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MARKOV CHAIN

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Abstract

Markov chain was named after Andrew Markov. It is a mathematical system, which moves from a particular form to the other. It has the property of merorylessness given that the subsequent form relies on the present form, but not the whole sequence involved. This is the Markov chain's characteristic. Markov chain is applicable in different real-world processes as statistical models and derived from random transitional process.

***Index terms*:** *continuous-time chain*, *discrete-time chain*, *Markov chain*, *Transition probabilities*.

Introduction

A random procedure or system having the attributes of Markov is a Markov chain. The chain in the Markov system is the sequence of a stochastic process in which the next stage is dependent on the current stage and not the whole sequence. Thus, it can be a useful tool in describing a system dependent on a linked chain of events [1]. Various literatures denote several types of Markov progressions as Markov chains. It is for a process with separate sets of times abbreviated DPMC (the distinct period Markov chains). A Markov system represents an endless-period Markov series. The Markov sequence may have varied extensions and generalizations [2]. This article will focus mostly on discrete state-space case and discrete time, as well as giving the various equations involved in the Markov chain.

Main body

Transition probabilities are the odds allied to different changes in a state. The process has a state space, a first state across the state space and a transition matrix. The transition matrix describes the probability of specific transitions. By assuming that all the states and variations are incorporated in the system's definition, there is a subsequent form and the progression hardly ends [3].

A system in which the state changes randomly between steps is a discrete –time process. The steps could be either natural numbers or integers. The Markov attribute asserts that every next step's probability distribution depends on the system's current state. This makes it difficult to make a prediction on the future of Markov's progression unless it is the statistical properties. An example of such a chain is the "drunkard walk". At each step on the number line, the position changes by either + 1 or – 1 with the same probability [4].

A Markov chain is given by the following equation:

$$Pr(X\_(n+1) = x|X\_1 = x\_1, X\_2 = x\_2, \dots, X\_n = x\_n) = Pr(X\_(n+1) = x|X_n = x_n$$

This is when the provisional likelihoods are clearly determined. That is, $\Pr(X_1 = x_1, \dots, X_n = x_n) > 0$. However, X_1, $X_2 \dots X_n$ should be a system of arbitrary variable. From the sequence, the likely values of X_i form S, which is the conditional phase of the Markov's system. In most cases, a system of fixed grids or charts is used to describe Markov chains. The edges of the graph, n, are marked by the probabilities of moving from a particular form at a given period, n, to a different form at period $n+1, \Pr(X_{n+1} = x \setminus X_n = x_n$. This can also be shown through transitional matrices. Both the matrix and graph are not dependent on n [5].

A Markov chain is time-homogenous or stationary when:

$$\Pr(x_{n+1} = x|x_n = y) = \Pr(x_{n-1} = x|x_{n-1} = y$$

The change in the probability should be autonomous of n. As a result, a Markov sequence having memory denoted by 'm' is a process that satisfies:

$$\Pr(x_n = x_n|x_{n-1} = x_{n-1}, x_{n-2} = x_{n-2}, \dots, x_1 = x_1) = \Pr(x_n = x_n|x_{n-1} = x_{n-1}, x_{n-2} = x_{n-2}, \dots, |x_{n-m} = x_{n-m})\, form > m$$

Future state is dependent on the past m state [6].

Markov chains have a number of properties. An irreducible chain is one that is able to shift to a particular form from any specific form. Every state has an even period. The states are always recurrent. A periodic-homogenous chain with a distinct periodic autonomous matrix is known as an immobile or static distribution [7].

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Conclusion

Markov chains are very significant in the society. It can be used in comprehensive or different kinds of fields such as sports, physics, music, medicine, game theory, and chemistry. A Markov chain involves an arbitrary process that move from a particular state to a very different form. The current state determines the next state. It cannot be used in making future prediction since it is not dependent on the entire sequence.

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