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Optimization methodology based on neural networks and reference point algorithm applied to fuzzy multiobjective optimization problems

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Abstract— Artificial neural networks are massively paralleled distributed computation and fast convergence and can be considered as an efficient method to solve real-time optimization problems. In this paper, we propose reference point based neural network algorithm for solving fuzzy multiobjective optimization problems MOOP. The target is to identify the Pareto-optimal region closest to the reference points. Our approach has two characteristic features. Firstly, fuzzy multiobjective optimization problem (F-MOOP) has been transformed to crisp multiobjective optimization problem (C-MOOP) by means of Alpha-cut. Secondly a neural networks based reference point algorithm is implemented to solve C-MOOP in such a way that they integrate the decision maker DM early in the optimization process instead of leaving him/her alone with the final choice of one solution among the whole Pareto optimal set. Such procedures will provide the DM with a set of solutions near her/his preference so that a better and a more reliable decision can be made. Simulation runs on engineering application problems demonstrate their usefulness in practice and show another use of a neural network methodology in allowing the DM to solve multiobjective optimization problems better and with more confidence.

Index Terms— Neural network; Reference point; Fuzzy numbers.

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1 Introduction

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. In many real world problems, there are situations where multiple objectives may be more appropriate rather than considering single objective. However, in such cases emphasis is on efficient solutions, which are optimal in a certain multiobjective sense[1-11]. The classical interactive multiobjective optimization methods demand the decision-makers to suggest a reference direction or reference points or other clues which result in a preferred set of solutions on the Pareto-optimal front. In these classical approaches, based on such clues, a single objective optimization problem is usually formed and a single solution is found. A single solution does not provide a good idea of the properties of solutions near the desired region of the front. By providing a clue, the DM is not usually looking for a single solution, ra ther she/he is interested in knowing the properties of solutions which correspond to the optimum and near-optimum solutions respecting the clue[1,12]. We here argue that instead of finding a single solution near the region of interest, if a number of solutions in the region of interest are found, the decision-maker will be able to make a better and more reliable decision. Moreover, if multiple such regions of interest can be found simultaneously, decision-makers can make a more effective and parallel search towards finding an ultimate preferred solution.

The classical reference point approaches will find a solution depending on the chosen weight vector and is therefore subjective. Moreover, the single solution is specific to the chosen weight vector and does not provide any information about how the solution would change with a slight change in the weight vector. To find a solution for another weight vector, a new achievement scalarizing problem needs to be formed again and solved. Moreover, despite some modifications [1], the reference point approach works with only one reference point at a time. However, the decision maker may be interested in exploring the preferred regions of Pareto-optimality for multiple reference points simultaneously. In the context of finding a preferred set of solutions, instead of the entire Pareto-optimal set, quite a few studies have been made in the past. The approach by Deb [13] was motivated by the goal programming idea, and required the DM to specify a goal or an aspiration level for each objective. Based on that information, Deb modified his NSGA approach to find a set of solutions which are closest to the supplied goal point, if the goal point is an infeasible solution and find the solutions which correspond to the supplied goal objective vector, if it is a feasible one. The method did not care finding the Pareto optimal solutions corresponding to the multiobjective optimization problem, rather attempted to find solutions satisfying the supplied goals.

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Recently, neural networks (NNs) have become widely used tools in many fields such as decision support tool, pattern recognition and secure communication [14-18]. Neural Networks is well-known as one of powerful computing systems to solve complex optimization problems. Due to the massive computing unit-neurons and parallel mechanism of neural network, large-scale optimization problem can be solved efficiently. Many neural network for solving constraint optimization problems can be found in [19-21]. With the above principles of reference point approaches and difficulties with the classical methods, we propose a reference point based on neural networks, by which a set of Pareto-optimal solutions near a supplied set of reference points will be found, thereby eliminating the need of any weight vector and the need of applying the method again and again. Instead of finding a single solution corresponding to a particular weight vector, the proposed procedure will attempt to a find a set of solutions in the neighborhood of the corresponding Pareto-optimal solution, so that the DM can have a better idea of the region rather than a single solution.

In this paper, an attempt is made to solve Fuzzy Multi-objective optimization with fuzzy parameters. Based on Alpha concept [22,23], F-MOOP can be transformed to crisp multiobjective optimization problem (C-MOOP) at certain degree of α (α-cut level). Also, we combine one such preference-based strategy with a neural network methodology and demonstrate how, instead of one solution, a preferred set of solutions near the reference points can be found parallel. Such procedures will provide the decision-maker with a set of solutions near her/his preference so that a better and a more reliable decision can be made.

