KALUZA-KLEIN COSMOLOGICAL MODELS WITH ANISOTROPIC DARK ENERGY AND SPECIAL FORM OF DECELERATION PARAMETER

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Abstract: The exact solutions of the Einstein field equations for dark energy in Kaluza-Klein metric under the assumption on the anisotropy of the fluid are obtained by applying special form of deceleration parameter in General Relativity. The geometrical and physical aspect of the model is also studied.

Keywords: Kaluza- Klein space-time, Anisotropic Fluid, Dark Energy, Isotropization, Special form of deceleration parameter.

Introduction:

Recent most remarkable observational discoveries have shown that our universe is currently accelerating [1] and co nfirmed later by cross checks from the cosmic microwave background radiation and large scale structure [2 , 3] strongly suggest that the Universe is spatially flat and dominated by an exotic component with large negative pressure, referred to as dark energy [4]. The first year result of the Wilkinson Microwave Anisotropy Probe (WMAP) shows that dark energy occupies about 73% of the energy of our Universe, and dark matter about 23%. The usual baryon matter which can be described by our known particle theory occupies only about 4% of the total energy of the Universe.

Today we can examine not only when the cosmic acceleration began and the current value of the deceleration parameter, but also how the acceleration (the deceleration parameter) varies with time. After the discovery of the late time acceleration of the universe, many authors have used CDP to obtain cosmological models in the context of dark energy (DE) in general relativity and some other modified theories of gravitation such as f(R) theory within the framework of spatially isotropic and anisotropic space-times. However, generalizing CDP assumption would allow us to construct more precise cosmological models.

Many authors [9], [10] proposed a linearly varying deceleration parameter (LVDP), which can be used in obtaining accelerating cosmological solutions. As a special case, LVDP also covers the special law of variation for Hubble parameter, which yields constant deceleration parameter (CDP) models of the universe, presented by Berman [11, 12] and references therein.

By choosing a particular form of the deceleration parameter q, which gives an early deceleration and late time acceleration for dust dominated model, [13] shows that this sign flip in q can be obtained by a simple trigonometric potential.

The quintessence model [14] with a minimally coupled scalar field by taking a special form of decelerating parameter q in such a way that which provides an early deceleration and late time acceleration for borotropic fluid and Chaplygin gas dominated models.

Motivated from the studies outlined above we choose a form of q as a function of the scale factor a so that it has the desired property of a signature flip.

In the present paper, Kaluza-Klein cosmological models with anisotropic dark energy and special form of deceleration parameter have been studied. To have a g eneral description of an anisotropic dark energy component, we consider a phenomenological parameterization of dark energy in terms of its Equation of State (ω) and skewness parameter (δ). The exact solutions of the Einstein field equations have been obtained by applying special form of deceleration parameter. Some features of the evolution of the metric and the dynamics of the anisotropic DE fluid have been examined

2. Metric and Field equations:

The Kaluza-Klein type metric is given by

$$ds^2 = dt^2 - a^2\left(dx^2 + dy^2 + dz^2\right) - b^2 d\psi^2 \quad , \tag{1}$$

where a and b are functions of cosmic time t only.

Here we are dealing only with an anisotropic fluid whose energy-momentum tensor is in the following form

$$T_v^u = diag\left[T_0^0, T_1^1, T_2^2, T_3^3, T_4^4\right] \quad .$$

We parametrize it as follows:

$$T_v^u = diag\left[\rho, -p_x, -p_y, -p_z, -p_\psi\right] = diag\left[1, -\omega_x, -\omega_y, -\omega_z, \omega_\psi\right]\rho \quad ,$$

where ρ is the energy density of the fluid; p_x, p_y, p_z and p_ψ are the pressures and $\omega_x, \omega_y, \omega_z$ and ω_ψ are the directional equation of state (EoS) parameters of the fluid.

Now, parametrizing the deviation from isotropy by setting $\omega_x = \omega_y = \omega_z = \omega$ and then introducing skewness parameter δ that is the deviation from ω on ψ-axis. Here ω and δ are not necessarily constants and can be functions of the cosmic time *t*.

