

Measurement of the Bit Error Rate (BER) and Eye Diagram of Interconnections through FDTD Algorithm

Houda Sanhaji, Nabih El Ouazzani

Abstract— The main objective of this paper is to assess the quality of signals transmitted at the output of interconnections in high speed circuits, subjected to Additive White Gaussian Noise (AWGN). We propose a numerical approach based on the technique of Finite Differences centered in the Time Domain (FDTD), to translate the behavior of voltages and currents along the line. The method deals with the variation of several parameters such as transmission rate, and losses. The impact of each of these parameters on the received signal is assessed. We use measurement tools like the bit error rate (BER) and the eye diagram obtained by means of both the FDTD based code and the Advanced Design System (ADS) software platform. This study allows predicting the effect of noise at early stages of the design.

Index Terms— AWGN, BER, Eye Diagram , FDTD, High speed circuits, Interconnections, Signal integrity, Transmission lines.

1 INTRODUCTION

In modern digital communication systems, preserving signal integrity is not only the most important aspect to account for, but also a growing concern for designers. Indeed, in these conditions, digital information can inevitably deteriorate making it difficult to have reliable transmission.

Knowing such circumstances, several phenomena affect considerably the data transmission within highly sophisticated circuits. Crosstalk, electromagnetic interferences, noise and skin effect are amongst the most influential parameters with huge impact on received data.

Though major improvements have been made in relation to circuit designing, and layout rules, interconnections are still investigated with emphasis on physical, geometrical and electrical proprieties and effects on the transmission of information.

In general, interconnections are considered as planar Multi-conductor Transmission Lines (MTL). They stand for transmission channels. MTL are basically governed by the well-known line equations expressed with respect to voltage and current matrices as well as per-unit-length (p.u.l) parameters such as resistance, conductance, capacitance and inductance matrices.

The assumption for all transmission line formulation and analysis is that the filed structure surrounding the conductors obeys a Transverse Electric-Magnetic (TEM) configuration.

In order to accurately describe the transmission problem, we have to include the noise effect into the proposed method. We focus essentially on internal noise that can be generated by either the line or any other connected elements especially non-linear ones.

Noise is usually the fundamental limitation for communication over physical channels. As a first step, we assume that the line is affected by Additive White Gaussian Noise (AWGN), that is this communication line is known as additive noise channel. As a result, both the line effects such as attenuation, propagation delay and crosstalk, and the Gaussian process of noise are taken into account and included into the same simu-

lation code.

Although, at present, there are various methods to solve MTL equations and produce voltages and currents at terminations, time domain results and measurements can be interpreted easily and are directly relevant to the function of digital devices. So, for a better understanding of such functions time-domain methods are basically the most appropriate and yield valuable information to predict the responses and overcome crucial problems.

The finite Difference Time Domain (FDTD) technique is of great interest to us since it is proven to be the simplest, both conceptually and in terms of solving complicated problems [1], [2]. FDTD method employs finite differences as approximations to both the spatial and temporal derivatives, it was first proposed by Yee [3] and later developed by Umashankar and Taflove [4], and the technique can produce very detailed solutions.

The proposed code consists of combining a numerical approach based on solving MTL equations by means of FDTD algorithm with the methods of computation of eye diagram and Bit Error Rate (BER), which are useful tools in interpreting received data especially when errors occur in the detection process.

The eye pattern allows the quality of the signal at the receiver to be characterized. The larger width of the eye, the better the data quality at the end of the line. The eye diagram width provides also information about the specific sampling time of the data at the receiver avoiding intersymbol interference (ISI) problems. As already mentioned, the other key parameter that measures high speed data streams and analyzes the integrity of received or transmitted data is the BER. It obviously enables us to compare the output of the device under study against the known data pattern. As a result, possible bit errors can be counted.

The electrical waveforms that represent binary digits "0" and "1" are 0 Volts and a positive pulse respectively. That is the Non-Return-to Zero (NRZ) code with logic "1" and logic

“0” having the same bit duration. The proposed method provides us with a starting point to develop the approaches necessary to design and analyze receivers.

This paper describes in **section 2** the proposed approach, highlighting the principals and the basic theory of the finite-difference time-domain in 1D. In order to implement the technique and deal with the MTL equations, we have chosen MATLAB as a coding language. In **section 3**, the eye diagram resulting of the FDTD approach for the coupled structure of lossless transmission line is studied and compared with the measured ADS data to validate the accuracy of the model. We present in **Section 4** the particular features of the noise source used and its integration into the system, to study subsequently its impact on transmitted signal. In **section 5** we added a White Gaussian Noise to a single transmission line and play on different parameters to include the transmission rates and metal losses. Then we observe their influence on the signal integrity via the Bit Error Rate (BER) and the eye diagram. **Section 6** provides a conclusion to the overall work.

2 FDTD-BASED APPROACH

As well known, the FDTD algorithm is simple, robust, and easy to understand. The technique deals with any physical system that is governed by a time-dependent partial differential equation in the time domain. In this work a planar uniform transmission line is studied through the use of the FDTD technique.