2- FUZZY MULTIOBJECTIVE OPTIMIZATION

Detailed A Multi-objective Optimization Problem (MOP) can be defined as determining a vector of variables within a feasible region to minimize a vector of objective functions that usually conflict with each other. The following fuzzy vector minimization problem (FVMP) involving fuzzy parameters in the objective functions and constraints such a problem takes the form:

$$Min\ \ \{f_1(X,\tilde{a}), f_2(X,\tilde{a}),....., f_m(X,\tilde{a})\}$$
$$subject\ to\ g(X,\tilde{a}) \le 0$$

where $f_1(X,\tilde{a})$ is the ith objective function; and $g(X,\tilde{a})$ is constraint vector, X is vector of decision variables; and $\tilde{a}=(\tilde{a}_1,\tilde{a}_2,....\tilde{a}_n)$ represented a vector of fuzzy parameters in the problem. Fuzzy parameters are assumed to be characterized as the fuzzy numbers. The real fuzzy numbers $\tilde{a}$ form a convex continuous fuzzy subset of the real line whose membership function $\mu_{\tilde{a}}(a)$ is defined by:

1) a continuous mapping from R^1 to the closed interval [0,1];
2) $\mu_{\tilde{a}}(a)=0$ for all $a \in (-\infty, a_1]$;
3) strictly increasing on $[a_1,a_2]$;
4) $\mu_{\tilde{a}}(a)=1$ for all $a \in [a_2,a_3]$;
5) strictly decreasing on $[a_3,a_4]$;
6) $\mu_{\tilde{a}}(a)=0$ for all $a \in [a_4,+\infty)$;

Assume that $\tilde{a}$ in the FM-RAP are fuzzy numbers whose membership functions are $\mu_{\tilde{a}}(a)$.

Definition 1. (α-level set). The α-level set or α-cut of the fuzzy numbers $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership functions exceeds the level $\alpha \in [0,1]$:

$$L_\alpha(\tilde{a}) = \{a \mid \mu_{\tilde{a}}(a) \ge \alpha\}.$$

For a certain degree α, the (FM-RAP) can be represented as a nonfuzzy α -VMP as follows:

$$Min\ \ \{f_1(X,a), f_2(X,a),....., f_m(X,a)\}$$
$$subject\ to\ g(X,a) \le 0$$
$$X=(x_1,x_2,...x_n),\ a=(a_1,a_2,....,a_n)$$
$$L_{\alpha i} \le a_i \le U_{\alpha i}$$

Where constraint $L_{\alpha i} \le a_i \le U_{\alpha i}$ gives the lower and upper bound for the parameters a_i

Definition 2. (α–Pareto optimal solution). $x^* \in X$ is said to be an α–Pareto optimal solution to the (α-VMP), if and only if there does not exist another $x \in X$, $a \in L_\alpha(\tilde{a})$ such that $f_i(x,a) \ge f_i(x^*,a^*), i=1,2,..,k,$ with strictly inequality holding for at least one *i*, where the corresponding values of parameters a_i^* are called α-level optimal parameters.

3- REFERENCE POINT BASED NEURAL NETWORK ALGORITHM

For papers In this section, a framework for the proposed approach that involves two phases was presented. The first one transforms the fuzzy multiobjective optimization problem (F-MOOP) to the crisp multiobjective optimization (C-MOOP) by means of Alpha-cut, while the other phase employs a reference point based on neural networks algorithm to solve the crisp optimization problem.

Phase I:

Step0: Formulate fuzzy multiobjective optimization problem

$$Min\ \ \{f_1(X,\tilde{a}), f_2(X,\tilde{a}),....., f_m(X,\tilde{a})\}$$
$$subject\ to\ g(X,\tilde{a}) \le 0$$

where $f_1(X,\tilde{a})$ is the ith objective function; and $g(X,\tilde{a})$ is constraint vector, X is vector of decision variables; and $\tilde{a}=(\tilde{a}_1,\tilde{a}_2,....\tilde{a}_n)$ represented a vector of fuzzy parameters in the problem

Step1: Transform fuzzy multiobjective optimization problem into crisp multiobjective optimization problem using Alpha-Level cut.

