $(p + 1)^n$ and b by x^n or $(-x)^n$ and $\equiv$ by $\equiv/$ for prime number of $(2p + 1)$

10.1 N-equation for generalized Fermat Number and its unit digit analysis.

Generalized Fermat Number can be defined as $F_n = \alpha^{2^n} + \beta^{2^n}$ where α, β are the combination of odd and even integers with $\gcd(\alpha, \beta) = 1$ and N-equation of a generalized Fermat Number can be written as
$a^2 + b^2 = (\alpha^{2^n} + \beta^{2^n})^2 = F_n^2$ where say, a is odd & b is even.
In general for $n > 1$, unit digit(UD) of $F_n = 7$ i.e. $F_n = (U_{4\text{ or }6})^2 + (U_{1\text{ or }9})^2$ [U_x means a number with unit digit x]
In all four cases UD of $\{(U_{4\text{ or }6})^2 \sim (U_{1\text{ or }9})^2\}$ is $5 \Rightarrow a = U_5$.
Say, $F_{n'}$ denotes all other wings of F_n when F_n is composite.
Now, $\mathrm{Conj}(F_n) = a = (\alpha^{2^n} - \beta^{2^n}) = F_{n-1}.\mathrm{Conj}(F_{n-1}) = F_{n-1}.F_{n-2}.\mathrm{Conj}(F_{n-2}) = \ldots\ldots\ldots$ and finally $\mathrm{Conj}(F_n) = F_{n-1}.F_{n-2}.F_{n-3}\ldots\ldots.F_1.F_0.\mathrm{Conj}(F_0)$. Here, any one of the last three factors must contain a sub-factor 5 depending upon the UD nature of (α, β) combination.
$\mathrm{Conj}(F_{n'})$ also contains a factor 5.

10.2 If F_n is composite say $F_n = (o_1^2 + e_1^2)(o_2^2 + e_2^2)$ then $\mathrm{Conj}(F_n) \pm \mathrm{Conj}(F_{n'}) = 2dd_1d_2$ where $d = (o_1o_2 + e_1e_2 + o_1e_2 + o_2e_1)$, $d_1 = |o_1 - e_1|$, $d_2 = |o_2 - e_2|$

We have $F_n = (o_1^2 + e_1^2)(o_2^2 + e_2^2) = (o_1o_2 \pm e_1e_2)^2 + (o_1e_2 \mp o_2e_1)^2$
Say, $F_n = (o_1o_2 - e_1e_2)^2 + (o_1e_2 + o_2e_1)^2$ & $F_{n'} = (o_1o_2 + e_1e_2)^2 + (o_1e_2 - o_2e_1)^2$
$\Rightarrow \mathrm{Conj}(F_n) = (o_1e_2 + o_2e_1)^2 - (o_1o_2 - e_1e_2)^2 = \pm (d - 2e_1e_2)(d - 2o_1o_2)$
& $\mathrm{Conj}(F_{n'}) = (o_1o_2 + e_1e_2)^2 - (o_1e_2 - o_2e_1)^2 = \pm (d - 2\,o_2e_1)(d - 2o_1e_2)$
$\Rightarrow \pm \mathrm{Conj}(F_n) = d^2 - (\omega_1 + \omega_2)d + \omega_1\omega_2$ & $\pm \mathrm{Conj}(F_{n'}) = d^2 - (\omega_3 + \omega_4)d + \omega_3\omega_4$ (say) where $\omega_1\omega_2 = \omega_3\omega_4$ and subtracting both $\mathrm{Conj}(F_n) \pm \mathrm{Conj}(F_{n'}) = 2d|o_1 - e_1||o_2 - e_2|$
Now, for F_n & $F_{n'}$ if the wing $(\text{odd})^2 + (\text{even})^2$ shows in both cases (odd) < (even) or (even) < (odd) then (+) sign is to be considered otherwise for opposite nature (–) sign is to be considered. It can be easily established by the following nature of integers.
$\mathrm{Conj}(F_n) \pm \mathrm{Conj}(F_{n'})$ must be of the form 2(odd no.).
Now, if P & Q both are of the form $4x + 1$ or $4x - 1$ then $P + Q$ is of the form 2(odd no.) & for opposite form $P - Q$ is of the form 2(odd no.). Moreover, $(\text{odd})^2 - (\text{even})^2$ is always of the form $4x + 1$ but $(\text{even})^2 - (\text{odd})^2$ is always $4x - 1$ form.
If F_n has r nos. of factors, obviously all are primes, then there will exist $(2^{r-1} - 1)$ nos. of sets of (o_1, o_2, e_1, e_2) for which above theorem will be true.

