Misalignment Considerations in Laser Diode to Single Mode Circular Core Dispersion-shifted/Dispersion-flattened Fiber Coupling via Hyperbolic Microlens on the Fiber Tip

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Abstract

We report the theoretical investigation of the coupling efficiency in presence of possible transverse and angular mismatches in case of laser diode to single-mode circular core dispersion-shifted/dispersion-flattened fiber coupling via hyperbolic microlens on the fiber tip. The study comprises first theoretical investigations of coupling optics involving the said type of coupler in presence of such mismatches. Employing ABCD matrix formalism for refraction of paraxial rays by a hyperbolic microlens on the fiber tip, we formulate analytical expressions for the coupling efficiencies in presence of the said two misalignments. Further, the lens transmitted spot size of the source should be equal to the spot size of the fiber in case of maximum coupling. In this connection, we use Petermann II spot size of the fiber in order to take care of non-Gaussian nature of field of such fibers and to make the prediction of the launch optics more realistic thereby. The investigations are made for two different wavelengths namely 1.3 µm and 1.5 µm in case of some typical dispersion managed optical fibers. Although, our simple method predicts the concerned coupling optics excellently, the evaluation of the concerned efficiencies and associated losses will involve little computations. The results present the relevant coupling efficiencies along with the tolerance with respect to the said kinds of mismatches and as such it will benefit the designers and packagers who are working in the field of optical technology.

Index Terms— laser diode, hyperbolic microlens, single-mode circular core dispersion-shifted/ dispersion-flattened fiber, optical coupling, coupling losses

1 INTRODUCTION

Microlenses on fiber tips have been found to be most efficient in respect of source to fiber coupling [1-5]. These microlenses, which are usually fabricated either in the conical or hemispherical shape, have the advantage of being self-centered. This is why tremendous interest has been generated for fabricating microlenses of different profiles in order to obtain maximum launch optics. Owing to limited aperture, mode mismatch and spherical aberration, the hemispherical microlens on the fiber tip is less efficient in terms of coupling[6-9], but it is still used worldwide since its fabrication is quite simple [3]. On the other hand, a hyperbolic microlens on the fiber tip emerges as the most suitable coupler in this context [2-4, 10-14] since hyperbolic microlens on the fiber tip has aperture large enough so as to collect the entire light emitted by laser diode and it is also free of spherical aberration. But the fabrication of hyperbolic microlens on fiber tip involves laser micro machining technique and as such it is not simple. The application of ABCD matrix formalism for the prediction of launch optics in case of laser diode to single-mode step index fiber coupling via hemispherical [6-9,15,16], hyperbolic [10-14] and upside down taper [17,18] lenses on the tips of the fibers has been found to have produced accurate results in a much simplified fashion. Further, graded index fiber being important in the light of its large bandwidth and negligible sensitivity to micro as well macro bending, investigations on the coupling optics involving both hyperbolic microlens [13] and hemispherical microlens [15, 16] on the tips of graded index fibers have been made recently. Further, also in this case the predictions on the basis of ABCD matrix formalism have been found to be excellent.

As regards optical communication, single-mode optical fiber has emerged as the most effective medium. The fiber material is silica for which the loss due to attenuation is minimum at the wavelength 1.55μm while the material dispersion is nil at the wavelength 1.3 μm. Accordingly, the operating wavelength for communication of information through optical fiber is restricted between 1.3 μm and 1.6 μm. If the zero dispersion wavelength is shifted to 1.55 μm by suitable choice of fiber parameters, one can obtain minimum attenuation and minimum dispersion simultaneously. Such fibers are termed as dispersion shifted fibers [19-21]. Obviously, dispersion shifted fibers provide large bandwidth and consequently fairly long repeater-less path. Again, taking into consideration that both Erbium doped fiber amplifier and Raman gain fiber amplifier perform efficiently around the wavelength 1.55 μm, one can realize the importance of dispersion-shifted fiber in the field of optical technology [22, 23]. Further, in another kind of fiber, known as dispersion-flattened fiber, the waveguide dispersion almost neutralises the material dispersion over a range of wavelength. This type of fiber can be employed to enhance the information carrying capacity by wavelength division multiplexing [24]. Therefore, prescription of simple but accurate expressions for fundamental modes of dispersion- shifted trapezoidal [19] as well as dispersion-flattened graded W [25] and step W fibers [26, 27] are necessary in respect of estimation of different propagation characteristics of such fibers. Very recently, simple power series expressions for fundamental modes in case of both dispersion-shifted trapezoidal and dispersion-flattened step and graded W fibers have been reported [28] and based on it, Petermann I and II spot sizes of such fibers have been predicted excellently [29].

In this paper, we present separately the coupling optics of laser diode to single-mode dispersion-shifted as well as dispersion-flattened fiber coupling via hyperbolic microlens on the fiber tip in presence of possible transverse and angular mismatches. Motivated by the accuracy of ABCD formalism, we have applied this simple but accurate method in order to formulate analytical expressions for coupling efficiencies in presence of said types of misalignments. Further, as regards spot size of such fiber which needs to be matched with lens transformed spot size for maximum coupling efficiency, we use estimated value of Petermann II spot size of the fiber in order to take care of non Gaussian nature of field of such fiber. For the present investigation, we employ two commonly used wavelengths, namely 1.3 μm and 1.5 μm [4]. The results found will assess the sensitivity of this coupler with respect to the said two types of misalignments. Thus such study of this type of coupler, which to the best of our knowledge is not available in literature till date, will be of immense importance in the field of the optimum launch optics.

2 THEORY

The basic coupling scheme has been presented in Fig. 1. The refractive indices of incident and lens media are n_1 and n_2 respectively. Elliptical intensity profile of optical beam emitted by the laser diode is characterized approximately by Gaussian spot sizes w_{1x} and w_{1y} along two mutually perpendicular directions x and y with x being parallel to the junction plane and y perpendicular to it. The laser diode field ψ_u at a distance u from the lens surface can be approximated as [30]

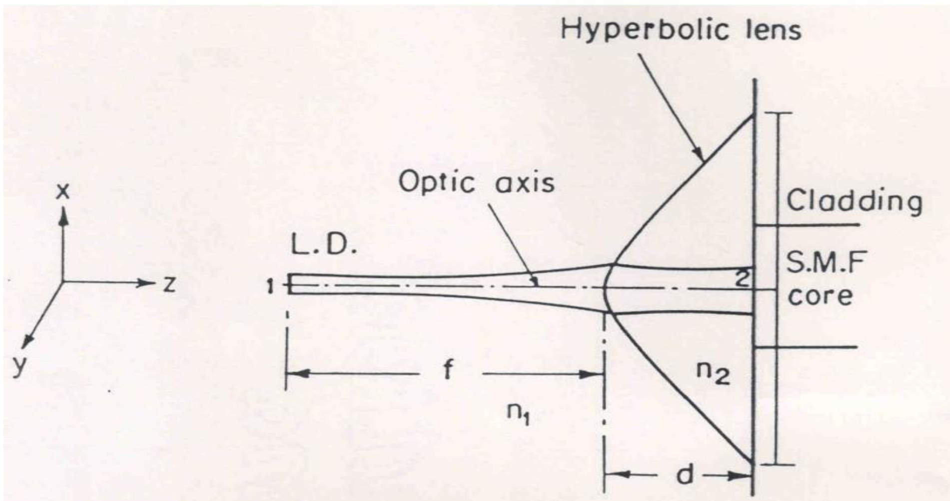


Fig. 1. Geometry of coupling of laser diode to circular core single-mode dispersion- shifted / dispersion flattened fiber via hyperbolic microlens on fiber tip.

$$\psi_u = \exp\left[-\left(\frac{x^2}{w_{1x}^2}+\frac{y^2}{w_{1y}^2}\right)\right]\times\exp\left[-\frac{jk_1(x^2+y^2)}{2R_1}\right] \tag{1}$$

Here, w_{1x}, w_{1y} represent the spot sizes of emitted optical beam from laser diode in X and Y directions respectively with R_1 and k_1 denoting the radius of curvature of the incident wavefront and the wave number in the incident medium respectively. Further, taking into consideration that Gaussian approximations for the fundamental mode in the circular core single-mode fiber have predicted the coupling optics exceedingly well [4, 6-18], we, here, also take similar expression [30] of the fundamental mode of the fiber as given below

$$\psi_f = \exp\left[-\frac{(x^2+y^2)}{w_f^2}\right] \tag{2}$$

where, w_f represents the spot size of the fiber. Further, it can be mentioned that though Gaussian nature of field inside the fiber simplifies the estimation of coupling optics, the field inside such dispersion managed is not strictly Gaussian and accordingly, w_f is regarded as Petermann II spot size in order to make the relevant study more realistic and simple as well [31]. Again concerned values of w_f are available in literature [29].

The hyperbolic lens transformed laser field ψ_v on the fiber plane 2 can be approximated as [30]

$$\psi_v = \exp\left[-\left(\frac{x^2}{w_{2x}^2}+\frac{y^2}{w_{2y}^2}\right)\right]\times\exp\left[-\frac{jk_2}{2}\left(\frac{x^2}{R_{2x}}+\frac{y^2}{R_{2y}}\right)\right] \tag{3}$$

here, k_2 is the wavenumber inside the lens medium with w_{2x}, w_{2y} presenting the lens transformed spot sizes along with the corresponding radii of curvature in x and y directions being respectively R_{2x} and R_{2y}. In the appendix, we have illustrated the method of evaluation of $w_{2x,2y}$ and $R_{2x,2y}$ in terms of $w_{1x,1y}$ and R_1 by ABCD matrix formalism [10]. Further, source to single-mode fiber coupling efficiency via hyperbolic microlens on the fiber tip is estimated by using the following well known overlap integral [6-18].

$$\eta = \frac{\left|\iint \psi_v \psi_f^* dxdy\right|^2}{\left|\iint \left|\psi_v\right|^2 dxdy \iint \left|\psi_f\right|^2 dxdy\right|} \tag{4}$$

For evaluation of coupling efficiency in presence of transverse misalignment in the X-Y plane, we assume that the centre of the fiber is displaced to a point having coordinates (d_1, d_2) as presented in Fig. 2. (a).