The parametrized energy-momentum tensor is

$$T_{\upsilon}^{\mu} = diag[1,-\omega,-\omega,-\omega-(\omega+\delta)]\rho \quad . \tag{2}$$

The Einstein field equations, in natural limits $(8\pi G = 1$ and $c = 1)$ are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -T_{\mu\nu} \quad , \tag{3}$$

where $g_{\mu\nu}u^{\mu}u^{\nu} = 1; u^{\mu} = (1,0,0,0,0)$ is the velocity vector; $R_{\mu\nu}$ is the Ricci tensor; R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor.

In a co-moving coordinate system, Einstein's field equations (3), for the anisotropic Kaluza-Klein space-time (1), with equation (2) yield

$$3\frac{\dot{a}\dot{b}}{ab} + 3\frac{\dot{a}^2}{a^2} = \rho \tag{4}$$

$$2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} = -\omega\rho \tag{5}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -(\omega+\delta)\rho \quad , \tag{6}$$

where the overhead dot (˙) denote derivative with respect to the cosmic time t .

3. Isotropization and the solution:

There are three linearly independent equations (4)-(6) with five unknowns a, b, ρ, ω and δ. In order to solve the system completely we impose a s pecial form of deceleration parameter as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -1 + \frac{\alpha}{1+R^{\alpha}} \, . \tag{7}$$

where R is mean scale factor of the universe , α (>0) is constant. This low has been recently proposed by Singha and Debnath (2009) [14] for FRW metric. From figure (i) we have seen that q decreases from $+1$ to -1 for evolution of the universe. Recently, Adhav et al. [31] has extended this law for Bianchi type-I, III, V, VIo and

Kantowski-Sachs cosmological models with dynamical equation of state (EoS) parameter.

From (7) after integrating, we obtain the Hubble parameter as

$$H = \frac{\dot{R}}{R} = m\left(1 + R^{-\alpha}\right) \quad , \tag{8}$$

where m is an arbitrary constant of integration.

Here we assume the deceleration parameter as given in (3.1) (7), which can be integrated twice to give $H = \frac{\dot{R}}{R}$ as in equation (3.2) (8) and the average scale factor as

$$R = (e^{m\alpha t} - 1)^{\frac{1}{\alpha}} \quad . \tag{9}$$

The spatial volume is given by

$$V = R^4 = a^3 b . \tag{10}$$

i.e. $$V = a^3 b = \left(e^{m\alpha t} - 1\right)^{\frac{4}{\alpha}} \quad . \tag{11}$$

The directional Hubble parameters in the direction of *x*, *y*, z and ψ respectively for the Kaluza- Klein metric are

$$H_x = H_y = H_z = \frac{\dot{a}}{a}, \text{ and } H_\psi = \frac{\dot{b}}{b} \tag{12}$$

The mean Hubble parameter is given as

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) \quad . \tag{13}$$

Subtracting equation (5) from equation (6), we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\frac{\dot{V}}{V} = -\delta\rho \quad .$$

Which on integrating gives

$$\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right)=\frac{\lambda}{V}\quad e^{\int\frac{\delta\rho}{\left(\frac{\dot b}{b}-\frac{\dot a}{a}\right)}dt}\;, \tag{14}$$

where λ is positive constant of integration.

In order to solve the above equation (14) we use the condition

$$\delta=\frac{\left(\frac{\dot b}{b}-\frac{\dot a}{a}\right)m\alpha}{\rho}\quad . \tag{15}$$

Using equation (15) in the equation (14), we obtain

$$\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right)=\frac{\lambda}{V}e^{m\alpha t}. \tag{16}$$

Using equation (11) in equation (16) and then integrating we get the scale factors as

$$a(t)=\left(e^{m\alpha t}-1\right)^{\frac{1}{\alpha}}\exp\left\{\frac{\lambda}{4m(\alpha-4)}\left(e^{m\alpha t}-1\right)^{\frac{\alpha-4}{\alpha}}\right\}. \tag{17}$$

$$b(t)=\left(e^{m\alpha t}-1\right)^{\frac{1}{\alpha}}\exp\left\{-\frac{3\lambda}{4m(\alpha-4)}\left(e^{m\alpha t}-1\right)^{\frac{\alpha-4}{\alpha}}\right\}. \tag{18}$$

