Planck's Quantum Hypothesis and Concept of Mass from Special Relativity

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Abstract— It is established that special relativity and quantum mechanics are two very wide apart theories of measurements in modern physics in terms of determinism versus indeterminism. Modern physics accepts indeterminism against classical determinism. But it is remarkable that Einstein's special relativity in its present form alone contains the basic ingredients of quantum theory. Einstein's theory can give a simple theoretical proof of the Planck's quantum hypothesis and can explain the origin of mass out of zero rest mass of photon. This article at the first place shows how to proceed in this path from relativistic energy momentum relations and at the second place it shows the reason of energy and momentum indeterminacy from the framework of relativity. At the last phase the article puts a question on the sustainability of special relativity itself before oscillation of any kind. This paper deals with the matter to the extent special relativity containing quantum theory.

Index Terms— Special relativity, simultaneity of events, Planck's quantum theory, energy momentum 4 vector, Heisenberg's indeterminacy principle, concept of mass, ensemble of photon.

1 Introduction

It is a common belief that the jurisdiction of special relativity and quantum theory are mutually exclusive. In 1900 Max Planck gave his famous quantum hypothesis of light together with the energy- frequency relation $E = h\nu$. In 1905 Einstein gave special relativity. Both the regime went their own way and ultimately clashed each other with the arrival of Heisenberg's indeterminacy principle in 1926. Einstein conceded defeat to Bohr ultimately but still believed that quantum theory lacks something very serious. But modern physics points clearly to the triumph of Heisenberg's indeterminacy principle in every context. Nonetheless it is worthwhile to mention that the very genesis of quantum theory of light prescribed by $E = h\nu$ and the very essence of mass can be derived even in the frame work of special relativity. Even Einstein could get it using his own energy momentum relation. Planck's quantum relation of photon and the construction of mass from an ensemble of photons are indeed inbuilt in Einstein's theory. We can even reconcile Heisenberg's principle of indeterminacy with the framework built by Einstein.

2 Proof of Planck's Energy Frequency Relation and Invariance of Planck's Constant:

Einstein's energy- momentum relation for a free particle in an inertial frame S is given by

$$E^2 = p^2c^2 + m_0^2c^4 \qquad (1)$$

If we have any particle whose rest mass is zero in frame S then its rest mass is zero in all frames and rest mass is invariant. For zero rest mass $E = pc$ and so

$$E/p = c = \omega/k \qquad (2)$$

In another inertial frame S' we have the following relation

$$E'/p' = c = \omega'/k' \qquad (3)$$

From equation (2) and (3) we can at once write

$$E/\omega = p/k = \hbar = E'/\omega' = p'/k' \qquad (4)$$

If we interpret E/ω as energy per angular frequency 1 rads^{-1} associated with photon then it must be same in all inertial frames otherwise we could get a preferred frame where E/ω equals a particular preferred value of $\hbar$. It means we get $E = \hbar\omega$ and $p = \hbar k$ for massless particle where $\hbar$ is invariant like c.

We can give an alternative proof using *E-p* transformation relation and frequency distribution (Doppler) relation.

$$E' = (E - up_x)\Big/\sqrt{1-\beta^2}$$
$$= (E - up\cos\theta)\Big/\sqrt{1-\beta^2} \qquad (5)$$

For particle with zero rest mass $E = pc$. So from above equation we get

$$E' = E(1-\beta\cos\theta)\Big/\sqrt{1-\beta^2} \qquad (6)$$

Here $\beta = u/c$ and u is the relative velocity of S' frame with respect to the S frame with coincident x and x' axis. But Doppler formula is given by

$$\nu' = \nu(1-\beta\cos\theta)\Big/\sqrt{1-\beta^2} \qquad (7)$$

From equation (6) and (7) we finally get

$$E'/\nu' = E/\nu = \text{invariant}(h)$$

We get Planck's relation $E = h\nu$ where *h* is inertial frame invariant. C. Moller gave a very cumbrous proof [1] of the invariance of h using plane monochromatic wave function in a lengthy and complicated 4 dimensional formalism and added more mathematics for simple physics.

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3 ENERGY MOMENTUM CONSERVATION IN SPECIAL RELATIVITY

To construct nonzero rest mass from an ensemble of particles of zero rest mass it is necessary that the conservation of energy and momentum in special relativity together with 4-energy- momentum formalism are to be analysed critically. There are many famous texts in this matter but hardly have they dealt with the matter in a systematic way starting from the basic equations. This is the reason for which the following portions have been developed.