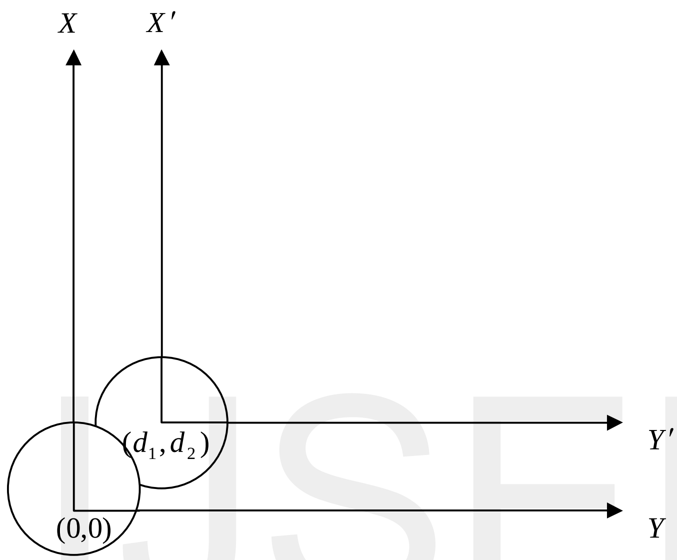


Fig. 2. (a) Transverse mismatch between the centre of the fiber and imaged laser spot.

Now the relation between the primed and unprimed coordinates can be expressed as

$$\begin{aligned} x &= x' + d_1 \\ y &= y' + d_2 \end{aligned} \tag{5}$$

Thus the fundamental mode in the fiber can be presented as

$$\psi_f = \exp\left[-\left(\frac{(x-d_1)^2 + (y-d_2)^2}{w_f^2}\right)\right] \tag{6}$$

Using Eqs. (3), (4) and (6) we get [14]

$$\eta_t = \eta \exp\left[\frac{2d_1^2}{w_f^2}\left\{\frac{w_{2x}^2\left(w_{2x}^2+w_f^2\right)}{\left(w_f^2+w_{2x}^2\right)^2+\left(k_2^2 w_{2x}^4 w_f^4\right)/4R_{2x}^2}-1\right\}\right]\times \exp\left[\frac{2d_2^2}{w_f^2}\left\{\frac{w_{2y}^2\left(w_{2y}^2+w_f^2\right)}{\left(w_f^2+w_{2y}^2\right)^2+\left(k_2^2 w_{2y}^4 w_f^4\right)/4R_{2y}^2}-1\right\}\right] \quad (7)$$

Where, $\eta = \dfrac{4w_{2x}w_{2y}w_f^2}{\left(\left[(w_f^2+w_{2x}^2)^2+\dfrac{(k_2^2 w_f^4 w_{2x}^4)}{(4R_{2x}^2)}\right]^{0.5}\times\left[(w_f^2+w_{2y}^2)^2+\dfrac{(k_2^2 w_f^4 w_{2y}^4)}{(4R_{2y}^2)}\right]^{0.5}\right)}$

In Fig. 2. (b), we show the angular misalignment of very small angle θ between the hyperbolic lens and the entrance of the fiber

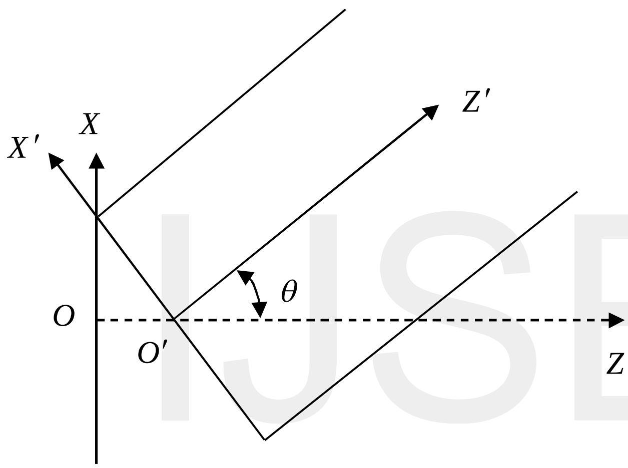


Fig. 2. (b) Angular offset between hyperbolic lens transformed input face and the end face of the fiber.

In this case the relation between primed and unprimed coordinates can be expressed as

$$\begin{aligned} x &= x'\cos\theta + z'\sin\theta \\ y &= y' \\ z &= -x'\sin\theta + z'\cos\theta \end{aligned} \quad (8)$$

In case of small angular mismatch, we can take $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ and thereby we obtain

$$\begin{aligned} x &= x' + z'\theta \\ y &= y' \\ z &= -x'\theta + z' \end{aligned} \quad (9)$$

The lens transformed field on the fiber can be approximated as [1]

$$\psi_v = \exp\left[-\left(\frac{x^2}{w_{2x}^2}+\frac{y^2}{w_{2y}^2}\right)\right]\exp\left[-\frac{jk_2}{2}\left(\frac{x^2}{R_{2x}}+\frac{y^2}{R_{2y}}\right)\right]\exp(-jk_2 z) \tag{10}$$

At the input end of the fiber z' being zero, Eq. (10) reduces to

$$\psi_v = \exp\left[-\left(\frac{x'^2}{w_{2x}^2}+\frac{y'^2}{w_{2y}^2}\right)\right]\exp\left[-\frac{jk_2}{2}\left(\frac{x'^2}{R_{2x}}+\frac{y'^2}{R_{2y}}\right)\right]\exp(jk_2 x'\theta) \tag{11}$$

Now, the fundamental modal field of the circular core fiber is given by

$$\psi_f = \exp\left[-\left(x'^2+y'^2\right)/w_f^2\right] \tag{12}$$

Using Eqs. (11) and (12) in (4), we obtain the coupling efficiency η_a in presence of small angular misalignment θ as

$$\eta_a = \eta\exp\left[-\frac{k_2^2\theta^2}{2}\left\{\frac{w_{2x}^2 w_f^2\left(w_{2x}^2+w_f^2\right)}{\left(w_f^2+w_{2x}^2\right)^2+\left(k_2^2 w_{2x}^4 w_f^4\right)/4R_{2x}^2}\right\}\right] \tag{13}$$

Employing the analytical formulations of η_t and η_a presented in Eq. (7) and (13) respectively; we investigate the coupling efficiencies and the corresponding losses owing to the said types of mismatches.

In this context, it is relevant to present the profile status of the dispersion managed fibers, we are using here.

The refractive index profile of graded index optical fiber is represented as

$$n^2(R) = \begin{cases} n_{co}^2(1-2\delta f(R)), & R\le 1 \\ n_{cl}^2\ , & R>1 \end{cases} \tag{14}$$

where, $R=\dfrac{r}{a}$, 'a' the core radius , 'r' the distance measured radially from the axis of the fiber and $\delta=\dfrac{(n_{co}^2-n_{cl}^2)}{2n_{co}^2}$, with n_{co} and n_{cl} representing the refractive indices of the core and cladding respectively.

The refractive index profile functions $f(R)$ for the concerned fibers are expressed as,

(I) $$\left.\begin{array}{ll} f(R)=0\ , & 0<R\le S \\ f(R)=\dfrac{R-S}{1-S}\ , & S<R\le 1 \end{array}\right\} \quad \text{dispersion} - \text{shifted trapezoidal fiber}$$ [19]

$$\text{(II)} \quad \left.\begin{aligned} f(R) &= \rho R^{q}, & R &\le \frac{1}{C} \\ &= \rho, & \frac{1}{C} &< R \le 1 \end{aligned}\right] \quad \text{dispersion} - \text{flattened graded W fiber [25]}$$

$$\text{(III)} \quad \left.\begin{aligned} f(R) &= 0, & R &\le \frac{1}{C} \\ f(R) &= \rho, & \frac{1}{C} &< R \le 1 \end{aligned}\right] \quad \text{dispersion} - \text{flattened step W fiber [26, 27]}$$

Here, 'S' stands for the aspect ratio for trapezoidal fiber. Again, q presents profile exponent in case of W fiber and clearly, $q=\infty$ for step index profile. Further, ρ, which quantifies the relative index depth of inner cladding of refractive index n_i, is given as $\rho = \frac{(n_{co}^2 - n_i^2)}{(n_{co}^2 - n_{cl}^2)}$