$$Min\ \ \{f_1(X,a), f_2(X,a),....., f_m(X,a)\}$$
$$subject\ to\ g(X,a) \le 0$$
$$X=(x_1,x_2,...x_n),\ a=(a_1,a_2,....,a_n)$$
$$L_{\alpha i} \le a_i \le U_{\alpha i}$$

Phase2:

Step2: Creating an achievement scalarizing problem using preferred reference point: Minimize and maximize the objective functions individually in the feasible region, and these information must given to the DM, the DM suggest preferred reference point, the reference point is a feasible or infeasible point in the objective space. When decision making is empha-

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sized, the objective of solving a multi-objective optimization problem is referred to supporting a decision maker in finding the most preferred Pareto optimal solution according to his/her subjective preferences. The underlying assumption is that one solution to the problem must be identified to be implemented in practice. Here, a human decision maker (DM) plays an important role. The DM is expected to be an expert in the problem domain. The reference point is used to derive achievement scalarizing functions as follows:

Given a reference point $\overline{z}$ for an M-objective optimization problem of minimizing $f_1(X), f_2(X),....., f_m(X)$ with $X \in S$ S, the following single-objective optimization problem is solved for this purpose:

$$Minimize \ \left(\sum_{i=1}^{m} w_i \left(f_i(x) - \overline{z}_i \right)^p \right)^{1/p}$$

$$subject \text{ to } X \in S$$

If p = 1, the sum of weighted deviations is minimized (and the problem to be solved is equal to the weighting method except a constant). If p = 2, we have a method of least squares. The proposed reference point approach discussed above, will find a solution depending on the chosen weight vector and is therefore subjective. Moreover, the single solution is specific to the chosen weight vector. To find a solution for another weight vector, a new achievement scalarizing problem needs to be formed again and solved. To make the procedure interactive and useful in practice, Wierzbicki **[24]** suggested a procedure in which the obtained solution z' is used to create M new reference points, as follows:

$$z^{(j)} = \overline{z} + (z' - \overline{z}).e^{(j)},$$

where $e^{(j)}$ is the j-th coordinate direction vector.

Step3: This step is a neural network phase [25] for solving convex nonlinear programming which formulated in the previous step. The distinguishing features of the proposed network are that the primal and dual problems can be solved simultaneously. The interested reader is referred to [25] where, all necessary and sufficient optimality conditions are incorporated, and no penalty parameter is involved. Also, based on Lyapunov, LaSalle and set stability theories, Chen K. z. [25] prove strictly an important theoretical result that, for an arbitrary initial point, the trajectory of the proposed network does converge to the set of its equilibrium points, regardless of whether a convex nonlinear programming problem has unique or infinitely many optimal solutions.

I- Let the following be a general convex Nonlinear Programming (CNLP) problem:

CNLP

$$\begin{aligned} & Min\ f(x),\ x \in R^n \\ & \text{s.t} \quad g_i(x) \geq 0, i = 1,2,.....,m \\ & h_i = a_j^T x - b_j,\ j = 1,2,....,p(p < n) \end{aligned}$$

where $f(x)$ and $g_i(x)$, (i=1,2,…, m) are convex functions.

II- According to the result in Mangasarian **[26],** the dual problem DNLP of CNLP is as follows:

$$\text{DNLP} \begin{cases} \max\limits_{x,\lambda,\mu} > L(x,\lambda,\mu),\ x \in R^n \\ \text{s.t} \quad \nabla_x L(x,\lambda,\mu) = 0 \\ \quad \lambda \geq 0 \end{cases}$$

Where $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)^T$, $\mu = (\mu_1, \mu_2, ..., \mu_p)^T$

$$L(x,\lambda,\mu) = f(x) - \sum_{i=1}^{m} \lambda_i g_i(x) - \sum_{j=1}^{p} \mu_j h_j(x) \equiv L(z),$$

$$\nabla_x L(x,\lambda,\mu) = \nabla f(x) - \sum_{i=1}^{m} \lambda_i \nabla g_i(x) - \sum_{j=1}^{p} \mu_j \nabla h_j(x)$$

III- Parameter Initialization, Let t=0. Arbitrary choose initial vector $x(t) \in R^n$, $\lambda(t) \in R^m$, $\mu(t) \in R^p$, $\Delta t > 0$ $(\Delta t = 0.0001)$ and error $\varepsilon = 10^{-9}$.