So the FDTD algorithm is applied to the MTL equations written as:

$$\left\{ \begin{array}{l} \frac{\partial V(z,t)}{\partial z} = -L \frac{\partial I(z,t)}{\partial t} - RI(z,t) \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} \frac{\partial I(z,t)}{\partial z} = -C \frac{\partial V(z,t)}{\partial t} - GV(z,t) \end{array} \right. \quad (2)$$

Where $V(z, t)$ and $I(z, t)$ represent the instantaneous voltage and current at position z and time t . The $R, L, G,$ and C terms represent the resistance, inductance, conductance, and capacitance matrices per unit length, respectively.

To evaluate the voltages and currents along the transmission line, the latter is at first divided into NDZ cells of equal length Δz , then we define a time step Δt so that the points of voltage and current solutions are separated in time and space respectively by $\Delta t/2$ and $\Delta z/2$ as shown in Fig1:

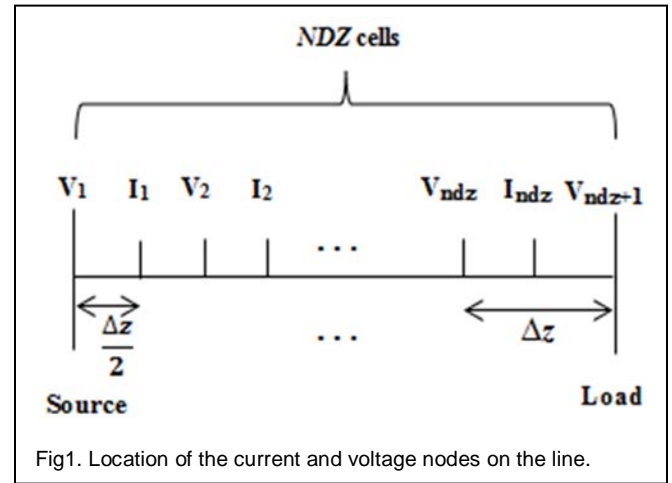


Fig1. Location of the current and voltage nodes on the line.

To insure stability, the position and time discretization must satisfy the CFL (Courant-Friedrichs-Lewy) condition which is expressed in one dimension [5], [6]:

$$\Delta t \leq \frac{\Delta z}{v_p} \quad (3)$$

Where v_p is the velocity of propagation of the waves. In the case of multiconductor lines, stability condition is satisfied by choosing the maximum of the mode velocities. Therefore, the condition will be satisfied for all the other smaller mode velocities.

Once, the discretization of space and time has been achieved, we develop (1) and (2) according to the FDTD algorithm [7]. So we obtain:

$$\frac{V_{k+1}^{n+1} - V_k^{n+1}}{\Delta z} = -L \frac{I_{k+\frac{1}{2}}^{n+\frac{3}{2}} - I_{k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta t} - R \frac{I_{k+\frac{1}{2}}^{n+\frac{3}{2}} + I_{k+\frac{1}{2}}^{n+\frac{1}{2}}}{2} \quad (4)$$

$$\frac{I_{k+\frac{1}{2}}^{n+\frac{1}{2}} - I_{k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} = -C \frac{V_k^{n+1} - V_k^n}{\Delta t} - G \frac{V_k^{n+1} + V_k^n}{2} \quad (5)$$

Expanding the equations (4) and (5), the general expression for the voltage and current at any point of the line can be written as:

$$V_k^{n+1} = \left(\frac{C}{\Delta t} + \frac{G}{2} \right)^{-1} \left[V_k^n \left(\frac{C}{\Delta t} - \frac{G}{2} \right) - \frac{1}{\Delta z} \left(I_{k+\frac{1}{2}}^{n+\frac{1}{2}} - I_{k-\frac{1}{2}}^{n+\frac{1}{2}} \right) \right] \quad (6)$$

• Houda Sanhaji is currently aPhD student in microwave electronic at Faculty of science and Technology of Fez in Morocco. E-mail: sanhaji.houda@gmail.com

• Nabih El Ouazzani is currently a research professor at the Faculty of science and Technology of Fez in Morocco, Its focus is the electromagnetic compatibility applied to Transmission Lines. E-mail: elouazzani.nabih@gmail.com

$$\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{3}{2}} = \left(\frac{\mathbf{L}}{\Delta t} + \frac{\mathbf{R}}{2} \right)^{-1} \left[\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{\mathbf{L}}{\Delta t} - \frac{\mathbf{R}}{2} \right) - \frac{1}{\Delta z} (\mathbf{V}_{k+1}^{n+1} - \mathbf{V}_k^{n+1}) \right] \quad (7)$$

We denote:

$$\mathbf{V}_k^n = \mathbf{V}[(k-1)\Delta z, n\Delta t] \quad (8a)$$

$$\mathbf{I}_k^n = \mathbf{V}[(k-1/2)\Delta z, n\Delta t] \quad (8b)$$

The general expression of the voltage and current becomes:

$$\mathbf{V}_k^{n+1} = \left(\frac{\mathbf{C}}{\Delta t} + \frac{\mathbf{G}}{2} \right)^{-1} \left[\mathbf{V}_k^n \left(\frac{\mathbf{C}}{\Delta t} - \frac{\mathbf{G}}{2} \right) - \frac{1}{\Delta z} \left(\mathbf{I}_k^{n+\frac{1}{2}} - \mathbf{I}_{k-1}^{n+\frac{1}{2}} \right) \right] \quad (9)$$

$$\mathbf{I}_k^{n+\frac{3}{2}} = \left(\frac{\mathbf{L}}{\Delta t} + \frac{\mathbf{R}}{2} \right)^{-1} \left[\mathbf{I}_k^{n+\frac{1}{2}} \left(\frac{\mathbf{L}}{\Delta t} - \frac{\mathbf{R}}{2} \right) - \frac{1}{\Delta z} (\mathbf{V}_{k+1}^{n+1} - \mathbf{V}_k^{n+1}) \right] \quad (10)$$