3.1 Energy Momentum Conservation for an Ensenbble of free non-interacting particles:

Newton's second law of motion for a single particle in relativistic mechanics can be written as

$$\boldsymbol{F}_{\text{ext}} = d\,\boldsymbol{p}/dt \quad (8)$$

Here

$$\boldsymbol{p} = m\boldsymbol{v} = \left(E/c^2\right)\boldsymbol{v} = \left(m_0\Big/\sqrt{1-v^2/c^2}\right)\boldsymbol{v} \quad (9)$$

If $\boldsymbol{F}_{ext} = \boldsymbol{0}$ then $\boldsymbol{p} = (m_0\Big/\sqrt{1-v^2/c^2})\boldsymbol{v}$ remains constant. As the particle is under no force field i.e., its interaction with field is zero

so $m = m_0\Big/\sqrt{1-v^2/c^2}$ and $E = mc^2$ also remain constant i.e., speed v and velocity v also remain constant therefore.

From the energy-momentum transformation relations it can be easily seen from any standard text that $\boldsymbol{p}_1 = \boldsymbol{p}_x, \boldsymbol{p}_2 = \boldsymbol{p}_y, \boldsymbol{p}_3 = \boldsymbol{p}_z, \boldsymbol{p}_4 = jE/c$ (where j=√-1) constitute the components of 4 energy-momentum vector p_μ where

$$p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = -(m_0 c)^2 = \text{invariant} \quad (10)$$

Now let us consider a system of non interacting or free particles in a frame at any particular instant. The following results hold.

$$E = \sum E_i = \sum m_i c^2 \quad = \text{constant} \quad (11)$$

$$\boldsymbol{P} = \sum \boldsymbol{p}_i \quad = \text{constant} \quad (12)$$

Equation (11) and (12) holds as every E_i and p_i remains constant. We may therefore say that linear momentum and relativistic masses or energies are additive for the ensemble of free particles. Moreover kinetic energy of the system is given by

$$T = \sum T_i = \sum (m_i - m_{0i})c^2$$
$$= \sum E_i - c^2 \sum m_{0i} \quad = \text{constant} \quad (13)$$

The reason for conservation of net kinetic energy is as follows. Here ΣE_i is constant. As each and every particle is free so each m_{0i} i.e., rest mass of each is conserved. Total kinetic energy of the ensemble of free particles therefore also remains conserved. So, kinetic energy is also additive in such case. One may argue that all E_i and p_i if are calculated in a particular frame simultaneously then their corresponding values are not simultaneous in any other frame and additivity given by equation (11) and (12) therefore might loose significance in respect of net energy E and net momentum P. But the situation is not that worse. As individual E_i' and p_i' again remain constant in any other frame S' so the corresponding sums calculated simultaneously do not differ from the sums calculated non-simultaneously.

Now we will consider rest mass M_0 of the system. It is clear that $M_0 \neq \sum m_{0i}$ because particles are moving with different velocities and so we can never find a single frame where all particles are at rest simultaneously. So to understand rest mass of a system we have to introduce the concept of C-O-M frame or centre of momentum frame [2]. It is such a frame where $\sum \boldsymbol{p} = \boldsymbol{0}$ i.e. net linear momentum $\boldsymbol{P}$ of the system of particles is a null vector. In such frame E given by equation (11) represents E_0 i.e., rest energy of the system. Equation (13) can now be written in such frame (C-O-M frame) in the following way

$$E_0 = c^2 \sum m_{0i} + \sum T_i \quad (14)$$

$$M_0 = \sum m_{0i} + 1/c^2 \sum T_i \quad (15)$$

So rest mass M_0 of the system is greater than the sum of rest masses $(\sum m_{0i})$ of the individual particles.

So rest mass of a system is no more additive effect of the rest masses of individual particles unlike relativistic mass or energy and linear momentum of a system given by equation (11) and (12). Non additivity of rest mass is an important conclusion of special relativity. The most important thing is that the introduction of a system's rest mass M_0 can make it possible to regard a system of noninteracting particles like a single particle with rest mass M_0 net energy $E = \sum m_i c^2$, and net linear momentum $\boldsymbol{P} = \sum \boldsymbol{p}_i$ in any inertial frame S if M_0 is invariant as is demanded by equation (10). So we are to prove the invariance of M_0.