IV- Computation of gradient:

$$\begin{aligned} & u(t) = \nabla_x E(z) = \lambda^T g(x).g(x)^T \lambda + \nabla g(x)^T [g(x) - |g(x)|] \\ & + \nabla_{xx}^2 L(z) \nabla_x L(z) + A^T (Ax - b) \\ & v(t) = \nabla_\lambda E(z) = \lambda^T g(x).g(x) - \nabla g(x) \nabla_x L(z) + [\lambda - |\lambda|] \\ & w(t) = \nabla_\mu E(z) = -A \nabla_x L(z) \end{aligned}$$

V- States Updating :

$x(t+\Delta t) = x(t) - \Delta t.u(t)$, $\lambda(t+\Delta t) = \lambda(t) - \Delta t.v(t)$, $\mu(t+\Delta t) = \mu(t) - \Delta t.w(t)$

VI- Calculation: $s = \sum_{i=1}^{n} u_i^2(t), r = \sum_{j=1}^{m} v_j^2(t), q = \sum_{j=1}^{p} w_j^2(t).$

VII- Stopping Rule:

if $s < \varepsilon, r < \varepsilon$ and $q < \varepsilon$, then output $x(t+\Delta t), \lambda(t+\Delta t), \mu(t+\Delta t)$ and draw the point $x_1(t+\Delta t), x_2(t+\Delta t)$ otherwise let $t = t + \Delta t$ and go to step **IV**.

New Pareto optimal solutions are then found by forming new achievement scalarizing problems. If the decision-maker is not satisfied with any of these Pareto-optimal solutions, a new reference point is suggested and the above procedure is repeated. It is interesting to note that the reference point may be a feasible one or an infeasible point. If a reference point is feasible and is not a Pareto-optimal solution, the decision-maker may then be interested in knowing solutions which are Pareto-optimal and close to the reference point. On the other hand, if the reference point is an infeasible one, the decision-maker would be interested in finding Pareto-optimal solutions which are close to the supplied reference point.

4- IMPLEMENTATION OF THE PROPOSED APPROACH

A case study of engineering application (The Environmental/Economic Dispatch EED multiobjective problem) , was carried out to verify the feasibility and efficiency of the proposed approach. EED seeks to simultaneously minimize both fuel cost and the emissions produced by power plants. Environmental concerns on the effect of SO2 and NOX emissions produced by the fossil-fueled power plants led to the inclusion of minimization of emissions as an objective in the OPF formulation. The economic emission load dispatch involves the simultaneous optimization of fuel cost and emission objectives[4,5,8]. The deterministic problem is formulated as described below.

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$$Min \quad f_1(x) = \sum_{i=1}^{n} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \ \$/hr$$

$$Min \quad f_2(\cdot) = \sum_{i=1}^{n} [10^{-2}(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \xi_i \exp(\lambda_i P_{Gi})] \ ton/hr$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{n} P_{Gi} - P_D - P_{Loss} = 0, \\ & P_{Gi\min} \le P_{Gi} \le P_{Gi\max} && i = 1,\ldots\ldots, n \\ & Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max} && i = 1,\ldots\ldots, n \\ & V_{i\min} \le V_i \le V_{i\max} && i = 1,\ldots\ldots, n \\ & S_\ell \le S_{\ell\max}, && \ell = 1,\ldots., n_{Line}, \end{aligned}$$

Where

$f_1(x)$ is The classical economic dispatch problem of finding the optimal combination of power generation, which minimizes the total fuel cost while satisfying the total required demand

$f_2(\cdot)$ is The emission function can be presented as the sum of all types of emission considered, such as NO_x, SO_2, thermal emission, etc., with suitable pricing or weighting on each pollutant emitted.

C: total fuel cost ($/hr), C_i: is fuel cost of generator i

P_{Gi}: power generated (p.u)by generator i,

a_i,b_i,c_i: fuel cost coefficients of generator i,

n: number of generator.

Constraints: The optimization problem is bounded by the following constraints:

Power balance constraint. The total power generated must supply the total load demand and the transmission losses.

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_{Loss} = 0$$

Where P_D: total load demand (p.u.), and P_{loss}: transmission losses (p.u.).