To find the expression of the voltage at the source, we base on the expression of the general tension in any point of the line, and we pose the following conditions:

$$k = 1; \quad \mathbf{V}_1^n = \mathbf{V}_s^n - \mathbf{R}_s \mathbf{I}_s^n \rightarrow \mathbf{I}_s^n = \frac{\mathbf{V}_s^n - \mathbf{V}_1^n}{\mathbf{R}_s}$$

With \mathbf{R}_s is the source resistance.

The voltage at the input is expressed as follows:

$$\mathbf{V}_1^{n+1} = \left(\frac{\mathbf{C}}{\Delta t} + \frac{\mathbf{G}}{2} \right)^{-1} \left[\mathbf{V}_1^n \left(\frac{\mathbf{C}}{\Delta t} - \frac{\mathbf{G}}{2} \right) - \frac{2}{\Delta z} \left(\mathbf{I}_1^{n+\frac{1}{2}} - \mathbf{I}_s^{n+\frac{1}{2}} \right) \right] \quad (11)$$

\mathbf{I}_1 and \mathbf{I}_s are separated by $\frac{\Delta z}{2}$

Assuming that:

$$\mathbf{I}_s^{n+\frac{1}{2}} = \frac{\mathbf{I}_s^{n+1} + \mathbf{I}_s^n}{2}$$

The final expression of the voltage at the input is then written in the form:

$$\mathbf{V}_1^{n+1} = \left(\frac{\mathbf{C}}{\Delta t} + \frac{\mathbf{G}}{2} - \frac{1}{\Delta z \mathbf{R}_s} \right)^{-1} \left[\mathbf{V}_1^n \left(\frac{\mathbf{C}}{\Delta t} - \frac{\mathbf{G}}{2} - \frac{1}{\Delta z \mathbf{R}_s} \right) - \frac{2}{\Delta z} \mathbf{I}_1^{n+\frac{1}{2}} + \frac{1}{\Delta z \mathbf{R}_s} (\mathbf{V}_s^n + \mathbf{V}_s^{n+1}) \right] \quad (12)$$

Likewise, the voltage at the load is expressed based on the general expression for the voltage at the line, and laying in a first time the following conditions:

$$k = ndz + 1; \quad \mathbf{V}_{ndz+1}^n = \mathbf{V}_L^n = \mathbf{R}_L \mathbf{I}_L^n \rightarrow \mathbf{I}_L^n = \frac{\mathbf{V}_{ndz+1}^n}{\mathbf{R}_L}$$

With \mathbf{R}_L is the load resistance, and \mathbf{I}_L and \mathbf{I}_{ndz} are separated

by $\frac{\Delta z}{2}$.

The final expression of the voltage at the load can then be written as:

$$\mathbf{V}_{ndz+1}^{n+1} = \left(\frac{\mathbf{C}}{\Delta t} + \frac{\mathbf{G}}{2} + \frac{1}{\Delta z \mathbf{R}_L} \right)^{-1} \left[\mathbf{V}_{ndz+1}^n \left(\frac{\mathbf{C}}{\Delta t} - \frac{\mathbf{G}}{2} - \frac{1}{\Delta z \mathbf{R}_L} \right) + \frac{2}{\Delta z} \mathbf{I}_{ndz+1}^{n+\frac{1}{2}} \right] \quad (13)$$

So, equations (9), (10), (12) and (13) make up the implemented FDTD algorithm that is aimed to provide \mathbf{V}_{ndz+1}^n in particular. Further operations, such as integrated calculations or the use of built-in functions of the programming code, will be carried out with respect to \mathbf{V}_{ndz+1}^n to determine the noise effect and other data transmission parameters.

Fig2 shows the flow chart of the whole procedure. Some computed results are compared with those obtained from Advanced Design System (ADS) in order to validate the proposed approach.

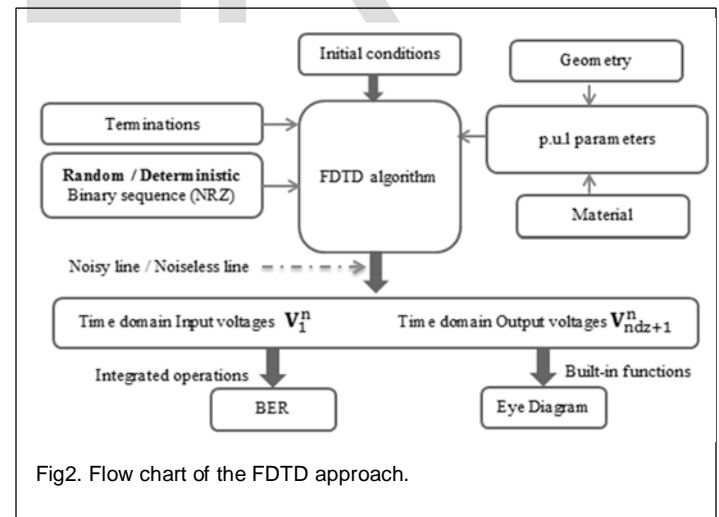


Fig2. Flow chart of the FDTD approach.