To prove invariance of M_0 we write 4 energy-momentum vectors of the r^{th} particle in any frame S.

$$\boldsymbol{p}_{r\mu} \equiv (m_{0r}\gamma_r v_{rk}, jm_{0r}\gamma_r c) \quad (16)$$

or, $\boldsymbol{p}_{r\mu} \equiv (m_{0r} v_{r\mu})$ (17)

where $\boldsymbol{v}_{r\mu} \equiv (\gamma_r v_{rk}, j\gamma_r c)$ (18)

Here k = 1, 2, 3 for x,y,z components and

$\gamma_r = 1\Big/\sqrt{1 - v_r^2/c^2}$ As the particles are non interacting so, $p_{r\mu}$ = constant i.e., $m_{0r}\gamma_r v_{rk}$ is constant or conserved for each one and also $m_{0r}\gamma_r c^2$ = constant for each. Hence we can write that, $P_\mu = \sum p_{r\mu}$

i.e. net 4 momentum of the system is also conserved in an inertial frame.

$$P_\mu^2 = \boldsymbol{P}_\mu\, \boldsymbol{P}_\mu$$
$$= \sum_r \boldsymbol{p}_{r\mu} \sum_s \boldsymbol{p}_{s\mu}$$
$$= \sum_{r=}\sum_s \boldsymbol{p}_{r\mu}\, \boldsymbol{p}_{s\mu} + \sum_{r\neq}\sum_s \boldsymbol{p}_{r\mu}\, \boldsymbol{p}_{s\mu}$$
$$= -\sum m_{0r}^2 c^2 - \sum_{r\neq}\sum_s m_{0r} m_{0s}\gamma_r\gamma_s (c^2 - \boldsymbol{v}_r \cdot \boldsymbol{v}_s) \quad (19)$$

It is negative and conserved quantity in any frame for ensemble of free particles and must be invariant as P_μ is systems 4 energy-momentum vector. It is the property of 4 vector that the sum of the squares of its components is invariant.

If particles are all photons and they are not going in the same direction then also P_μ^2 is negative. The question is whether the negative invariant quantity given by equation (19) really represents $-M_0^2 c^2$. The answer is in affirmative. At least this is expected. To check it let us write the following relation treating the system like a single particle.

$$E^2 = P^2 c^2 + M_0^2 c^4$$
$$\text{or,} -M_0^2 = P^2 - E^2/c^2 \quad (20)$$
$$= \sum_r p_r^2 + \sum_{r\neq}\sum_s p_r \cdot p_s - \sum_r E_r^2/c^2 - \sum_{r\neq}\sum_s E_r E_s/c^2 \quad (21)$$

As $E_r^2 = p_r{}^2 c^2 + m_{0r}^2 c^4$ so

$$\sum_r p_r^2 - \sum_r E_r^2/c^2 = -\sum_r m_{0r}^2 c^2 \quad (22)$$

From equation (21) and (22) we get

$$-M_0^2 c^2 = -\sum_r m_{0r}^2 c^2 + \sum_{r\neq}\sum_s \left(\boldsymbol{p}_r \cdot \boldsymbol{p}_s - E_r E_s/c^2\right)$$
$$= -\sum m_{0r}^2 c^2 - \sum_{r\neq}\sum_s m_{0r} m_{0s}\gamma_r\gamma_s (c^2 - \boldsymbol{v}_r \cdot \boldsymbol{v}_s) \quad (23)$$

Comparing equation (19) and (23) we finally prove

$$P_\mu^2 = -M_0^2 c^4 = \text{Invariant} \quad (24)$$

This is the same relation as is given by equitation (10) for a single particle. We therefore have following important conclusions

$$\sum E_r^2 - \sum p_r^2 c^2 = \sum m_{0r}^2 c^4 = \text{Invariant} \quad (25)$$

as each $E_r^2 - p_r^2 c^2 = m_{0r}^2 c^4$ is invariant for an ensemble of non interacting particles.

And $$\left(\sum_r E_r\right)^2 - \left(\sum_r \boldsymbol{p}_r\, c\right)^2 = M_0^2 c^4 = \text{invariant} \quad (26)$$

which is the same equation as equation (20). Moreover all E_r and $\boldsymbol{p}_r$ are additive to produce energy $E = \sum E_r$ and net linear momentum $\boldsymbol{P} = \sum \boldsymbol{p}_r$. Both of E and P remain conserved in a frame as has been shown by equation (11) and equation (12).On the other hand all m_{0r} do not add to give systems rest mass M_0 as has been shown by equation (15).