The transmission losses are given by[27]:

$$P_{Loss} = \sum_{i=1}^{n} \sum_{j=1}^{n} [A_{ij}(P_i P_j + Q_i Q_j) + B_{ij}(Q_i P_j - P_i Q_j)]$$

Where $P_i = P_{Gi} - P_{Di}$, $Q_i = Q_{Gi} - Q_{Di}$,

$$A_{ij} = \frac{R_{ij}}{V_i V_j}\cos(\delta_i - \delta_j), \quad B_{ij} = \frac{R_{ij}}{V_i V_j}\sin(\delta_i - \delta_j)$$

n : number of buses

R_{ij}: series resistance connecting buses *i* and *j*

V_i : voltage magnitude at bus *i*

δ_i : voltage angle at bus *i*

P_i : real power injection at bus *i*

Q_i : reactive power injection at bus *i*

Maximum and Minimum Limits Of Power Generation. The power generated P_{Gi} by each generator is constrained between its minimum and maximum limits, i.e.,

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max}, \quad Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max},$$
$$V_{i\min} \le V_i \le V_{i\max}, \qquad i = 1,\ldots\ldots, n$$

where $P_{Gi\min}$: minimum power generated, and $P_{Gi\max}$: maximum power generated.

- **Security Constraints.** For secure operation, the transmission line loading S_l is restricted by its upper limit as

$$S_\ell \le S_{\ell\max}, \ell = 1,\ldots., n_\ell$$

Where n_ℓ is the number of transmission line.

For comparison purposes with the reported results, the system is considered as losses and the security constraint is released.

The proposed approach has been applied to the standard IEEE 30-bus 6-generator test system. The single-line diagram of this system is shown in figure **1** and the detailed data are given in [28-30].

Table 1: Generator cost and emission coefficients

		G1	G2	G3	G4	G5	G6
Cost	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
Emission	α	4.091	2.543	4.258	5.426	4.258	6.131
	β	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	γ	6.490	4.638	4.586	3.380	4.586	5.151
	ζ	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
	λ	2.857	3.333	8.000	2.000	8.000	6.667

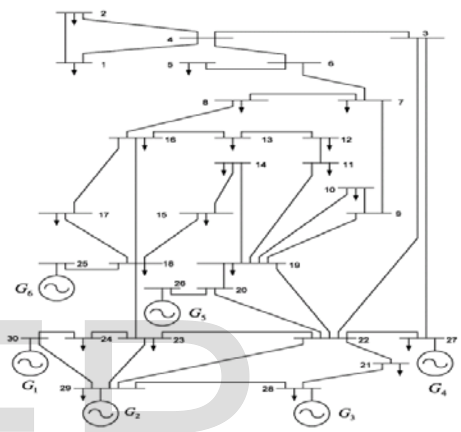

Fig. 1: Single line diagram of IEEE 30-bus 6-generator test system

The values of fuel cost and emission coefficients are given in Table 1. Naturally, these data (cost and emission) involve many controlled parameters whose possible values are vague and uncertain. Consequently each numerical value in the domain can be assigned a specific "grade of membership" where 0 represents the smallest possible grade of membership, and 1 is the largest possible grade of membership. Thus fuzzy parameters can be represented by its membership grade ranging between 0 and 1.

The fuzzy numbers shown in figure 2 have been obtained from interviewing DMs or from observing the instabilities in the global market and rate of prices fluctuations. The idea is to transform a problem with these fuzzy parameters to a crisp version using α-cut level. This membership function can rewrite as follows:

$$\mu(a_{ij}) = \begin{cases} 1, & a = a_{ij} \\ \dfrac{20a}{a_{ij}} - 19 & 0.95 a_{jk} \le a \le a_{ij} \\ 21 - \dfrac{20a}{a_{ij}} & a_{ij} \le a \le 1.05 a_{ij} \\ 0 & a < 0.95 a_{ij} \ or \ a > 1.05 a_{ij} \end{cases}$$

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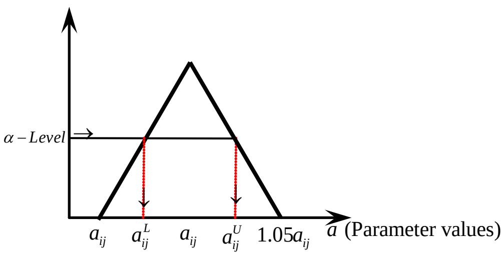

Fig.2. Fuzzy numbers of the effectiveness of resource

So, every fuzzy parameter $\tilde{a}_{ij}$ can be represented using the membership function. By using α -cut level, these fuzzy parameters can be transformed to a crisp one having upper and lower bounds $[a_{ij}^L, a_{ij}^U]$, which declared in figure 2. Consequently, each α -cut level can be represented by the two end points of the alpha level.