4. Physical behavior of the model:

Using equations (17) and (18) the directional Hubble parameters are found as

$$H_x=H_y=H_z=\frac{\dot a}{a}=\frac{\lambda}{4}e^{m\alpha t}\left(e^{m\alpha t}-1\right)^{\frac{-4}{\alpha}}+m\left(1-e^{-m\alpha t}\right)^{-1}. \tag{19}$$

And $$H_\psi=\frac{\dot b}{b}=-\frac{3\lambda}{4}e^{m\alpha t}\left(e^{m\alpha t}-1\right)^{\frac{-4}{\alpha}}+m\left(1-e^{-m\alpha t}\right)^{-1}. \tag{20}$$

The mean Hubble parameter H for Kaluza-Klein metric may given by

$$H=\frac{m}{\left(1-e^{-m\alpha t}\right)}. \tag{21}$$

The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta\equiv\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2\;,$$

where $H_i\left(i=1,2,3,4\right)$ represent the directional Hubble parameters in the directions of x , y, z and ψ respectively and is found as

$$\Delta=\frac{3\lambda^2}{16m^2}\left(e^{m\alpha t}-1\right)^{\frac{2(\alpha-4)}{\alpha}} . \tag{22}$$

The expansion scalar θ is given by

$$\theta=4H=\frac{4m}{\left(1-e^{-m\alpha t}\right)}. \tag{23}$$

The shear scalar σ^2 is given by

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{4}H_i^2-4H^2\right)=\frac{4}{2}\Delta H^2$$

$$=\frac{3\lambda^2}{8}e^{-2m(4-\alpha)t}\left(1-e^{-m\alpha t}\right)^{\frac{-8}{\alpha}} . \tag{24}$$

Using equations (19) and (20) in equation (4), we obtain the energy density for the model as

$$\rho=\left[6m^2(1-e^{-m\alpha t})^{-2}-\frac{3\lambda^2}{8}e^{-2m(4-\alpha)t}(1-e^{-m\alpha t})^{\frac{-8}{\alpha}}\right] . \tag{25}$$

Using equations (25) in equation (15), we obtain the deviation parameter as

$$\delta=-\left\{\frac{m\alpha\lambda\, e^{-m(4-\alpha)t}}{6m^2(1-e^{-m\alpha t})^{\frac{4}{\alpha}-2}-\frac{3\lambda^2}{8}e^{-2m(4-\alpha)t}(1-e^{-m\alpha t})^{\frac{-4}{\alpha}}}\right\} . \tag{26}$$

Using equations (17), (19), (25) and (26) in equation (6), we obtain the deviation-free parameter as

$$\omega=-\left\{\frac{\frac{3m^2\alpha}{(1-e^{-m\alpha t})}+\frac{3m^2(2-\alpha)}{(1-e^{-m\alpha t})^2}-\frac{m\alpha\lambda}{2}\frac{e^{-m(4-\alpha)t}}{(1-e^{-m\alpha t})^{\frac{4}{\alpha}}}+\frac{3\lambda^2}{8}\frac{e^{-2m(4-\alpha)t}}{(1-e^{-m\alpha t})^{\frac{8}{\alpha}}}}{6m^2(1-e^{-m\alpha t})^{-2}-\frac{3\lambda^2}{8}e^{-2m(4-\alpha)t}(1-e^{-m\alpha t})^{\frac{-8}{\alpha}}}\right\}. \tag{27}$$

5. Discussion and conclusion:

The spatial volume is finite at $t=0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. The directional Hubble parameters are infinite at $t=0$ and finite at $t=\infty$. It is observed that this space-time expands anisotropically since the shear scalar $\sigma^2 \to \frac{3\lambda^2}{8}$ (non-zero) as time $t \to 0$ and become isotropic as time increases.

The dynamics of energy density (ρ) for Kaluza-Klein space-time is as shown in figure (ii). The energy density of the DE component $\rho \to \infty$ as $t \to 0$ and as $t \to \infty$, the energy density $\rho \to 3m^2 > 0$. Here we observe that the model start with big bang having infinite density and as time increase (for finite time) the energy density ρ tends to finite value. Hence after some finite time the models approaches to steady state. In figure (iii) we plot anisotropy parameter of expansion Δ against cosmic time *t*. It is observed that in this model anisotropy increases as time increases and then decreases to zero after some time and remains zero after some finite time. Hence, the model reaches to isotropy after some finite time which matches with the recent observation as the universe is isotropic at large scale. The evolution of expansion scalar θ for $\alpha = 1$ is as shown in figure (iv). It is observed that the expansion is infinite at $t=0$ but as cosmic time t increases it decreases and remains constant throughout the evolution of the universe ($\theta = 4m$). Also at rest when $t \to 0$, the EoS parameter ω tends to infinity and as time increases, the EoS parameter ω tends to -1 which gives the strong support to the existence of dark energy.