3 EYE DIAGRAM OF A LOSSLESS PLANAR TRANSMISSION LINE

As a first step in the implemented process, and in order to validate the accuracy and efficiency of the proposed method, we analyze a 2-conductors microstrip line representing coupled interconnections. Crosstalk effect is illustrated thanks to the eye pattern obtained using both the FDTD approach and ADS software.

Fig3 displays the structure and parameters of the transmission line case. Conductors of rectangular cross section of width $w = 20\mu\text{m}$ are separated by $s = 20\mu\text{m}$, the total length is $\ell = 10\text{cm}$ and is terminated at the near and far ends in 50Ω resistors. The transmission line is on Printed Circuit Board (PCB) with Silicon substrate ($\epsilon_r = 12$) of thickness $h = 100\mu\text{m}$. The source is a pulse rising to a level of 1V with a rise and fall times of 0.25ns , and a period of 1ns . The per-unit-length inductance and capacitance matrices are computed using the numerical method of [5]:

$$L = \begin{bmatrix} 0.73232 & 0.333004 \\ 0.333004 & 0.73232 \end{bmatrix} \mu\text{H} / \text{m}$$

$$C = \begin{bmatrix} 110.856 & -51.1177 \\ -51.1177 & 110.856 \end{bmatrix} \text{pF} / \text{m}$$

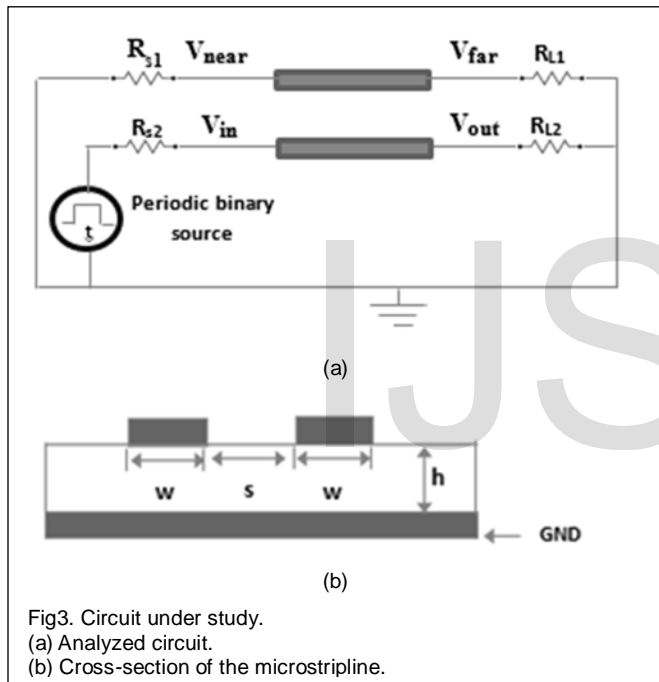


Fig3. Circuit under study.
 (a) Analyzed circuit.
 (b) Cross-section of the microstripline.

The entire analysis consists of performing mainly two major operations. First, we run the time domain simulation through the proposed FDTD approach leading to a set of values of far end crosstalk voltage $V_{far}(t)$. Secondly, a built-in eye diagram function of MATLAB is executed with $V_{far}(t)$ as an input parameter. The FDTD algorithm parameters of simulation are:

$$\Delta z = 1 \text{ mm}, \quad \Delta t = 5 \text{ ps}, \quad \text{NDZ} = 100,$$

$$U_{pmax} = 1.2 \times 10^8 \text{ m/s}.$$

$$V_{far}(t) = V_{(ndz+1)}^n(I)$$

Thus, we obtain the eye diagram which is compared to that produced by ADS as shown in Fig4:

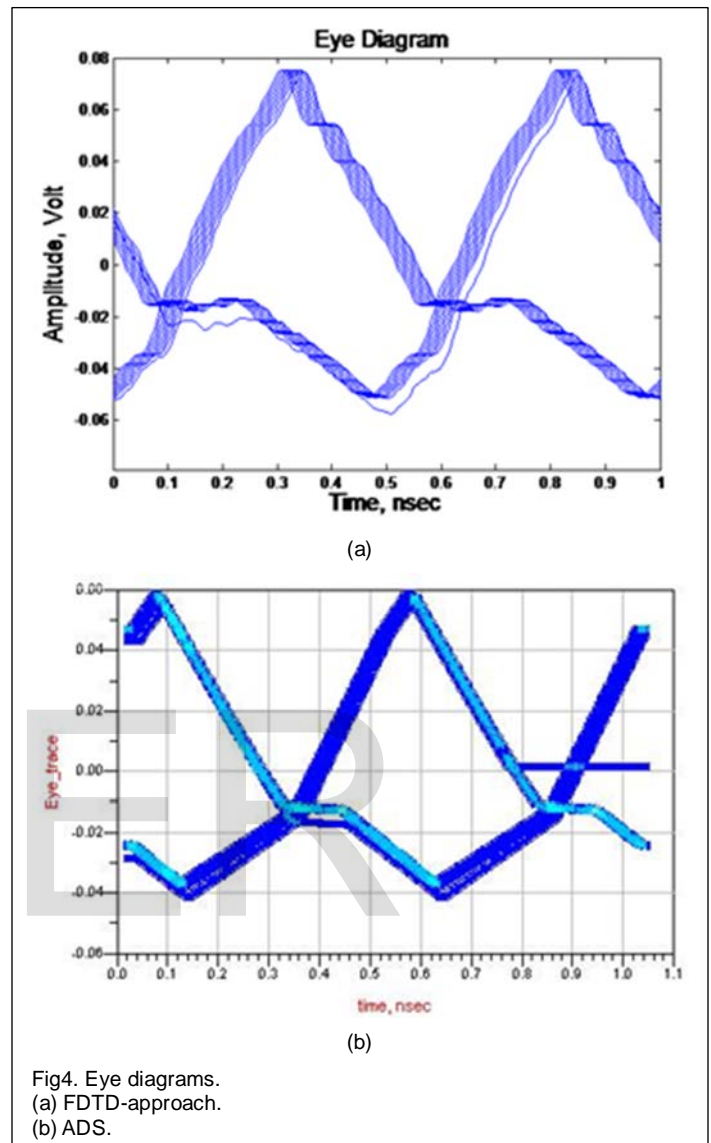


Fig4. Eye diagrams.
 (a) FDTD-approach.
 (b) ADS.

As expected, we notice that the eye width is reduced indicating that the sampling time at the receiving end becomes a bigger problem. This happens whenever crosstalk affects data transmission leading, of course, to poor detection.

The comparison between both eye patterns shows a good level of agreement. As a result, the accurate simulation validates and illustrates the effectiveness of the proposed FDTD approach.

4 NOISE CONTEXT

White noise is prevalent among disturbing parameters in all active and passive components of data transmission systems operating at high bit rate.

The impact of noise becomes more detrimental as amplifiers and line drivers multiply its energy.

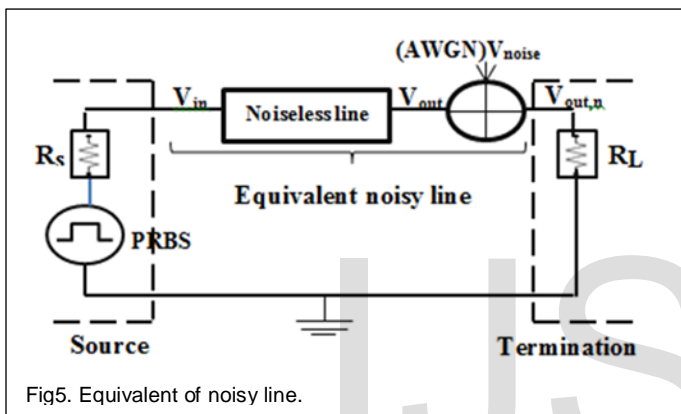
By its energy distribution, white noise in electronics systems is Gaussian, so it can be described by the following density function [8], [9]:

$$PDF_n(x) = \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left[-\frac{(x-\mu)^2}{2\sigma_n^2}\right] \quad (14)$$

Where x is the independent value, σ_n is the standard deviation and μ the mean of the distribution. In this paper, we set the noise to be zero-mean, white and Gaussian having $\sigma_n = 0.7$.

Due to its wide frequency spectrum, white noise is very difficult to suppress or even attenuate without impacting the actual signal.

In order to simulate a noisy line, we set a distributed signal by adding the noise vector to the output signal vector of the line according to the procedure shown in Fig5:



Where:

$$V_{noise} = \frac{n_0}{E_b / N_0} \quad (15)$$

Where n_0 is randomly determined according to $PDF_n(x)$, and E_b/N_0 is the measure of signal to noise ratio for a digital communication system, it is classically defined as the ratio of Energy per Bit E_b to the Spectral Noise Density N_0 . This ratio is measured at the input to the receiver, and the Energy per is given by [10]:

$$E_b = \frac{1}{N * f_{bit}} \sum_{n=1}^N X^2(n) \quad (16)$$

Where N is the total number of samples of the signal, and f_{bit} is the bit rate in bits-per-second.

The load in Fig5 stands for the input impedance of the data receiver.

Once the noise parameters are established, the following vectors operation allows us to have the AWGN line model:

$$V_{out,n}(t) = V_{out}(t) + V_{noise}(t) \quad (17)$$

Since signals are usually random rather than deterministic, a random NRZ signal is used to simulate the interconnection

case. So, the excitation source is a Pseudo Random Binary Sequence (PRBS) with R_s as the internal resistance.

5 EYE PATTERN AND BER OF A NOISY LINE

5.1 Circuit Under Study

We investigate a single-end microstrip transmission line shown in Fig6. A copper conductor, of rectangular cross section of width $w = 0.01\text{cm}$ and thickness $t_m = 5\mu\text{m}$ and conductivity $\sigma = 5.8 \times 10^7 \text{ S/m}$, placed on one side of a silicon substrate of thickness $h = 10\mu\text{m}$. The length of the line is $\ell = 10\text{cm}$ and the characteristic impedance is 150Ω . We apply the method developed in [5] to extract the p.u.l parameters which are $L = 6.54739 \times 10^{-7} \text{ H/m}$ and $C = 2.90910 \times 10^{-11} \text{ F/m}$.