3 RESULTS AND DISCUSSIONS

For estimation of coupling efficiencies of the given set up in presence of possible transverse as well as angular mismatches, we use [4] two laser diodes– one emitting wavelength λ = 1.5 µm with w_{1x}= 0.843 µm, w_{1y} = 0.857 µm and another emitting wavelength λ = 1.3 µm with w_{1x}= 1.081 µm, w_{1y} = 1.161 µm. Further, the material refractive index of microlens and the maximum or axial depth (d) of microlens are taken as 1.55 µm and 6.0 µm respectively [4, 10-14]. Again, the estimation of coupling efficiency on the basis of plane wavefront model for the input beam from the laser facet being almost identical with that on the basis of spherical wavefront model [4, 6-18], we restrict our present investigation to planar wavefront model for the sake of simplicity and accuracy as well. As typical example of single-mode dispersion-shifted fiber, we use three trapezoidal fibers, each of same V number as 2.5 but of different aspect ratio S namely 0.25, 0.50 and 0.75 respectively [19]. It is found that at excitation wavelength 1.5 µm, the maximum coupling efficiencies for the said fibers(taken in ascending order of aspect ratio) in absence of mismatches are nearly 79.93% (corresponding loss 0.972902dB), 70.57%(corresponding loss 1.513799dB), and 72.80% (corresponding loss 1.378686dB), with the corresponding effective focal lengths of hyperbolic microlenses being11.4 µm, 10.7 µm and 10.9 µm respectively. Further, it deserves mentioning in this context that the wavelength 1.5 µm is appropriate for dispersion-shifted fiber and further, this wavelength is significant in all optical technology taking into consideration that erbium doped fiber amplifier and Raman gain fiber amplifier work efficiently around this wavelength[22,23,32]. We, however, repeat the same investigation for λ = 1.3 µm for the sake of technical importance only. It is found that at excitation wavelength 1.3 µm, the maximum coupling efficiencies for the said fibers (taken in ascending order of aspect ratio) in absence of mismatches are nearly 99.4% (corresponding loss 0.026136dB), 96.01%(corresponding loss 0.176835dB), and 97.27% (corresponding loss 0.120211dB), with the corresponding effective focal lengths of hyperbolic microlenses being 11.6 µm, 11.1µm and 11.3 µm respectively. Further, it deserves mentioning in this connection that presently designers do not exceed transverse mismatch beyond 2 µm and angular mismatch beyond 2^0 and thus our study is restricted to 0- 2 µm for transverse mismatch and 0^0- 2^0 for angular mismatch. Moreover, the focal length in each case has been optimized so as to produce maximum coupling efficiency [31]. In Figs. 3. 1(a), 3. 2(a) and 3. 3(a), we present the variation of coupling loss versus d_1 or d_2 for three trapezoidal fibers of same V number but of aspect ratio 0.25, 0.50 and 0.75 respectively at excitation wavelength λ = 1.3 µm while Figs. 3. 1(b), 3. 2(b) and 3. 3(b) present the same kind of variation for the said fibers in the same order at excitation wavelength λ = 1.5 µm. Thus it is seen the trapezoidal fibers are most efficient in terms of coupling at wavelength 1.3 µm and further, such fibers show more tolerance with respect to transverse mismatch at both the wavelengths 1.3 µm and 1.5 µm.

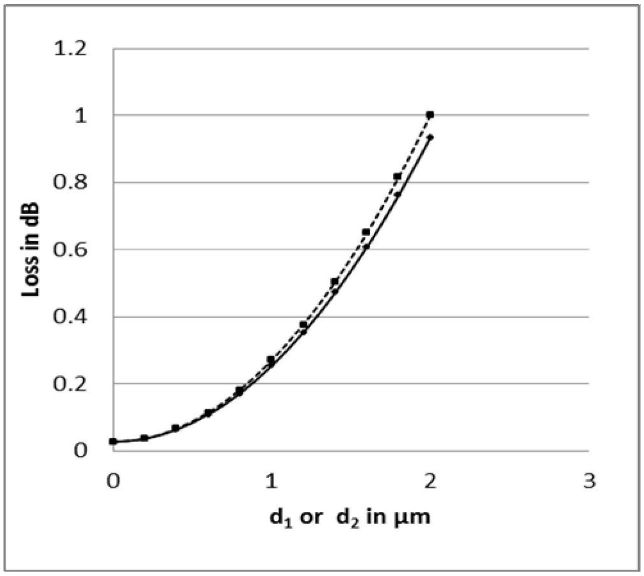


Fig. 3. 1(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (μm) for effective focal length f=11.6 μm in case of Trapezoidal fiber (V=2.5, S=0.25, w_f=4.15735) excited by wavelength λ=1.3 μm having w_{1x}=1.081 μm and w_{1y}=1.161 μm.
——— d_1; -------- d_2

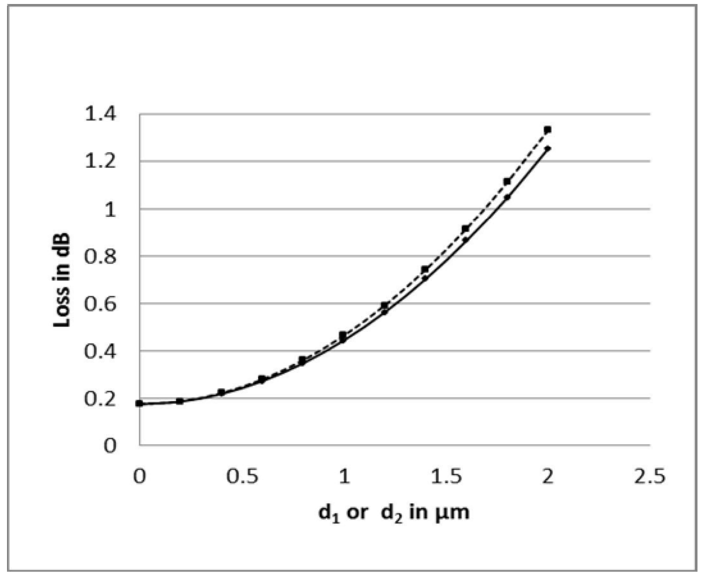


Fig. 3. 2(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (μm) for effective focal length f=11.1 μm in case of Trapezoidal fiber (V=2.5, S=0.50, wf=3.59452) excited by wavelength λ=1.3 μm having w1x=1.081 μm and w1y=1.161 μm.
——— d_1; -------- d_2

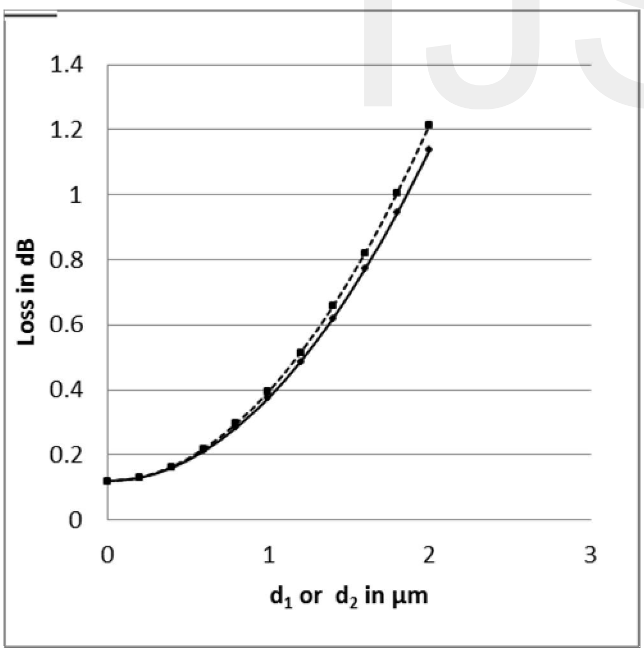


Fig. 3. 3(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (μm) for effective focal length f=11.3 μm in case of Trapezoidal fiber (V=2.5, S=0.75, w_f=3.7489) excited by wavelength λ=1.3 μm having w_{1x}=1.081 μm and w_{1y}=1.161 μm.
——— d_1; -------- d_2

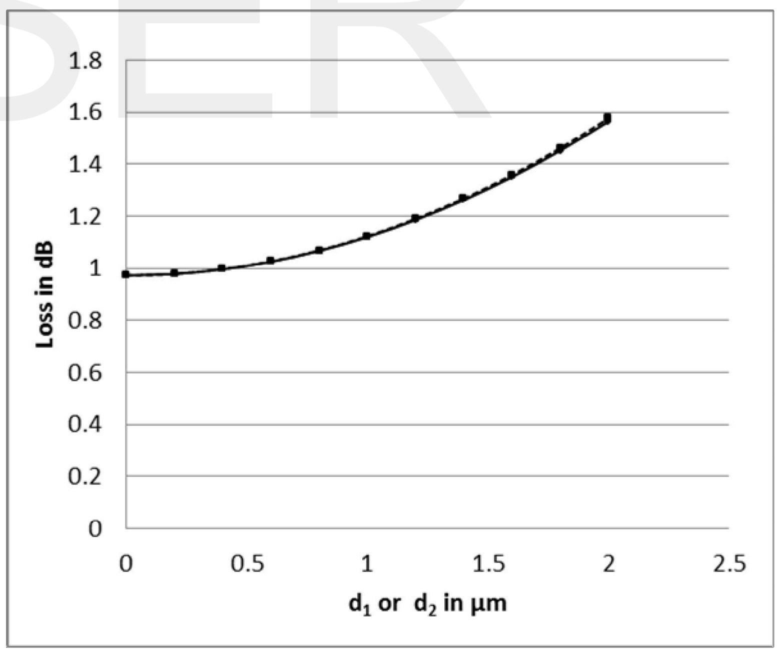


Fig. 3. 1(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (μm) for effective focal length f=11.4 μm in case of Trapezoidal fiber (V=2.5, S=0.25, w_f=4.1573) excited by wavelength λ=1.5 μm having w_{1x}=0.843 μm and w_{1y}=0.857 μm.
——— d_1; -------- d_2

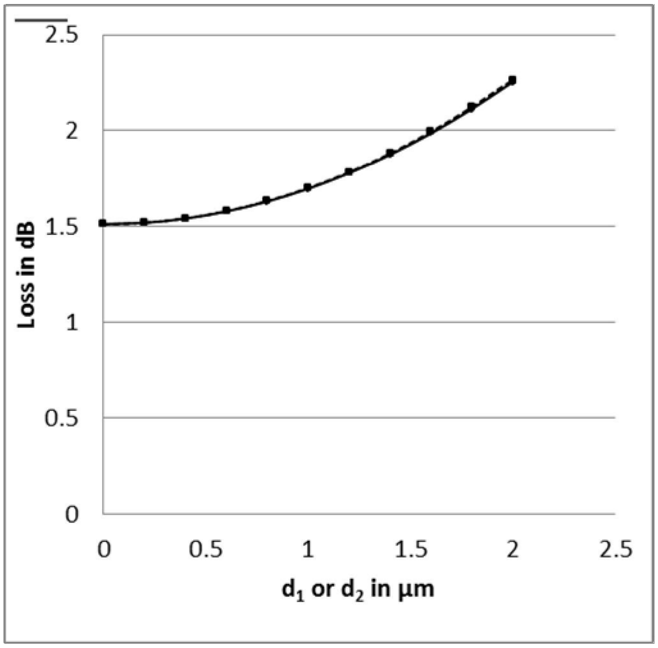


Fig. 3. 2(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=10.7 µm in case of Trapezoidal fiber (V=2.5, S=0.50, w_f=3.5945) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