3.2 Energy momentum conservation for an ensemble of interacting particles:

This section has been divided into two sub-sections for clarity.

a). Simultaneous collision of particles at asingle point:

This classical picture of collision is a pure abstraction. Real interactions never approach such a classical situation. But the same situation is intended for a brief presentation only to comprehend the real circumstances. Let Δt is the duration of collision and it is assumed that the point particles interact only when they collide at a single point. Particles therefore just before and after collisions are totally free. As net external force on the system is zero and action -reaction at a point being simultaneous, equal and opposite, net linear momentum P of the system remains conserved in the process. In this situation calculation of force is not barred by relativity of simultaneity as forces of interaction at a single point are all simultaneous in all frames. When particles collide at a point then they form a single mass collectively. If this collection has rest mass M_0 which has been shown to remain invariant by virtue of 4 vector character $P\mu$, then net energy E of the system also remains conserved in the process as is evident from equation (26). It is however not true in general that rest mass m_{0i} of individual remains constant in the process. If the particles do not exchange rest mass from each other then identity of them is not lost in the process and individual rest mass remains same. Such thing occurs in elastic collision. It is now seen from equation (13) that total kinetic energy T of the system is conserved in the process. So for elastic point collision all the equations from (11) to (26) hold. The only difference from the ensemble of free particle is that here individual E_r and p_r do not remain conserved due to mutual interactions in time Δt. We see therefore that in elastic collision net energy E , net kinetic energy T and individual rest mass m_{0i} remain same but redistribution of kinetic energy among the colliding particles occur only by the process of collision exclusively from the kinetic energy pool T.

In partly elastic point collision or in case of inelastic point collision (collision where colliding particles merge together to a lump) all the equations from (11) to (26) again hold with individual E_r and p_r not remaining conserved once more as it is in case of elastic collision but with one more significant reservation. Kinetic energy T as given by equation (13) is no longer a conserved quantity. It means that the sum of individual rest mass $(\sum m_{0i})$ is also no longer a conserved quantity. From equation (13) it is seen that $\Delta T = -c^2 \Delta \sum m_{0i}$. Therefore a part of kinetic energy which is lost or gained (ΔT) is compensated by a corresponding gain or loss $(\Delta \sum m_{0i})$ in the sum of individual rest mass. So change in the sum of rest masses comes from kinetic energy pool. In such collisions new particles are therefore generated. It has already been seen that the value of *E* in C-O-M system is $E_0 = M_0 c^2$ and is an invariant quantity. It is also seen from equation (14) or from equation (15) that $M_0 > \sum m_{0i}$ and therefore rest mass is not any additive quantity in any case. Partly elastic collision or inelastic collision provides a change with $\sum m_{0i}$ and this thing plays a singular role in generation of new particles in nuclear reactions although rest mass M_0 of the system does never change.Another very important thing is worth mentioning. Not only net energy balance *E* and net linear momentum *P* remain as conserved quantities before and after the point collision but also at every instant during the interaction time Δt. It happens because simultaneity of action-reaction pairs has absolute meaning in all frames due to spatial pointness of collision. So in such case $\sum_{i \neq j} \boldsymbol{F}_{ij}$ representing the net mutual action-reaction always vanish in all frames at every instant in point classical collision. Therefore $\boldsymbol{F}_{ext}$ being a null force on the system, $\boldsymbol{F}_{ext} + \sum_{i \neq j} \boldsymbol{F}_{ij} = d(\sum \boldsymbol{p}_i)/dt$ becomes a null vector at each instant and $\sum \boldsymbol{p}_i$ remain conserved at each instant during collision.