10.3 Generalized Fermat Number (F_n) always represents a composite number for n > m from a different angle of view.

We have $\{Conj(F_{n+1})\}^2 + (b_{n+1})^2 = (F_{n+1})^2$ and $\{Conj(F_n)\}^2 + (b_n)^2 = (F_n)^2$
$\Rightarrow \{(F_{n+1})^2 - (b_{n+1})^2\} / \{(F_n)^2 - (b_n)^2\} = \{Conj(F_{n+1})\}^2 / \{Conj(F_n)\}^2 = (F_n)^2$
Or, $\{Conj(F_{n+1})\}^2 = (F_n)^4 - (F_n b_n)^2$ or, $\{Conj(F_{n+1})\}^2 + (F_n b_n)^2 = (F_n)^4$
Now, $Conj(F_{n+1})$ has several negative wings and with respect to any wing $Conj(F_{n+1})$ & $(F_n)^2$ cannot be conjugate to each other unless F_n is replaced by $F_n{}'$ for equivalent wing of $(F_n)^2$ i.e. $\{Conj(F_{n+1})\}^2 + (F_n{}' b_n)^2 = (F_n{}')^4$
$\Rightarrow$ this N-equation exists when F_n is composite. Similarly, the equation $\{Conj(F_{n+2})\}^2 + (F_{n+1}{}' b_{n+1})^2 = (F_{n+1}{}')^4$ will exist when F_{n+1} is composite and so on.
So, once F_n is found to be composite for n = m, it will remain composite for any integer of n > m.

10.4 Generalized Fermat Number (F_n or $F_n{}'$) is always expressible in the form of $\{(15x \pm 2y)/13\}^2 + \{(10x \mp 3y)/13\}^2$

As the left hand odd element of a N-equation for a generalized Fermat Number is always multiple of 5, let us consider the N-equation $(p)^2 + b_1{}^2 = \{ a^{2^n} + b^{2^n} \}^2 = (F_n)^2$ where $p = (\alpha^2 - \beta^2)(3^2 - 2^2) = (3\alpha \pm 2\beta)^2 - (2\alpha \pm 3\beta)^2$
$\Rightarrow (3\alpha \pm 2\beta)^2 + (2\alpha \pm 3\beta)^2 = F_n$
$\Rightarrow 13\alpha^2 + 13\beta^2 \pm 24\alpha\beta - F_n = 0 \Rightarrow \alpha = 1/13[\pm 12\beta \pm \sqrt{(13F_n - 25\beta^2)}] \Rightarrow 13 F_n - 25\beta^2 = I^2$
Or, $F_n = \{I^2 + (5\beta)^2\}/(3^2 + 2^2)$ or, by division rule $F_n = \{(15\beta \pm 2I)/13\}^2 + \{(10\beta \quad 3I)/13\}^2$
$\Rightarrow$ Generalized Fermat Number is always expressible in the form of $\{(15x \pm 2y)/13\}^2 + \{(10x \quad 3y)/13\}^2$

Conclusion:

Like N-equation there also exists another equation i.e. N-equation with irrational zygote element and may be denoted by Nir-equation which produces two elements in power form. It was elaborately discussed in Aug-publication 2013. I believe that the existence of Nir-equation should not cast any reverse shadow to the proof & truthfulness of all the theorems that have been discussed here. But regarding fractional power formation (i.e. odd integer/2) of elements in an equality of two prime wings like $a^x + b^y = c^z + d^\omega$, Nir-equation may also play a positive role. It needs further investigation. Regarding newly born Power-wing's N-equation i.e. Np-equation question is whether it at all exists or not? If exists, it may also play a vital role to unveil so many properties of Generalized Fermat Number. Examples towards the fact that both the elements of one Product wing and one Produced wing are in power form can be frequently noticed. Fermat's first composite number (F_5) is an example of it i.e. $(2^4 + 5^4)(2556^2 + 409^2) = (2^{2^4})^2 + 1^2 = 20449^2 + 62264^2 = 4294967297$. It seems that to form a Generalized Fermat number it does not matter whether Np-equation exists or not.

References

[1] Academic text books of Algebra.

[2] In August edition, Vol-4 2013 of IJSER, Author: self

[3] In May edition of IJSER, Author: self

[4] In June edition of IJSER, Author: self

[5] In Sept edition, Vol-6, 2015 of IJSER, Author: self

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