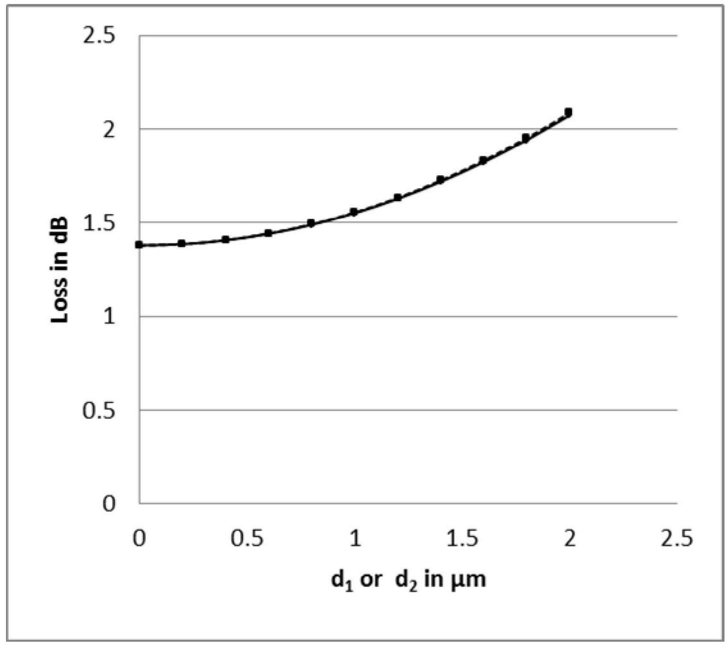


Fig. 3. 3(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=10.9 µm in case of Trapezoidal fiber (V=2.5, S=0.75, w_f=3.7489) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

Further, Figs. 3. 1(c), 3. 2(c) and 3. 3(c) present how coupling loss varies with angular mismatch in case of the said three trapezoidal fibers in the same order at excitation wavelength λ =1.3 µm and similarly Figs. 3. 1(d), 3. 2(d) and 3. 3(d) depict the same study at wavelength λ = 1.5 µm.

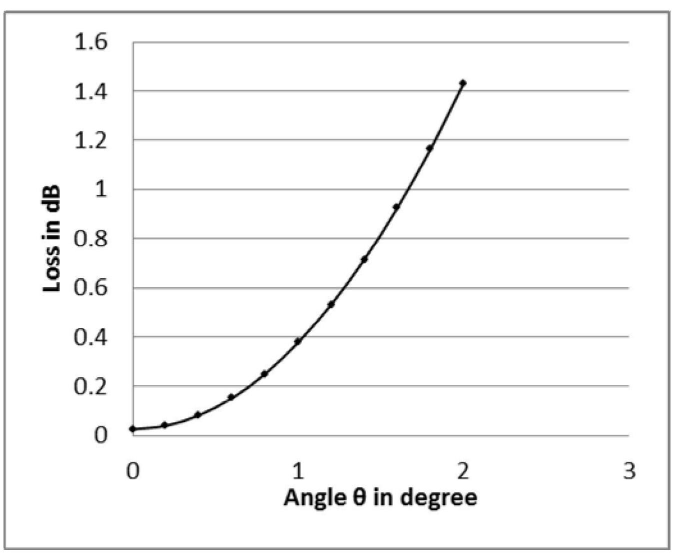


Fig. 3. 1(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.6 µm in case of Trapezoidal fiber (V=2.5, S=0.25, w_f=4.15735) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.

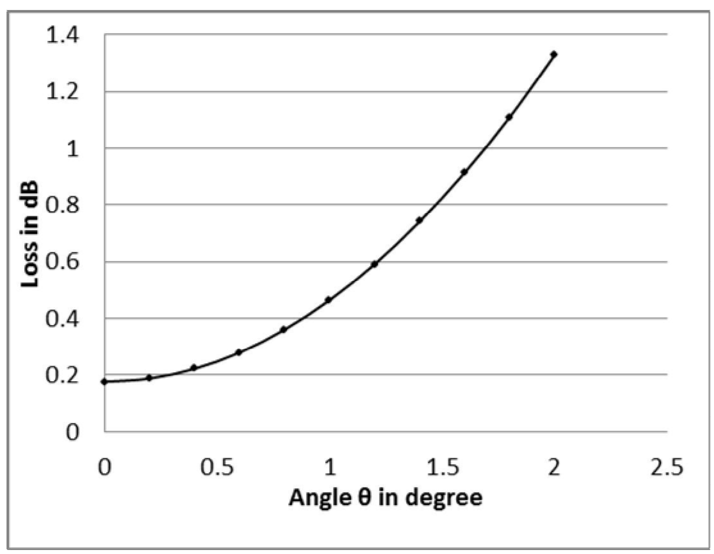


Fig. 3. 2(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.1 µm in case of Trapezoidal fiber (V=2.5, S=0.5, w_f=3.59452) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.

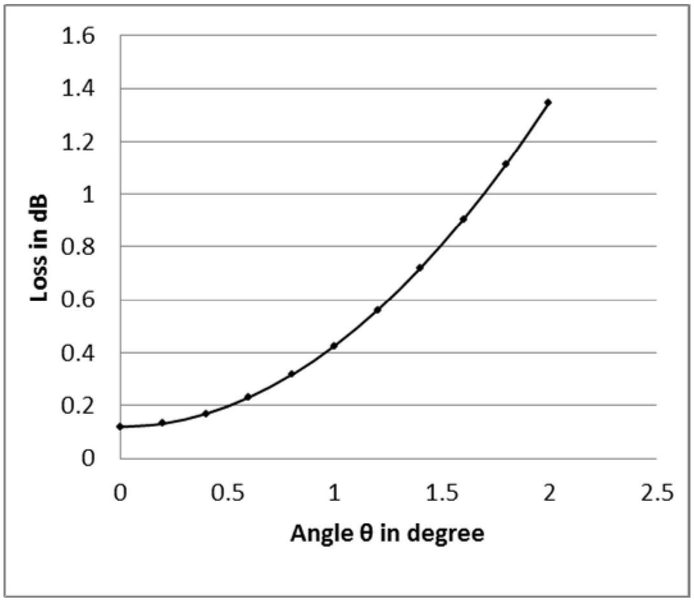


Fig. 3. 3(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.3 µm in case of Trapezoidal fiber (V=2.5, S=0.75, w_f=3.7489) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.

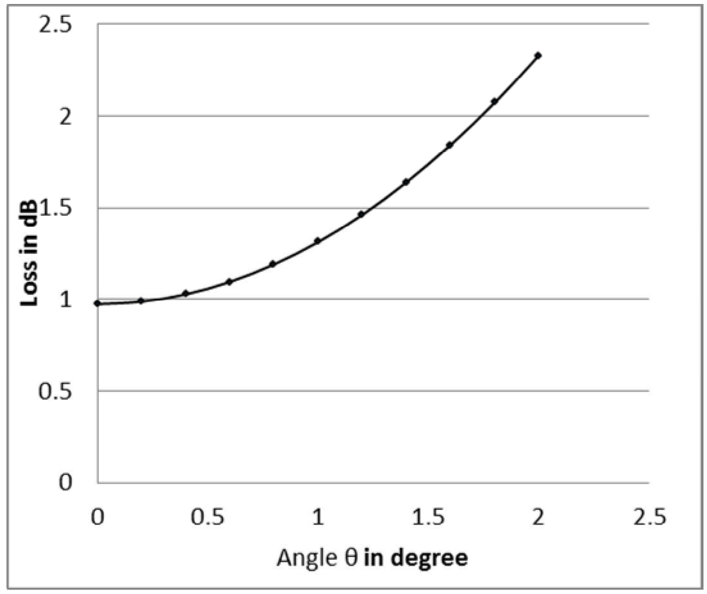


Fig. 3. 1(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.4 µm in case of Trapezoidal fiber (V=2.5, S=0.25, w_f=4.1573) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

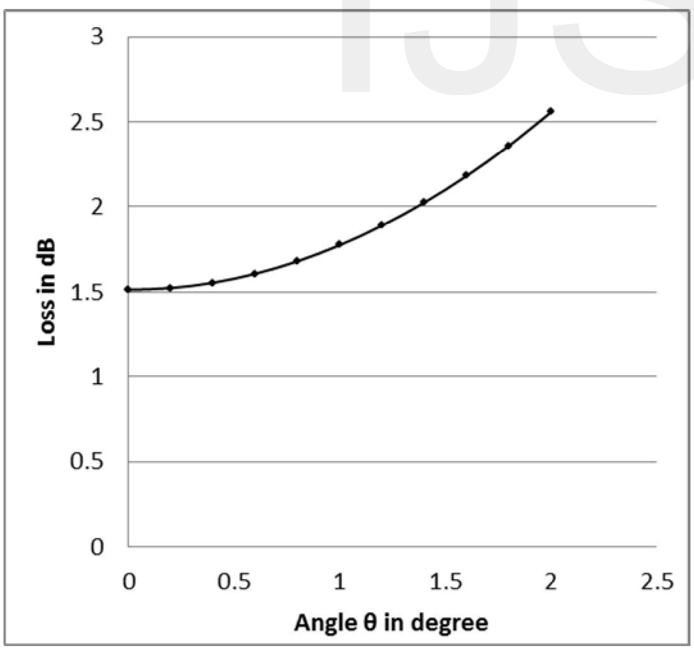


Fig. 3. 2(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=10.7 µm in case of Trapezoidal fiber (V=2.5, S=0.50, w_f=3.5945) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