To conclude this section we consider the status of energy momentum conservation in special relativity. Homogeneity of space as stated by Noether's theorem is the fundamental reason for linear momentum conservation. But in special relativity it is not 'space' alone or 'time' alone , it is 'space-time' which is the real stage and homogeneity of 'space-time' should be responsible for linear momentum and energy conservation given by equation(11) and (12) simultaneously even to a system of interacting particles with in a very short region of space-time. In special relativity this is inbuilt for almost point variables like *E* and *P*. Starting from only equation (11) one can arrive at equation (12). Relativistic energy conservation in a process is not a separate issue from net linear momentum conservation. Let us see the reason. Let us consider *r* number of particles just before interactions in a short region space-time and let *s* number of particles is produced after the process. By principle of momentum conservation we can write

$$\sum_r \boldsymbol{p}_r = \sum_s \boldsymbol{p}_s$$

or , $$\boldsymbol{p}_{initial} = \boldsymbol{p}_{final} \tag{27}$$

We now can see from equation (26) that $\sum_s E_s = \sum_r E_r$.
It means net relativistic mass remain conserved in any interaction.

b). Real interactions-Einstein versus Heisenberg:

Real interactions among particles may be elastic or not but never occur at a single point. Interacting particles together with field quanta may be close but the interaction region is finite spatially. As the interacting particles in any stage of interaction are at different locations, so simultaneity is now a relative concept. Therefore net energy and momentum of an ensemble can be calculated simultaneously in a frame but the corresponding values in other frames are not simultaneous. So conservation of energy if can be applied in one frame before and after specified interactions, cannot then automatically lead to the applicability in other frame. A set of complete specified simultaneous interactions in one frame can never be considered a complete set in other frame. Same is true for the quantity *P* i.e., for net momentum of the system. The relativity of simultaneity simply deprives the quantities *P* and *E* of the properties of 4 vector components. Therefore all the equations from (19) to (26) don't apply at all during interaction time Δt. Apart from this inapplicability there appears a very important junction. Net energy and net momentum conservation given by equations (11) and (12) raise questions on conservation principles. If conservation principle applies to one frame, it will apply in all frames by principle of relativity. The relativity of simultaneity however does not permit simultaneous calculations of net energy and net momentum for a set of specified configuration in all frames. So it might appear that conservation principles are not meaningful in any frame during interaction. This is shocking but conclusion is inevitable. In the following section we will see the same thing in respect of Heisenberg's indeterminacy principle. Neither the relativity of simultaneity nor Heisenberg's principle upholds the conservation of energy and momentum as laws of nature during interaction for ensemble of interacting particles. The principle of momentum conservation for any type of interaction however is known to obey universally. The collision may be elastic or inelastic but none has ever reported any evidence against momentum conservation. A natural question is immediately posed. Why nothing against momentum conservation has not yet been reported? To answer this question it is required to concentrate on the interaction. It was assumed that interaction time is Δt. One may consider the reactant particles as nearly free ones before interaction starts and similarly the product particles may also be considered as free ones after the interactions. Einstein believed in objective reality. He considered things to exist even when they are not observed. But quantum mechanics is too rational and radical both in this context. Particles exist because they interact. When they are not observed, they just do not exist. Obviously this is interaction time during which we are to apply the conservation laws. Even a 'free particle' interacts but this interaction is too weak. Without any kind of interaction however weak, one has no right to say that there is a 'particle'. It can naturally be assumed that during the entire interaction time Δt, strength of interaction is not uniform but gets humped only for a much shorter time Δt^* when interaction becomes strong and then ΔE versus Δt curve might look like delta function. If this happens then during most of the life time $\Delta t - \Delta t^*$ interaction is too weak and energy uncertainty is negligible and we are virtually in the domain of conservation laws. Therefore we may apply conservation laws for E and $\boldsymbol{P}$ separately outside the time domain Δt^* as we have done it for free particles in section 3.2.a). In this outside domain we may safely apply equation (24). Net rest mass M_0 of the system is assumed to remain constant before and after the specified time domain Δt^* on the ground that nothing from outside has joined the system and nothing has leaked out of the defined system. The trouble lies however with the domain Δt^*. In this domain interaction is strong and the particles are not free and everything from equation (11) to equation (27) breaks down. This is the domain where Heisenberg's principle of indeterminacy shows up its effect and the relativity of simultaneity put a great block to calculate systems energy E and linear momentum $\boldsymbol{P}$ simultaneously in all frames for a set of specified interaction configuration. Theory of Einstein and principle of Heisenberg both tell remarkably the same thing but in different ways. It is discussed in brief in the following paragraph.