A voltage source with $R_s = 150\Omega$ produces the input signal, which is a PRBS, at the driver side. At the receiver side a 150Ω resistor is connected as the termination. That is a matching case to be analyzed. These dimensions are typically of current high speed packaging technologies.

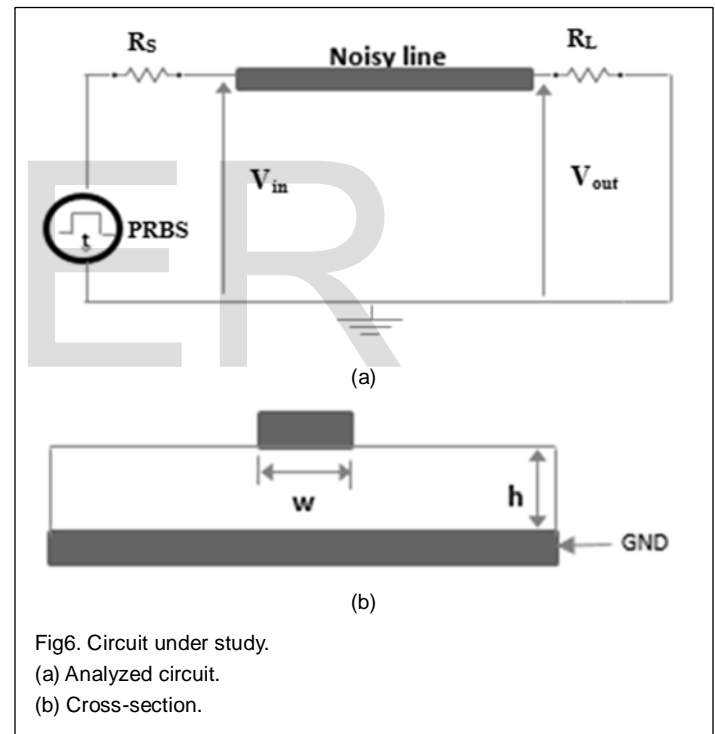


Fig6. Circuit under study.

(a) Analyzed circuit.

(b) Cross-section.

5.2 Calculation of the BER

As the name implies, the bit error rate (BER) is defined as the number of erroneous bit divided by the total number of transferred bits during a studied time interval. The BER is calculated by comparing the transmitted sequence of bits with the received bits, by counting the number of errors. The definition of bit error rate can be translated into a simple formula [10]:

$$BER = \frac{N_e}{N_{bits}} \quad (18)$$

Where N_e is the number of erroneous bits, and N_{bits} is the total number of bits sent.

In order to establish the BER we have followed the procedure shown in the following steps:

Step 1:

First, we create a vector containing the values of the transmitted signal collected at a sampling period, and transforme into logic state '0' or '1' as shown in Fig7.

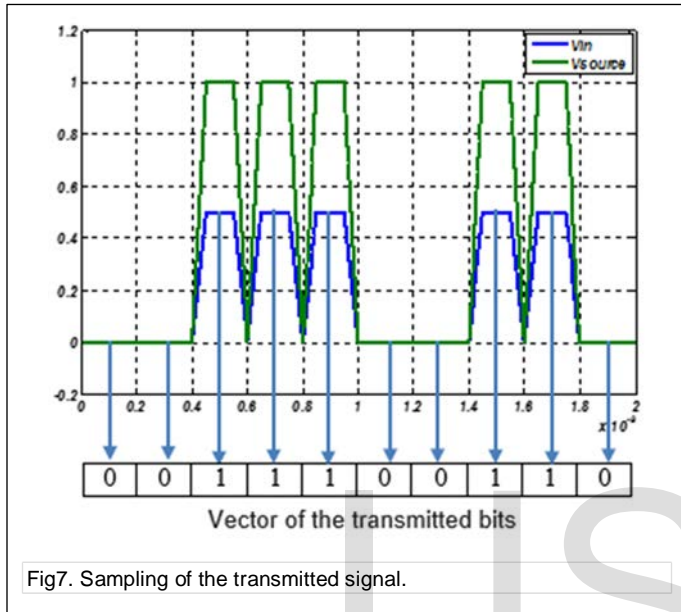


Fig7. Sampling of the transmitted signal.

This vector represents the logic states of symbols transmitted, and constitutes the referential basis for calculating the BER.

Step 2:

We recover the signal affected by the added noise and we define a threshold depending on the level of requirement of the receiver. This threshold converts the signal assigned to the logic level '0' or '1' Fig8. The resulting vector is compared to the transmitted signal vector, and the difference between the logic states of two vectors (logic state vector transmitted signal & logic state vector affected received signal) leads to the number of erroneous bits.