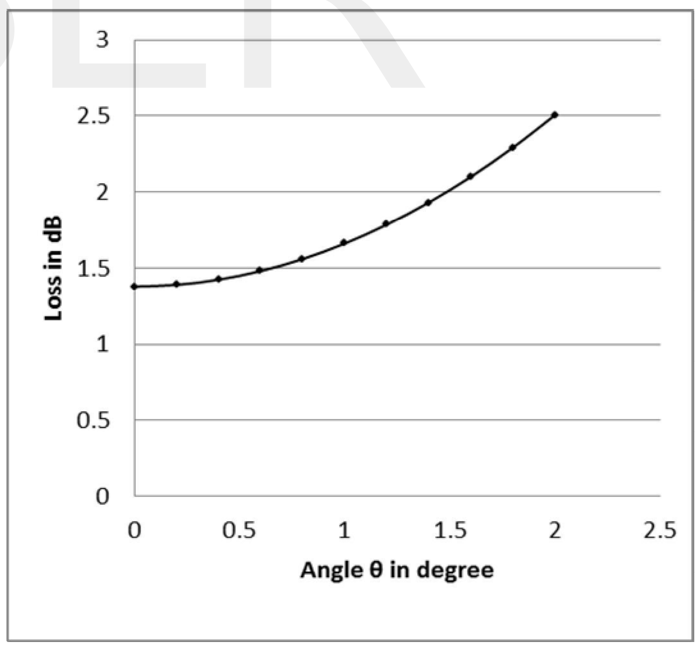
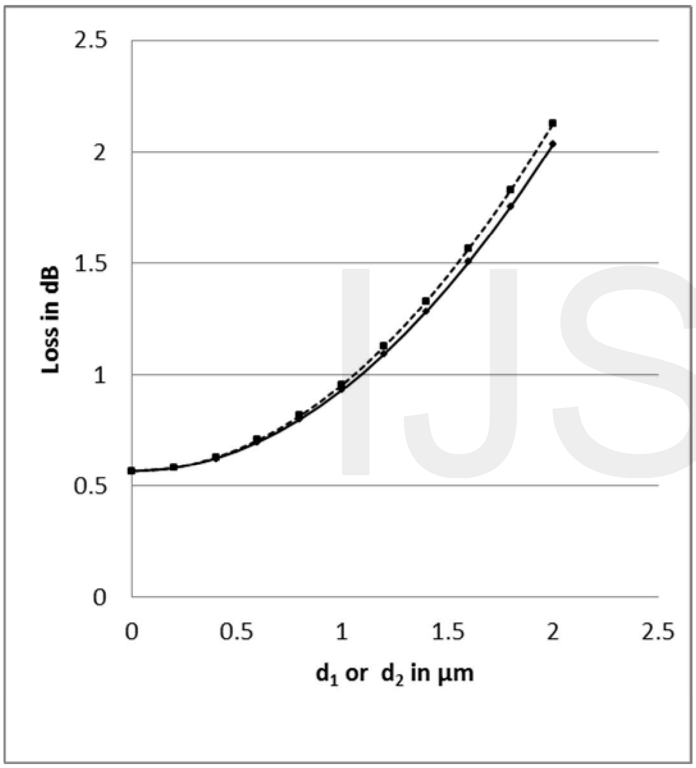


Fig. 3. 3(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=10.9 µm in case of Trapezoidal fiber (V=2.5, S=0.75, w_f=3.7489) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

As typical example of single-mode dispersion-flattened fiber, we choose two parabolic index (q=2) W fibers each of the same V number as 3.0 and same C value as 1.5 but having slightly different relative index depth (ϱ) of values 1.4975 and 1.5000 respectively [25]. It is found that at excitation wavelength 1.5 µm in absence of mismatches, the maximum coupling efficiencies for these fibers (taken in ascending order of relative index depth) are found to be nearly 58.96% and 58.95% respectively with the corresponding effective focal lengths of hyperbolic microlenses being 9.1 µm in each case. Further, when the excitation wavelength is changed to 1.3 µm, the maximum coupling efficiency without consideration of mismatch for each of the said fibers becomes nearly 87.79% with the corresponding effective focal lengths of hyperbolic microlenses being 9.8 µm & 9.7 µm respectively. The variation of coupling loss versus d_1 or d_2 has been represented in Figs. 4. 1(a) and 4. 2(a) for the two kinds of Graded W fiber having the same V number, profile exponent and C value but of different relative index depth (ϱ) as 1.4975 and 1.50 respectively at excitation wavelength λ = 1.3 µm.



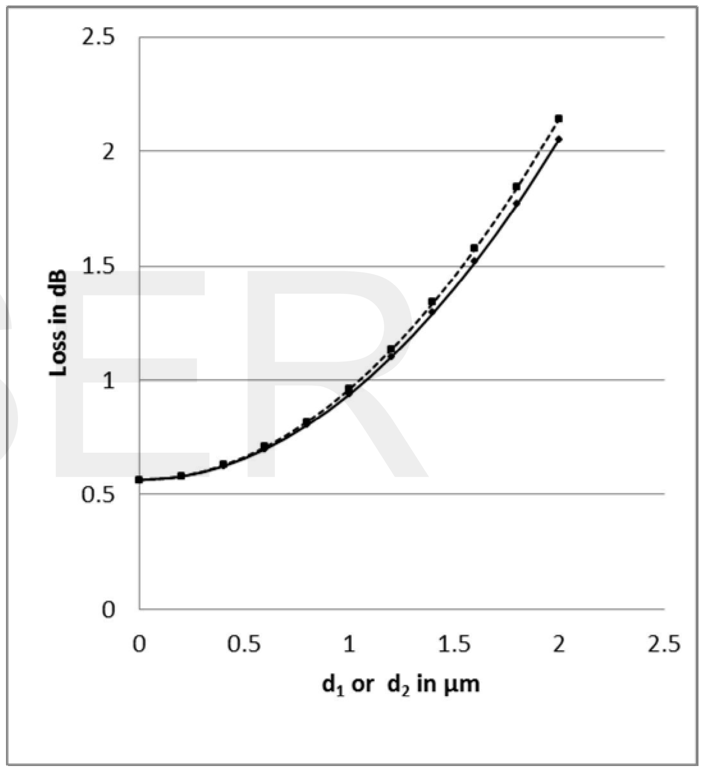


Fig. 4. 1(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=9.8 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.4975,w_f=2.92876) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.
——— d_1; -------- d_2

Fig. 4. 2(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=9.7 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.50, w_f=2.928395) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.
——— d_1; -------- d_2

In the same way, Figs. 4. 1(b) and 4. 2(b) show similar variation for the same fibers when 1.5 µm is used as excitation wavelength.

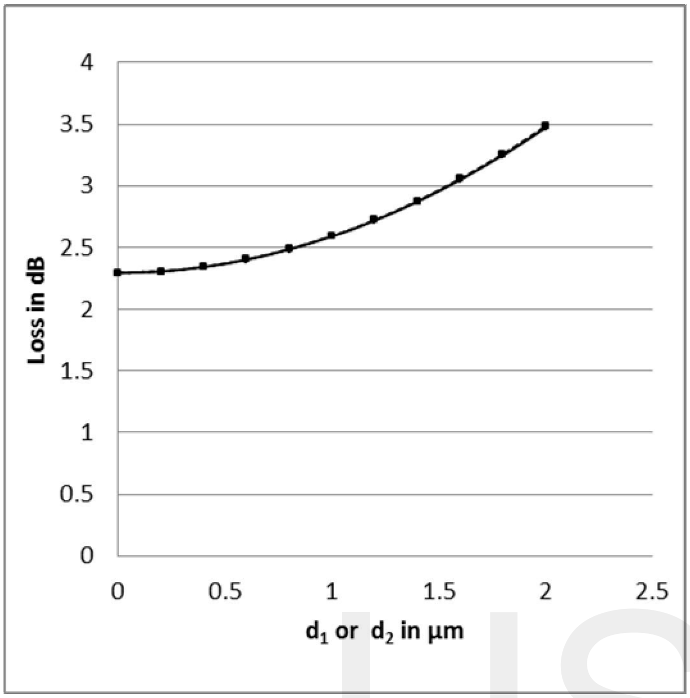


Fig. 4. 1(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length *f*=9.1 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.4975, w_f=2.9287) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

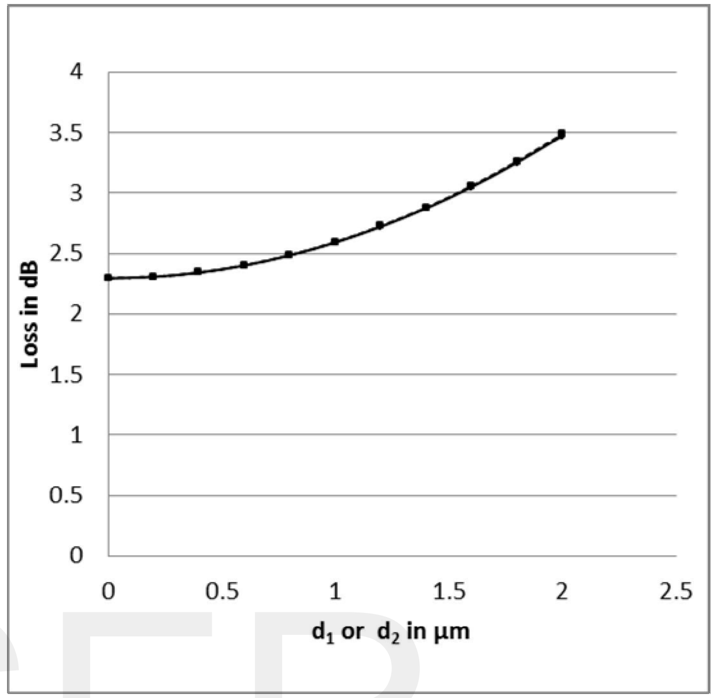
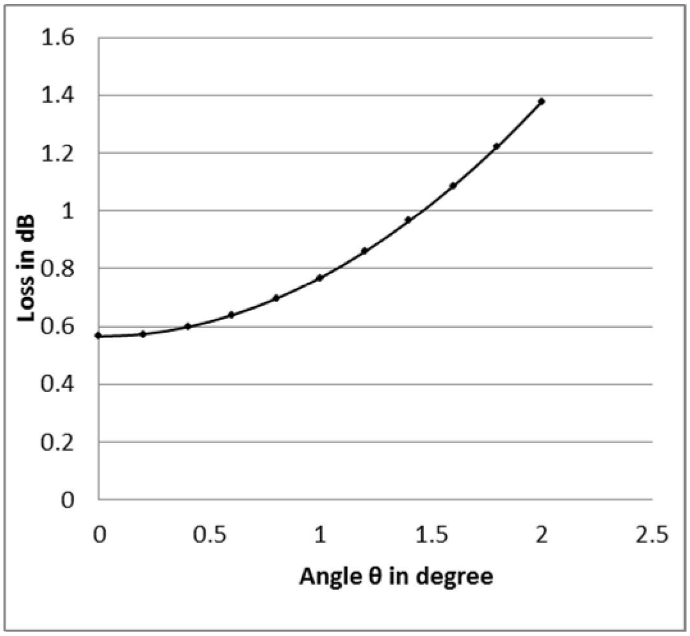


Fig. 4. 2(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length *f*=9.1 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.50, w_f=2.9283) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

On the other hand Fig. 4. 1(c), 4. 2(c) and 4. 1(d), 4. 2(d) represent the said variations in case of angular mismatch while the excitation wavelengths are λ = 1.3 µm λ = 1.5 µm respectively.