Let us consider an instant t' during close interaction between two particles at coordinates (x_1', y_1, z_1, t'), (x_2', y_2, z_2, t') with linear momenta p_1' and p_2' in S' frame. In S frame this configuration is not however simultaneous. Their corresponding coordinates will be (x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) with momenta p_1 and p_2 according to Lorentz transformations. If one wants to observe both at t_1 time then net momentum of the two particles were

$$\boldsymbol{p}_1 + \left\{ \boldsymbol{p}_2 + \left(d\, \boldsymbol{p}_2 / dt \right)_{t_2} (t_1 - t_2) \right\}.$$

Similarly if one wants to observe both at time t_2 then net momentum were $\left\{ \boldsymbol{p}_1 + \left(d\, \boldsymbol{p}_1 / dt \right)_{t_1} (t_2 - t_1) \right\} + \boldsymbol{p}_2$.

How ever neither of the two configurations was observed in S'. It is now seen that net momenta of the two particles are not same at S frame time t_1 and t_2. If we punch $t - t'$ and $x - x'$ transformation relations we get $t = \sqrt{1-\beta^2}\, t' + \left(u / c^2 \right) x$. This indicates that choice of same t' renders different t at different x coordinates. In this case

$$t_2 - t_1 = \left(u / c^2 \right)(x_2 - x_1) \text{ i.e. } \Delta t = \left(u / c^2 \right) \Delta x .$$

So net linear momentum of the two interacting particles at t_1 and t_2 time given by $\boldsymbol{p}_1 + \boldsymbol{p}_2 - \left(d\, \boldsymbol{p}_2 / dt \right)_{t_2} \left(u / c^2 \right) \Delta x$ and $\boldsymbol{p}_1 + \boldsymbol{p}_2 + \left(d\, \boldsymbol{p}_1 / dt \right)_{t_1} \left(u / c^2 \right) \Delta x$ were equal if either Δx equals zero (for point interaction) or

$$\left(d\, \boldsymbol{p}_1 / dt \right)_{t_1} = -\left(d\, \boldsymbol{p}_2 / dt \right)_{t_2} \text{ i.e. } \left(\boldsymbol{F}_1 \right)_{t_1} = -\left(\boldsymbol{F}_2 \right)_{t_2}.$$

The first option is ruled out on ground of pure abstraction. The second possibility is ruled out on ground of faster than light propagation of influence. As $t_2 - t_1 = \left(u / c^2 \right)(x_2 - x_1)$ so force F_1 on first particle at time t_1 could produceequal and opposite reaction (Newton's third law) $-F_2$ at time t_2 on second particle only when the influence were transferred with speed $(x_2 - x_1)/(t_2 - t_1) = c^2/u > c$. So it is clear that during interaction as observed in S frame, net linear momentum of the interacting particles does not remain conserved. One may say that this is not unexpected as the particles are under force field and one can not expect momentum conservation without considering field momentum as we see in classical electrodynamics. In terms of particle physics this sounds that not only these two particles are involved, there are also field quanta involved in interaction. If the field quanta had momenta p_{1f} and p_{2f} at time t_1 and t_2 and then linear momentum conservation will apply in the close system consisting of the two particles and field quanta. Therefore net linear momentum of the system at time t_1 and t_2 will be given by

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 - \left(d\, \boldsymbol{p}_2 / dt \right)_{t_2} \left(u / c^2 \right) \Delta x + \boldsymbol{p}_{1f} \text{ and}$$

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 + \left(d\, \boldsymbol{p}_1 / dt \right)_{t_1} + \left(u / c^2 \right) \Delta x + \boldsymbol{p}_{2f} \text{ must be equal.}$$

So the necessity of momentum conservation as demanded by Noether's theorem requires field quanta which we missed to consider in S' frame. This thing is inbuilt in homogeneous space time structure of special relativity. When interaction is almost over i.e. particles are almost free, p_1 and p_2 does not change and the corresponding momentum of field quanta pf ceases to exist. We see then $\boldsymbol{p}_1 + \boldsymbol{p}_2$ as conserved quantity. But within close interaction domain Δt^* net linear momentum of the system becomes

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 - \left(d\, \boldsymbol{p}_2 / dt \right)_{t_2} \left(u / c^2 \right) \Delta x + \boldsymbol{p}_{1f}$$

$$\text{or } \boldsymbol{p}_1 + \boldsymbol{p}_2 + \left(d\, \boldsymbol{p}_1 / dt \right)_{t_1} + \left(u / c^2 \right) \Delta x + \boldsymbol{p}_{2f} .$$

So interaction renders the system's momentum to fluctuate from its classical value $(\boldsymbol{p}_1 + \boldsymbol{p}_2)$ in the spatial domain Δx. So energy also fluctuates in the closed interaction domain Δt^*. All these things happen by virtue of force field (in classical term) or by field quanta (in quantum language). So Einstein's special relativity can predict this energy or momentum indeterminacy. But it was only Einstein's belief in the abstraction of passive observation that swept him far away from the basic principles of quantum mechanics.