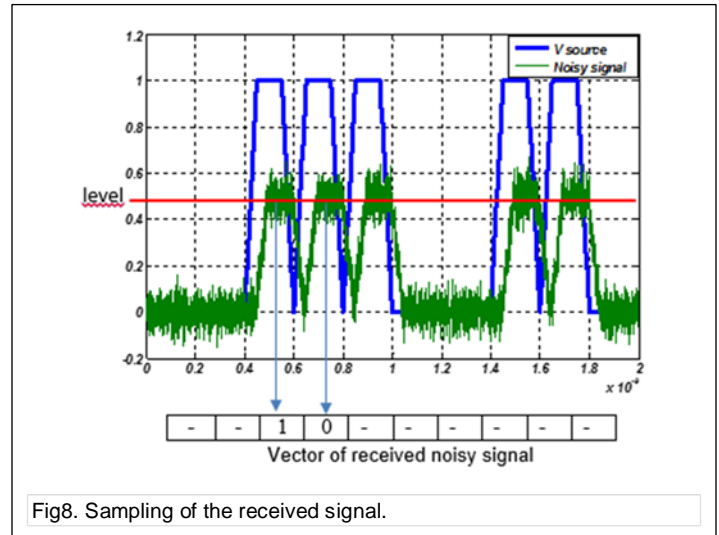


Fig8. Sampling of the received signal.

The FDTD algorithm parameters of simulation are then:

$$\Delta z = 1mm, \quad \Delta t = 4.27 ps, \quad NDZ = 100$$

$$v_p = 2.2913 \times 10^8 m / s.$$

5.3 The line BER and Eye diagram

At this point, it is of interest to analyze the BER with respect to the transmission rate. To achieve this we propose an example that deals with two different rates which are 1Gbits/s and 2Gbits/s.

We easily compute the BER by means of the previously developed procedure. Fig9 shows two curves related to the considered data rates. The BER is typically expressed as 10 to the negative power. The impact is obviously degrading the quality of data. So, we notice that the BER follows the speed of data transmission, it provides a higher number of errors when the transmission rate becomes more significant. Therefore the transmission line loses its performances at high transmission rates.

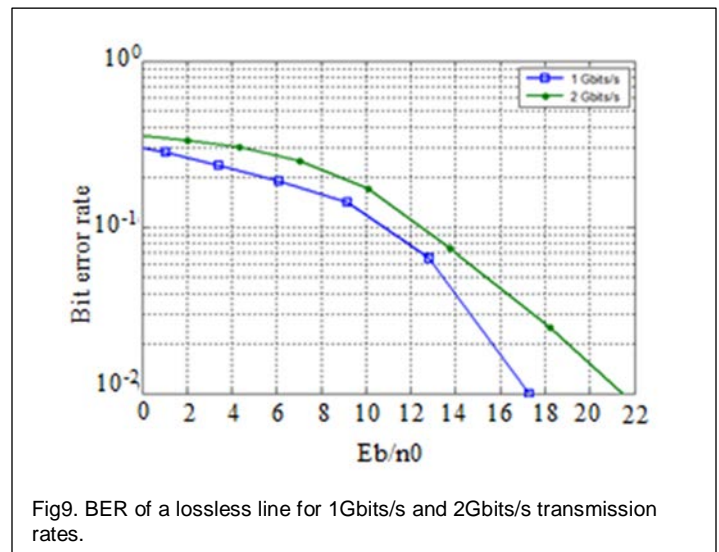


Fig9. BER of a lossless line for 1Gbits/s and 2Gbits/s transmission rates.

The degradation of the transmission is also confirmed thanks to the eye diagram Fig10. The impact of higher transmission rate causes the closure of the eye, showing that the transmission quality has deteriorated.

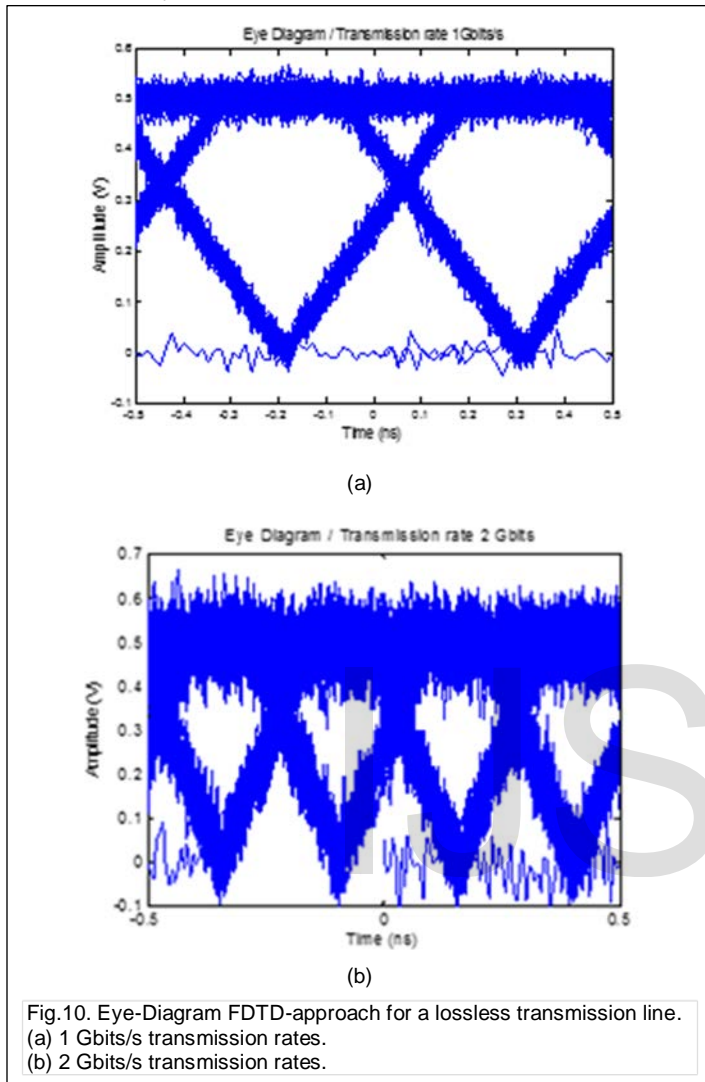


Fig.10. Eye-Diagram FDTD-approach for a lossless transmission line.
(a) 1 Gbits/s transmission rates.
(b) 2 Gbits/s transmission rates.