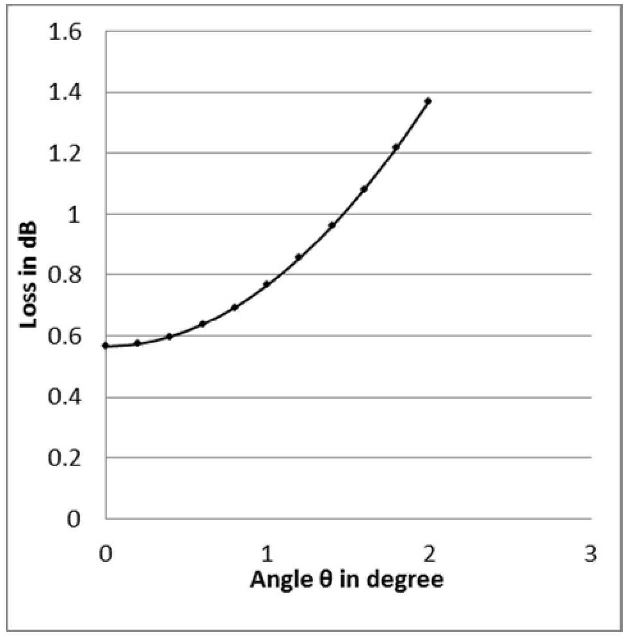
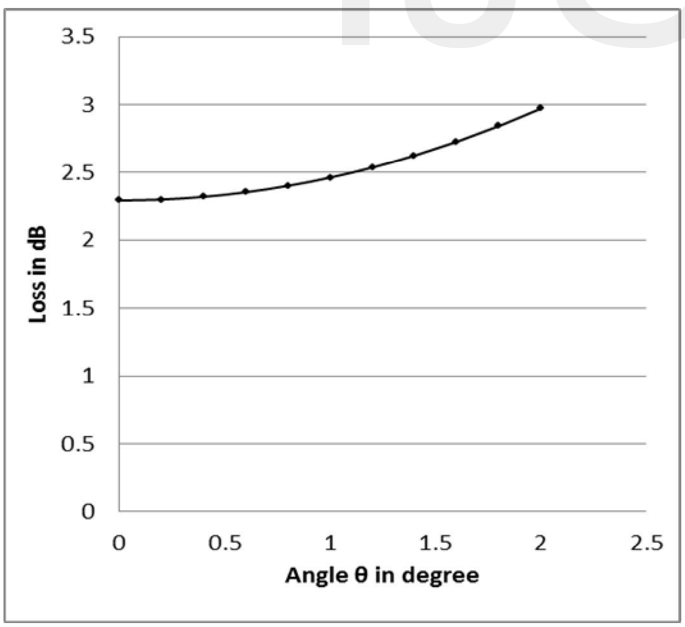


Fig. 4. 1(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=9.8 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.4975, w_f=2.92876) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.

Fig. 4. 2(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=9.7 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.50, w_f=2.928395) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.



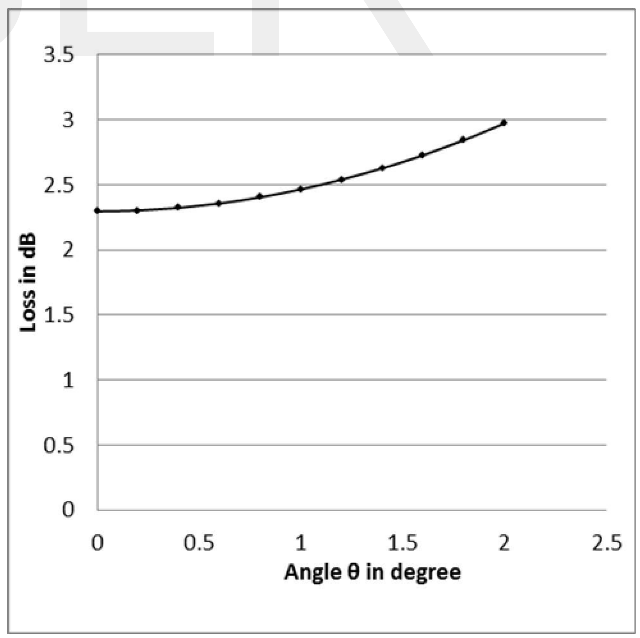


Fig. 4. 1(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=9.1 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.4975, w_f=2.9287) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

Fig. 4. 2(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=9.1 µm in case of Graded W fiber (V=3, q=2, C=1.5, ϱ=1.50, w_f=2.9283) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

Again, as regards dispersion- flattened fiber, we choose two Step W fibers each being of same V number 2.0 and C value 2.0 but different relative index depth (ϱ) as 1.3333 and 1.2500 respectively [26, 27]. It is found that for excitation-wavelength 1.5 µm and in absence of any possible mismatch, the maximum coupling efficiency for the said fibers (taken in descending order of relative index depth) are almost 88.83% and 88.16% with the corresponding focal lengths of hyperbolic microlenses being 11.8 µm & 11.7µm respectively. When the excitation wavelength becomes 1.3µm, the maximum coupling efficiencies for the said fibers becomes almost 99.41% and 99.55% with the corresponding focal length of hyperbolic microlenses in each case being 11.9 µm.

Further, the corresponding variation of coupling loss versus d_1 or d_2 for excitation wavelength 1.3 µm has been depicted for the said two Step W fibers taken in the descending order of relative index depth in Figs. 5. 1(a) and 5. 2(a) and Figs. 5. 1(b) and 5. 2(b) correspond to similar variation at excitation wavelength 1.5 µm.

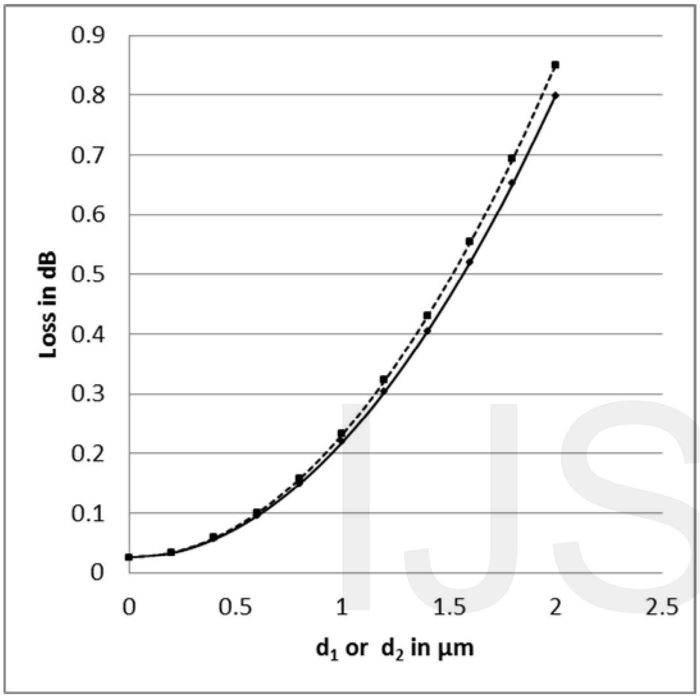


Fig. 5. 1(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=11.9 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.3333, w_f=4.81143) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.
——— d_1; -------- d_2

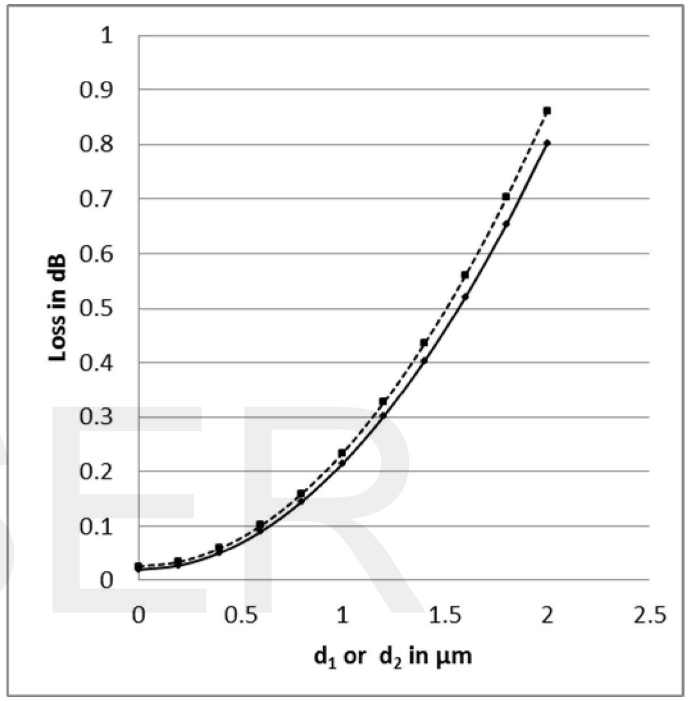


Fig. 5. 2(a) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=11.9 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.25, w_f=4.75558) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.
——— d_1; -------- d_2