So there must remain energy indeterminacy ΔE for interaction Δt^* and momentum indeterminacy ΔP for the corresponding spatial dimension Δx of interaction. If however interaction is weak enough when Δt or Δx is large one may consider the interacting particles almost as good as free ones and can apply Einstein determinism safely. If we consider that 'particle is there because it is interacting' then very close inspection means interaction domain Δx small but finite and therefore the inspected particle is never free. In such situation momentum indeterminacy comes into play. Same is true for energy indeterminacy for a small time domain of relatively stronger interaction.

4 REST MASS OF PHOTON ENSEMBLE

Conservation of rest mass from photons' energy was touched upon by V.A. Ugrarov [3]. Here the matter will be taken from an elaborate but brief mathematical description remaining completely in the special relativistic frame work. Let us consider n number of photons having frequencies $\nu_1, \nu_1, \ldots, \nu_n$ and moving in different directions

$$E = \sum E_r = h\sum_r \nu_r \tag{28}$$

$$\boldsymbol{P} = \sum_r \boldsymbol{p}_r == (h/c)\left(\sum_r \nu_r\, \boldsymbol{s}_r\right) \tag{29}$$

Here s_r is the unit vector in the direction of momentum of r th photon. Equation (26) can now be written as

$$M_0^2 = E^2/c^2 - \boldsymbol{P}\cdot\boldsymbol{P}$$

$$= \left(h^2/c^2\right)\{(\sum_r \nu_r)^2 - (\sum_r \nu_r\, \boldsymbol{s}_r)^2\}$$

$$\text{or,}\ M_0^2 = (h/c^2)^2\{(\sum_r \nu_r)^2 - \sum_i \nu_i\, \boldsymbol{s}_i \cdot \sum_j \nu_j\, \boldsymbol{s}_j\}$$

$$\text{or,}\ M_0 = (h/c^2)\sqrt{(\sum \nu_r)^2 - \sum_r (\nu_r^2) - 2\sum_{\substack{i\\ i\neq j}}\sum_j \nu_i\nu_j\, \boldsymbol{s}_i\cdot\boldsymbol{s}_j} \tag{30}$$

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If we denote σ for standard deviation of the non-dimensional number $x_r = \nu_r / \nu_{\max}$ then we have

$$M_0 = \frac{h\nu_{max}}{c^2}\sqrt{n\sigma^2 + (\sum_r x_r)^2(1+\frac{1}{n}) - 2\sum_{\substack{i\\ i\neq j}}\sum_j x_i x_j\, \boldsymbol{s}_i\cdot\boldsymbol{s}_j} \tag{31}$$

This is a remarkable result. A cloud of photons possesses a positive rest mass M_0. Only when all photons having same frequency propagate in the same direction M_0 equals zero. In fact if we consider virtual photons around elementary particles, we may recognise that a substantial amount of rest mass comes from photon cloud. In reality a 'bare' electron does not exist. Any 'particle' is always an interacting system with photons [4]. In the C-O-M system net linear momentum of photon cloud is a null vector and it means that the system of photons around any particle is completely diffuse in nature in the C-O-M system.

5 REMARKS

At present there are two theories of measurements in physics. One is relativity and the other is quantum mechanics. These two theories are at odd face of the other. There must be a single consistent theory of measurements. It has been shown that Einstein's special relativity contains the basic ingredients of quantum theory. Einstein's general relativity with far richer structure therefore is expected to produce much more things in its energy momentum tensor and differential conservation law. Einstein was swept away from the basic principles of quantum mechanics in the same way his belief in static universe took him away from predicting expanding universe from his own mathematical framework of general relativity. But there is a loose end. Special relativity based on Lorentz transformations of coordinates can't transform oscillatory sequence of events meaningfully [5]. One can see that SHM equation $x' = a'\sin\omega' t'$ if is substituted in Lorentz transformation relations we get $x - ut = a\sin k(c^2 t / u - x)$. This transformed relation is physically impossible in the sense that both the sides repeat simultaneously as wave function only when the frame velocity u equals c. No physical interpretation can save the situation. So special relativity too needs a modification in spite of its epoch making success to incorporate the principles of quantum mechanics based entirely on wave function.

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