The accuracy of the proposed method can be improved by means of introducing conductor losses. The dc losses are calculated using the following formula:

$$R_{\text{static}} = \frac{l}{\sigma \omega t_m} \quad (19)$$

We compared the BER of lossy and lossless lines for the same signal transmission rate 2 Gbits/s.

The Fig11 shows the simulation of the BER for the two cases of the same transmission line. We note that the number of erroneous bit becomes more important in the case of lossy line; it confirms that the line with losses undermine the transmission quality and therefore presents a major problem for data at a high transmission rate.

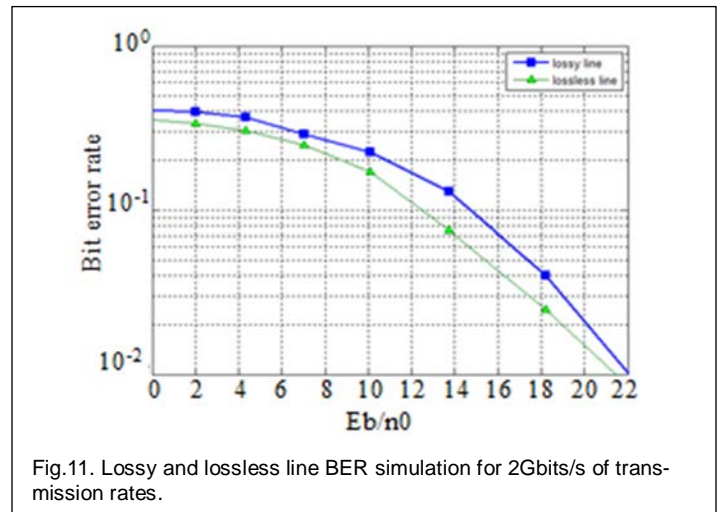


Fig.11. Lossy and lossless line BER simulation for 2Gbits/s of transmission rates.

6 DISCUSSIONS AND CONCLUSION

Through this paper, we have developed a technique to simulate the integrity of signals affected by a noise source (AWGN in our case) along planar transmission lines. The method used to predict the behavior of currents and voltages on the line is mainly based on the FDTD algorithm applied to the MTL equations.

The FDTD results have been confronted to those obtained from ADS validating the proposed approach. A random binary source with various transmission rates such as 1Gbits/s and 2Gbits/s, has enabled us, in the noisy case, highlight the impact of such rates on the data processing time and also the quality of the transmitted signal at the receiving end. We have also introduced the metal losses of the line into the algorithm. The simulation clearly has shown that the number of erroneous bits becomes substantial in the case of lossy line. The graphs of eye diagram and BER confirm that the data transmission is more efficient with the solution of an adapted and lossless line with average transmission rate.

REFERENCES

- [1] Ngendakumna. G, Elouazzani. N, "Laguerre-FDTD analysis of the sinus microstrip fed-path antenna for boardband matching," IEEE-Multimedia computing and systems, April 2014, pp.1279-1289
- [2] Dennis. Sullivan. M., "Electromagnetic Simulation Using the FDTD Method" IEEE Press series on RF And Microwave Technology, New York, 2000.
- [3] Yee. Kane. S, "Numerical Solution of Initial Boundary Value Problem Involving Maxwell's Equations in Isotropic Media" IEEE Trans. Antennas and Propagation, vol.14, no.124, 1966, pp. 302-307
- [4] Taflove. A, Umashankar. K. R, "The finite-difference time domain method for numerical modeling of electromagnetic wave interactions with arbitrary structures" PIER 2, Ch. 8, Elsevier Science Publishing Co., Inc., 1990, pp. 547-582.
- [5] Clayton. Paul. R, "Analysis of Multiconductor Transmission Lines" New York, Wiley Interscience, 1994.
- [6] Alan. Roden. J, Clayton. Paul. R, William. Smith. T, et al, "Finite-Difference, Time-Domain Analysis of Lossy Transmission Lines" IEEE Transactions on electromagnetic compatibility, Vol.38, NO.1, February 1996.

- [7] Elouazzani. N, Tahri. D, Kabbaj. H, "Transient analysis of nonuniform transmission lines with skin effect" *Annals of telecommunication-54*, n°5-6, 1999, pp 331-340.
- [8] Robert. G, B, "Introduction to Random Signal Analysis and applied Kalman Filtering". John Wiley and Sons, 1983.
- [9] Athanasios. P, "Probability, Random Variables, and Stochastic Processes, 3rd ed". WCB/McGraw-Hill, 1991, pp.285-344.
- [10] James. Gilley. E, "Bit-Error-Rate simulation using MATLAB" Transcript International, Inc., Aug-2003.

IJSER