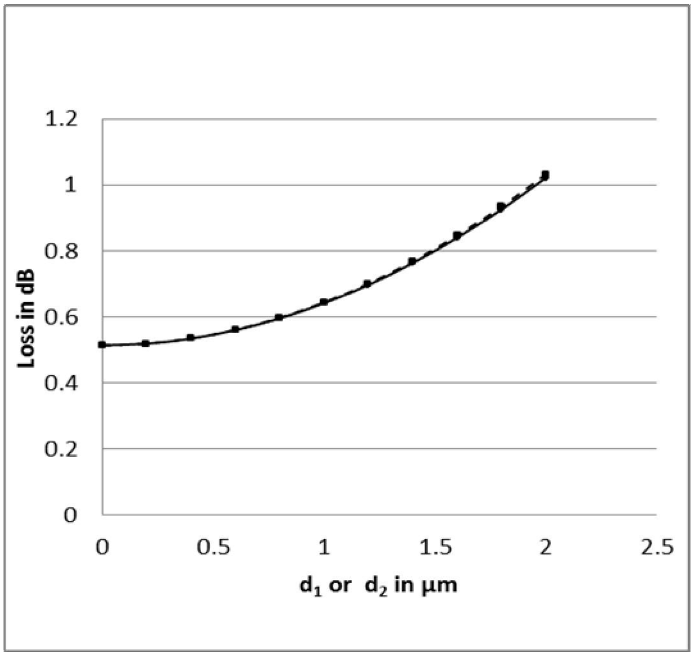


Fig. 5. 1(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=11.8 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.3333, w_f=4.8114) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

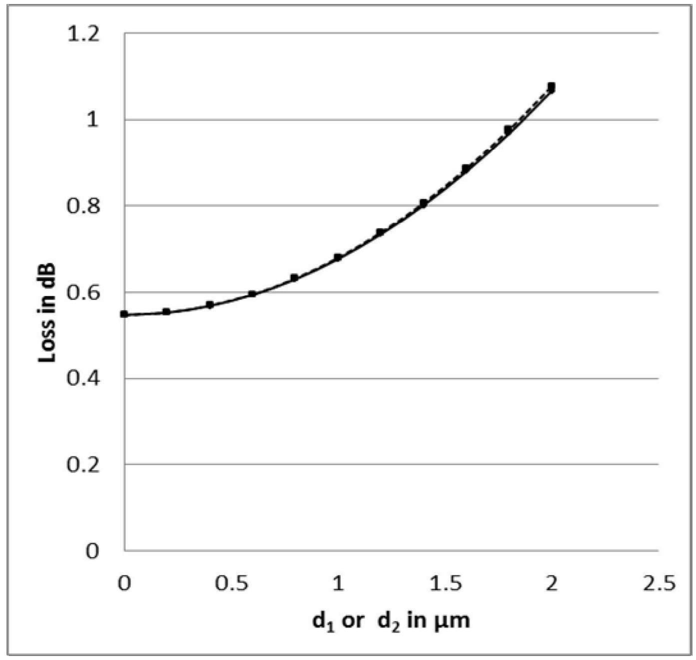


Fig. 5. 2(b) Variation of coupling loss (dB) with transverse offset d_1 or d_2 (µm) for effective focal length f=11.7 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.25, w_f=4.7555) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.
——— d_1; -------- d_2

Again, Figs. 5. 1(c), 5. 2(c) and 5. 1(d), 5. 2(d) show the variation of coupling loss with angular mismatch for the said fibers taken in the said order for excitation wavelengths 1.3 µm and 1.5 µm respectively.

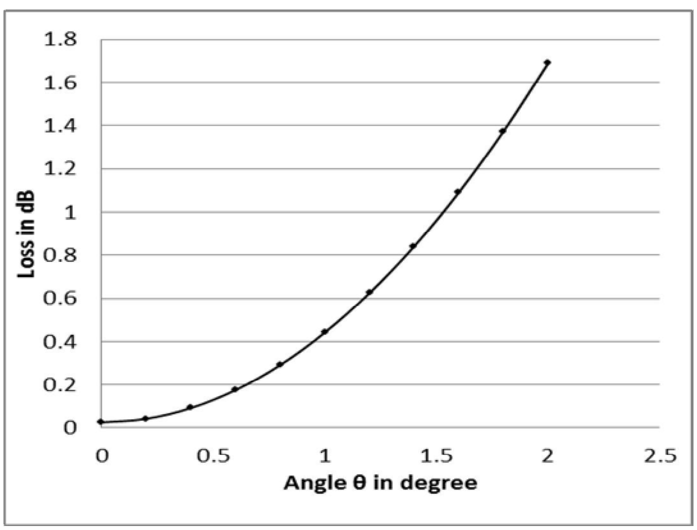


Fig. 5. 1(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.9 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.3333, w_f=4.81143) excited by wavelength λ=1.3 µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.

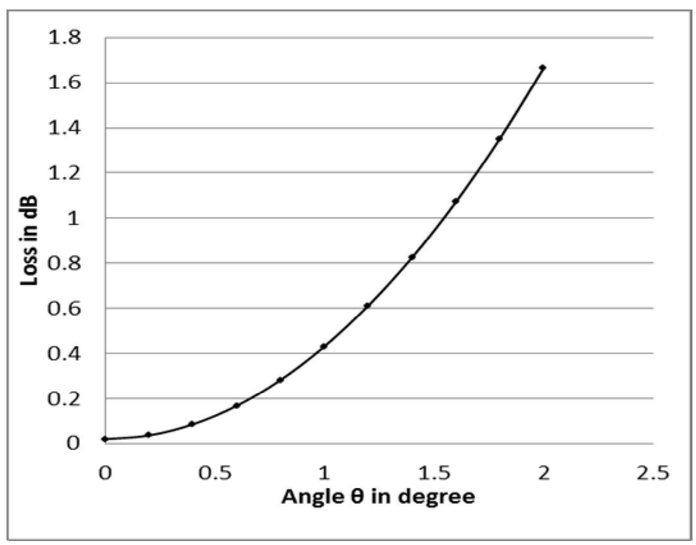
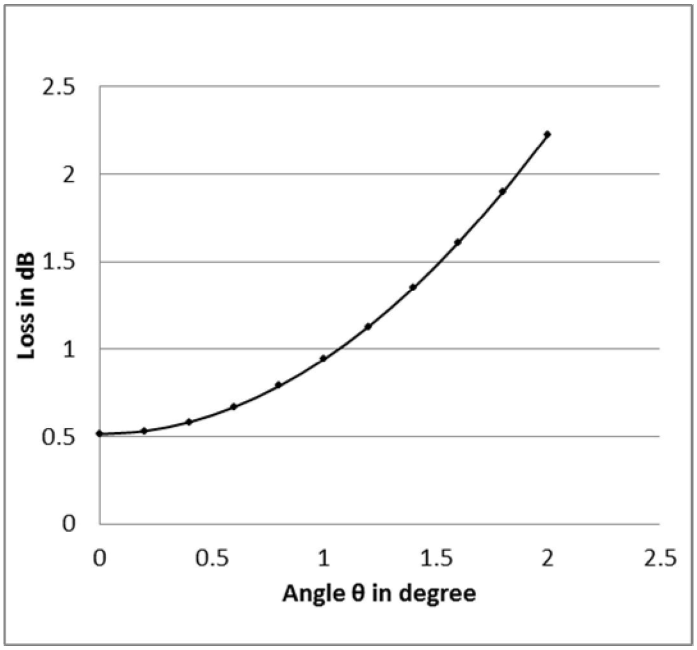


Fig. 5. 2(c) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.9 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.25, w_f=4.75558) excited by wavelength λ=1.3µm having w_{1x}=1.081 µm and w_{1y}=1.161 µm.



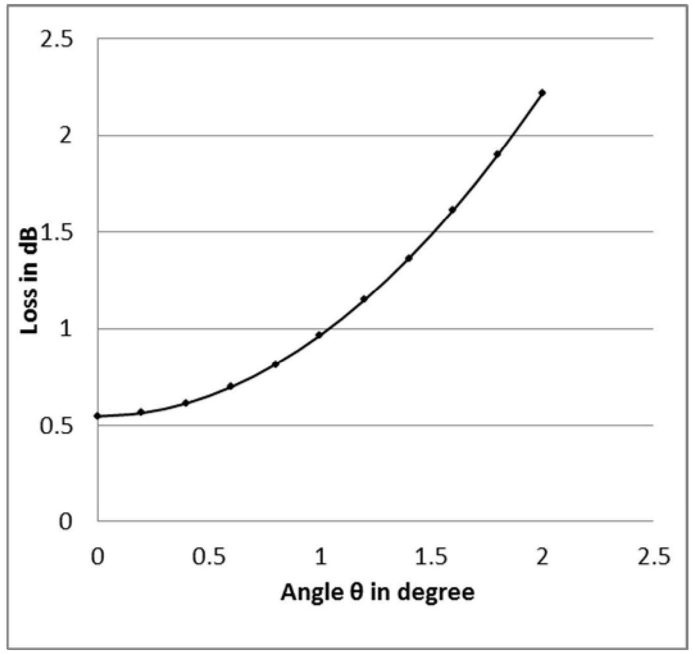


Fig. 5. 1(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.8 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.3333, w_f=4.8114) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

Fig. 5. 2(d) Variation of coupling loss (dB) with angular offset θ in degree for effective focal length f=11.7 µm in case of Step W fiber (V=2, C=2.0, ϱ=1.25, w_f=4.7555) excited by wavelength λ=1.5 µm having w_{1x}=0.843 µm and w_{1y}=0.857 µm.