5- RESULTS AND DISCUSSION

Here, the problem is how to determine the optimal power flow for considering the minimum cost and the minimum emission objectives simultaneously. In order to efficiently and effectively obtain the solution, the search for the optimal solution is carried out in two steps. Firstly transforming the fuzzy multiobjective optimization problem (F-MOOP) to the crisp multiobjective optimization (C-MOOP) by means of Alpha-cut, In order to study the influence of fuzzy parameters on the obtained Pareto optimal solutions, all the range of the parameter fluctuation were scanned, two bounds of Alpha value have been considered $\alpha = 0,1$, and also we take some values between these bounds $\alpha = 0.2, 0.4, 0.6, 0.8$. While the other phase employs a neural networks based reference point algorithm to solve C-MOOP, where the decision maker (DM) plays an important role. The DM is expected to be an expert in the problem domain and provide us with different preferred reference point for each case as in figures(3-8) . A partial set of nondominated solutions is obtained by exploring the optimal Pareto frontier using different α cut level and certain preferred reference point. Graphical presentations of the experimental results are presented in figures (3-8) for six cases with different three preferred reference point. It is obvious from figures (3-8) that the results maintain the diversity and convergence for all α cut level. On the basis of the application, we can conclude that the proposed method can provide a sound optimal power flow by simultaneously considering multiobjective problem. On the basis of the application, we can conclude that the proposed method can provide a sound optimal power flow by simultaneously considering multiobjective problem.

6- CONCLUSIONS

In this paper, we have addressed an important task of combining neural network methodologies with a classical reference point approach to not find a single optimal solution, but to find a set of solutions near the desired region of decision-maker's interest. With a number of trade-off solutions in the region of interests we have argued that the decision-maker would be able to make a better and more reliable decision than with a single solution. An attempt is made to solve Fuzzy Multiobjective optimization with fuzzy parameters. Based on Alpha concept, F-MOOP can be transformed to crisp multi-objective optimization problem (C-MOOP) at certain degree of α (α-cut level). Also, we combine one such preference-based strategy with a neural network methodology and demonstrate how, instead of one solution, a preferred set of solutions near the reference points can be found parallel. Such procedures will provide the decision-maker with a set of solutions near her/his preference so that a better and a more reliable decision can be made.

The main features of the proposed algorithm could be summarized as follows:

(a) The main crux of this paper is exploitation of reference point base neural network procedure in finding more than one solutions not on the entire Pareto-optimal frontier, but in the regions of Pareto-optimality which are of interest to the DM.

(b) With a number of trade-off solutions in the region of interests we have argued that the decision-maker would be able to make a better and more reliable decision than with a single solution

(c) Such methodology allows the DM to first make a higher-level search of monitoring a region of interest on the Pareto-optimal front, rather than using a single solution to focus on a particular solution.

(d) Since there is instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, for this reasons a fuzzy representation of economic emission load dispatch problem has been defined.

(e)Eliminating the need of any weight vector and the need of applying the method again and again.

(f)The trade-off solutions in the obtained Pareto-optimal set are well distributed and have satisfactory diversity characteristics. This is useful in giving a reasonable freedom in choosing operating point from the available finite alternative.

(g) If a reference point is feasible and is not a Pareto-optimal solution, the decision-maker may then be interested in knowing solutions which are Pareto-optimal and close to the reference point. On the other hand, if the reference point is an infeasible one, the decision-maker would be interested in finding Pareto-optimal solutions which are close to the supplied reference point.

(h) On the basis of the application, we can conclude that the proposed method can provide a sound optimal power flow by simultaneously considering multiobjective problem.

For future work, we intend to test the algorithm on more complex real-world applications. Also, conduct research on the parallel mechanism of multi-reference point algorithms and multi-criteria decision group problems so that it improves the efficiency of such approaches which are very relevant for real- world scenarios.