Further, the typical fibers used in our study have been mentioned in references [19, 25-27] and in each case V numbers selected are below the first higher order mode cut-off value, the relevant cut-off values being presented in references[19, 25-27]. Moreover, our investigations as presented in Tables 1 and 2, show that the refractive index profile is an important parameter in dispersion managed fibers as regards efficiency of coupling. Accordingly, it is seen that out of all the fibers chosen for our study, Step W fiber, which is also important for the purpose of multiplexing, gives maximum coupling efficiency for both the wavelengths. Further, dispersion- flattened fiber having W type parabolic refractive index profile is not so suitable for excitation at 1.5 µm while it is nicely efficient at 1.3 µm producing nearly 88% coupling efficiency. Moreover, it is seen that at both the wavelengths, the fibers are more tolerant with respect to transverse mismatches in comparison to angular mismatches. Thus the present study will benefit the system designers who are concerned with such coupling devices. Further, for ready reference of coupling efficiency and tolerance for the fibers taken with respect to the said kind mismatches, we present some relevant data in Tables 1 and 2.

Table 1. Results for maximum coupling efficiency (η) in case of hyperbolic microlens on the tip of fiber (Excitation wavelength λ=1.3 µm, laser diode spot size in µm: w1x=1.081: w1y=1.161)

Fiber Type	Fiber Specification	w_f in µm	f in µm	η in %	Due to transverse mismatch along X direction			Due to transverse mismatch along Y direction			Due to angular mismatch		
					d_1 µm	η_t	Loss dB	d_2 µm	η_t	Loss in dB	θ in degree	η_θ	Loss in dB
Trapezoidal V=2.5	S=0.25	4.15735	11.6	99.40	0	0.994	0.026136	0	0.994	0.02613	0	0.994	0.026136
					1	0.94338	0.253135	1	0.939812	0.26959	1	0.916729	0.377588
					2	0.806467	0.934132	2	0.794337	0.99995	2	0.719127	1.431945
	S=0.50	3.59452	11.1	96.01	0	0.9601	0.176835	0	0.9601	0.17683	0	0.9601	0.176835
					1	0.902547	0.445303	1	0.898403	0.46528	1	0.898536	0.464644
					2	0.749772	1.250705	2	0.736098	1.33064	2	0.736534	1.328072
	S=0.75	3.7489	11.3	97.27	0	0.9727	0.120211	0	0.9727	0.12021	0	0.9727	0.120211
					1	0.917342	0.374685	1	0.913317	0.39378	1	0.906398	0.426811
					2	0.769466	1.138108	2	0.756048	1.21450	2	0.733397	1.34661
Graded W V=3	q=2 ρ=1.4975 C=1.5	2.92876	9.8	87.79	0	0.8779	0.56555	0	0.8779	0.56555	0	0.8779	0.56555
					1	0.806878	0.931922	1	0.802621	0.95489	1	0.837806	0.768563
					2	0.626464	2.031039	2	0.613347	2.12293	2	0.728182	1.377604
	q=2 ρ=1.50 C=1.5	2.928395	9.7	87.79	0	0.8779	0.56555	0	0.8779	0.56555	0	0.8779	0.56555
					1	0.805876	0.937315	1	0.801712	0.95981	1	0.838137	0.766848
					2	0.62336	2.052613	2	0.610573	2.14262	2	0.729333	1.370742
Step W V=2	ρ=1.3333 C=2.0	4.81143	11.9	99.41	0	0.9941	0.025699	0	0.9941	0.02569	0	0.9941	0.025699
					1	0.950799	0.219114	1	0.948015	0.23184	1	0.903304	0.44166
					2	0.831887	0.799358	2	0.822188	0.85028	2	0.677713	1.689542
	ρ=1.25 C=2.0	4.75558	11.9	99.55	0	0.9955	0.019587	0	0.9941	0.02569	0	0.9955	0.019587
					1	0.951627	0.215331	1	0.947438	0.23449	1	0.905557	0.430841
					2	0.831273	0.802561	2	0.820187	0.86087	2	0.681616	1.664602

Table 2. Results for maximum coupling efficiency (η) in case of hyperbolic microlens on the tip of fiber (Excitation wavelength λ=1.5 µm, laser diode spot size in µm: w1x=0.843 : w1y=0.857)

Fiber Type	Fiber Specification	w_f in µm	f in µm	η in %	Due to transverse mismatch along X direction			Due to transverse mismatch along Y direction			Due to angular mismatch		
					d_1 µm	η_t	Loss dB	d_2 µm	η_t	Loss in dB	θ in	η_θ	Loss in dB
Trapezoidal V=2.5	S=0.25	4.1573	11.4	79.93	0	0.7993	0.972902	0	0.7993	0.97290	0	0.7993	0.972902
					1	0.772497	1.121033	1	0.771967	1.12401	1	0.7391917	1.312429
					2	0.69736	1.565427	2	0.695449	1.57734	2	0.5846542	2.331009
	S=0.50	3.5945	10.7	70.57	0	0.7057	1.513799	0	0.7057	1.51379	0	0.7057	1.51379
					1	0.676274	1.698771	1	0.675775	1.70197	1	0.6645574	1.77467
					2	0.595156	2.253688	2	0.593401	2.26652	2	0.5549702	2.5573
	S=0.75	3.7489	10.9	72.80	0	0.728	1.378686	0	0.728	1.37868	0	0.728	1.37868
					1	0.699503	1.552106	1	0.698997	1.55524	1	0.6822883	1.66032
					2	0.620531	2.072364	2	0.61874	2.08491	2	0.5616653	2.50522
Graded W V=3	q=2 ϱ=1.4975 C=1.5	2.9287	9.1	58.96	0	0.5896	2.294425	0	0.5896	2.29442	0	0.5896	2.29442
					1	0.550955	2.588835	1	0.550552	2.59201	1	0.567037	2.463887
					2	0.449566	3.472064	2	0.448252	3.48477	2	0.504397	2.972272
	q=2 ϱ=1.50 C=1.5	2.9283	9.1	58.95	0	0.5895	2.295162	0	0.5895	2.29516	0	0.5895	2.29516
					1	0.550861	2.589578	1	0.550458	2.59275	1	0.566944	2.464595
					2	0.449487	3.472827	2	0.448173	3.48554	2	0.504325	2.972893
Step W V=2	ϱ=1.3333 C=2.0	4.8114	11.8	88.83	0	0.8883	0.514403	0	0.8883	0.51440	0	0.8883	0.51440
					1	0.862738	0.64121	1	0.862223	0.64380	1	0.805136	0.94131
					2	0.790382	1.021631	2	0.788495	1.03200	2	0.599511	2.222031
	ϱ=1.25 C=2.0	4.7555	11.7	88.16	0	0.8816	0.547284	0	0.8816	0.54728	0	0.8816	0.54728
					1	0.855655	0.677013	1	0.855144	0.67960	1	0.800856	0.964455
					2	0.782312	1.066198	2	0.780444	1.07658	2	0.600348	2.215969

4 CONCLUSION

Employing the prescribed ABCD matrix for refraction of paraxial rays by hyperbolic lens, we predict the coupling optics involving excitation of single-mode circular core dispersion-shifted and dispersion-flattened fiber via hyperbolic microlens on the tip of the fiber, taking into consideration possible transverse and angular mismatches. Analytical expressions for the efficiencies in presence of the said mismatches are formulated. The investigation is made for some typical commonly used dispersion-shifted as well as dispersion-flattened fibers for two practical wavelengths namely 1.3 µm and 1.5 µm. It has been found that trapezoidal fibers as well as step W and graded W fibers are more tolerant with respect to transverse mismatches in case of both the wavelengths 1.3 µm and 1.5 µm. Further, it is also found that wavelength 1.3µm is more efficient in terms of optical coupling. The technique developed will immensely benefit the designers and packagers in this particular field of optimum launch optics.

APPENDIX

The relation between output parameter q_2 and input parameter q_1 is given by the following expression

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}, \tag{A1}$$

where,

$$\frac{1}{q_{1,2}} = \frac{1}{R_{1,2}} - \frac{j\lambda_0}{\pi w_{1,2}^2 n_{1,2}} \tag{A2}$$

here, R, n, w and λ_0 represent the radius of curvature of wavefront, refractive index, spot size and the wavelength in free space respectively.

The relevant matrix of refraction by the hyperbolic microlens on the fiber tip has been formulated as [10, 11]

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1-n}{n\dfrac{b^2}{a}} & \dfrac{1}{n} \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \tag{A3}$$

Thus we have

$$A=1+\frac{d\left(1-n\right)}{n\,\frac{b^2}{a}},$$

$$B=f+\frac{fd\left(1-n\right)}{n\,\frac{b^2}{a}}+\frac{d}{n},$$

$$C=\frac{\left(1-n\right)}{n\,\frac{b^2}{a}}, \qquad \text{(A4)}$$

$$D=\frac{1}{n}+\frac{\left(1-n\right)f}{n\,\frac{b^2}{a}}$$

Here, a and b represent the lengths of semi- axes of the hyperbolic lens while *f* is the focal length of this microlens. Further, the focal length of microlens (*f*) also happens to be the distance of separation between the laser diode and microlens for optimum launch optics. Here, d represents the axial depth of the microlens and the refractive index of the material of the lens with respect to outside medium is denoted by *n* where $n=\frac{n_2}{n_1}$.

The lens transformed spot sizes $w_{2x,2y}$ and radii of curvature $R_{2x,2y}$ are evaluated by using Eqs. (A1), (A2) and (A4) and those are given by

$$w_{2x,2y}^2=\frac{A_1^2w_{1x,1y}^2+(\lambda_1^2B^2)/(\pi^2w_{1x,1y}^2)}{n\left(A_1D-BC_1\right)} \qquad \text{(A5)}$$

$$\frac{1}{R_{2x,2y}}=\frac{A_1C_1w_{1x,1y}^2+\left(\lambda_1^2BD\right)/(\pi^2w_{1x,1y}^2)}{A_1^2w_{1x,1y}^2+\left(\lambda_1^2B^2\right)/(\pi^2w_{1x,1y}^2)} \qquad \text{(A6)}$$

where, $\lambda_1=\lambda_0/n_1,\ \ A_1=A+B/R_1 \ and\ \ C_1=C+D/R_1$.

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