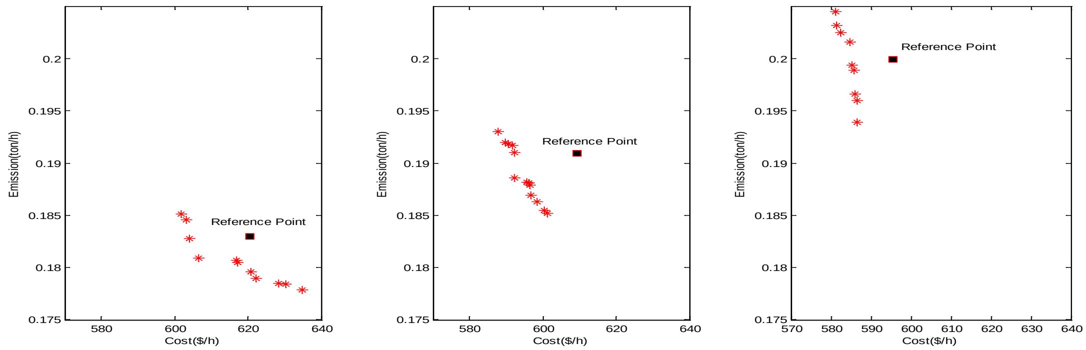

Fig. 3. Pareto optimal set for α cut level =0

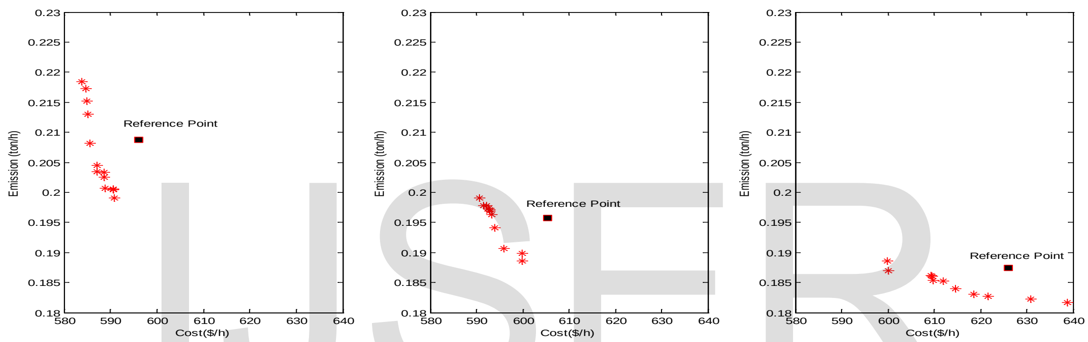

Fig. 4. Pareto optimal set for α cut level =0.2

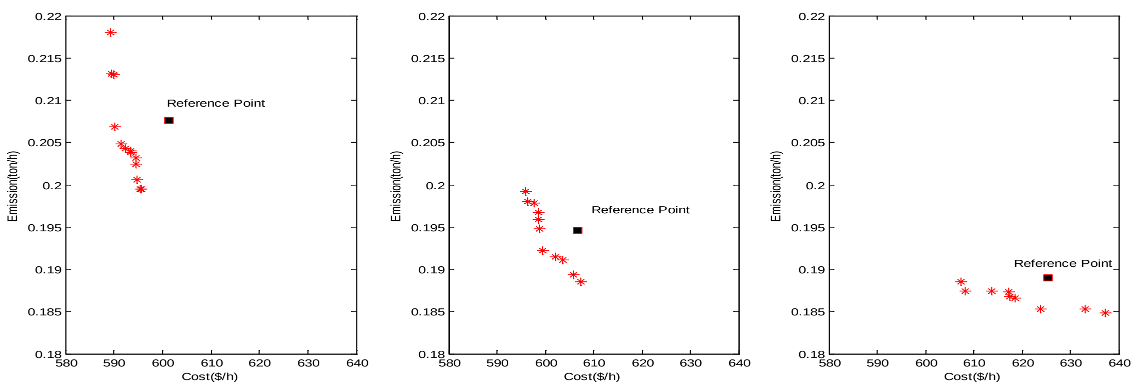

Fig. 5. Pareto optimal set for α cut level =0.4

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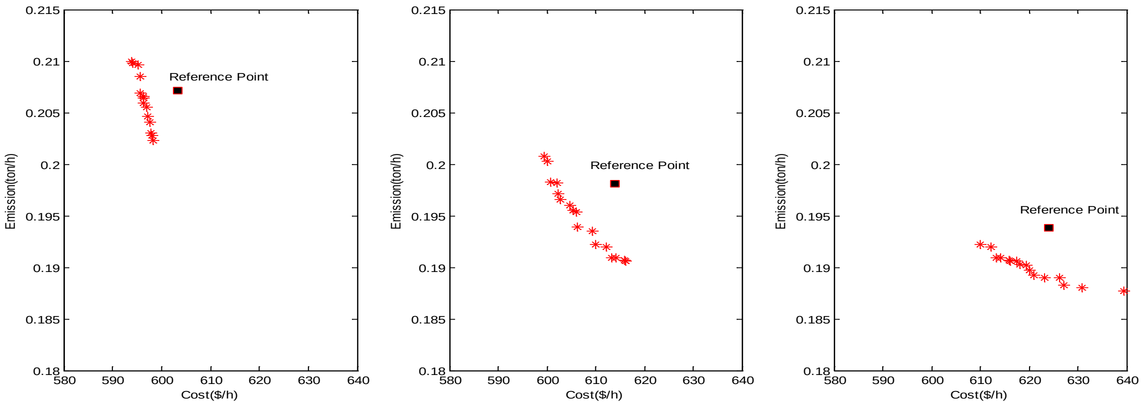

Fig. 6. Pareto optimal set for α cut level =0.6

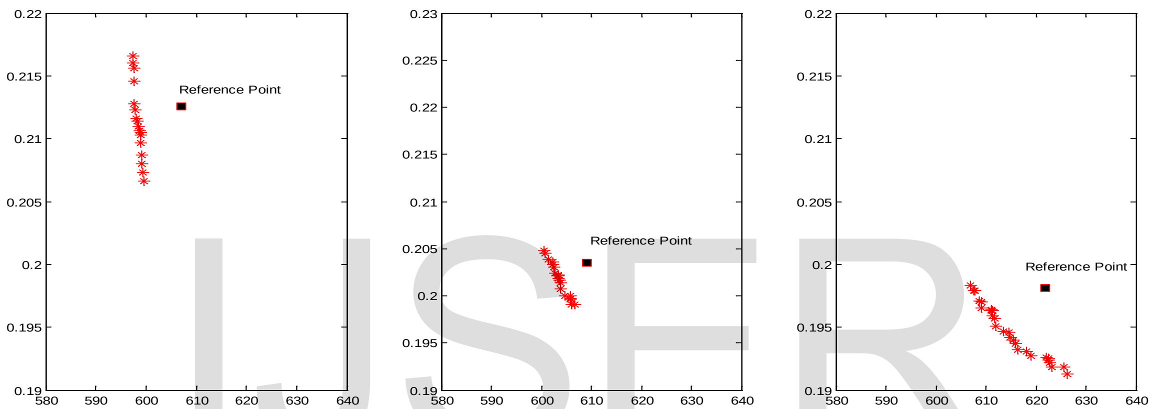

Fig. 7. Pareto optimal set for α cut level =0.8

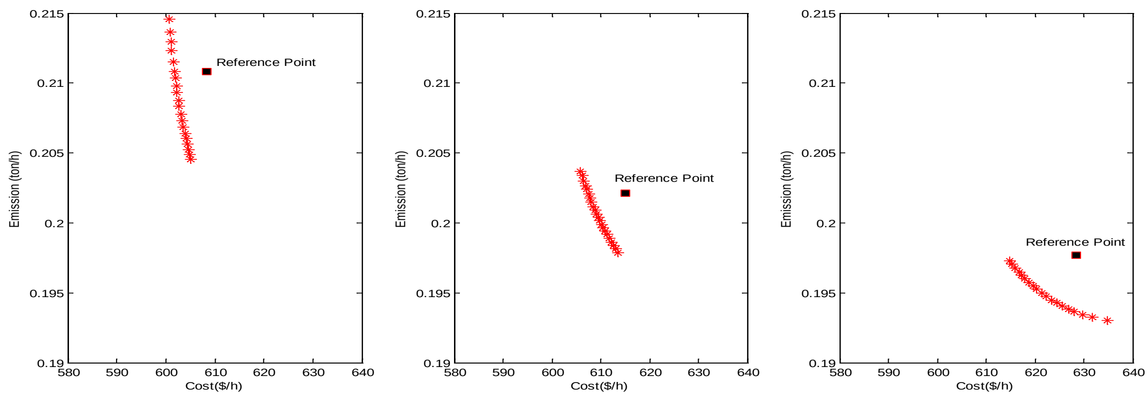

Fig. 8. Pareto optimal set for α cut level =1.0

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