The energy density of the fluid ρ, the deviation-free EoS parameter ω and the deviation parameter δ are dynamical. As $t \to \infty$, the anisotropic fluid isotropizes and mimics the vacuum energy which is mathematically equivalent to the cosmological constant (Λ) i.e. $\delta \to 0, \omega \to -1$ and $\rho \to 6m^2 > 0$.

Here the anisotropy of the model isotropizes after finite time t which matches with the observation as the universe is initially (at the time of Big-bang) anisotropic and soon after time it isotropizes.

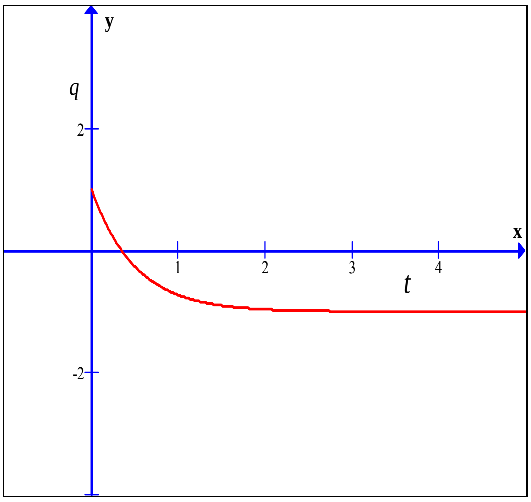

Fig. (i) The variation of q vs. t for $\alpha = 2$.

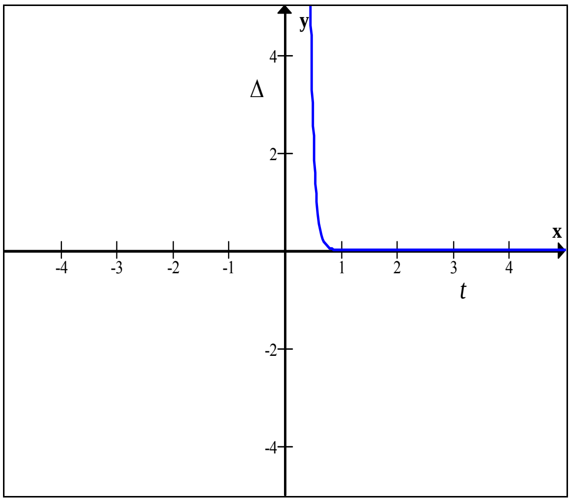

Fig. (iii) Anisotropy parameter Δ vs. cosmic time t for $\lambda = \alpha = m = 1$.

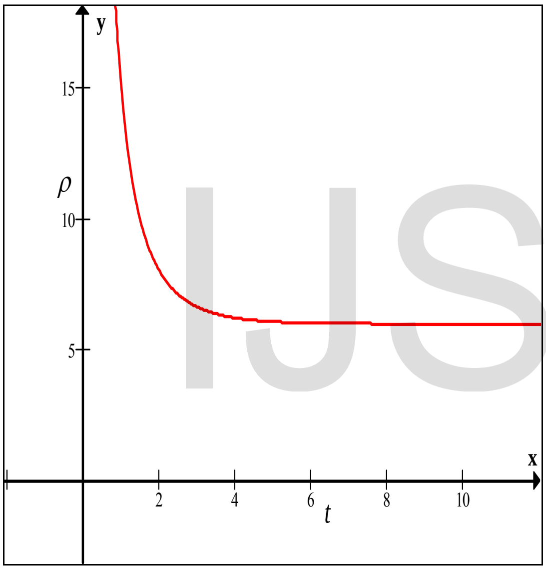

Fig. (ii) Energy density ρ vs. cosmic time t for $\lambda = \alpha = m = 1$.

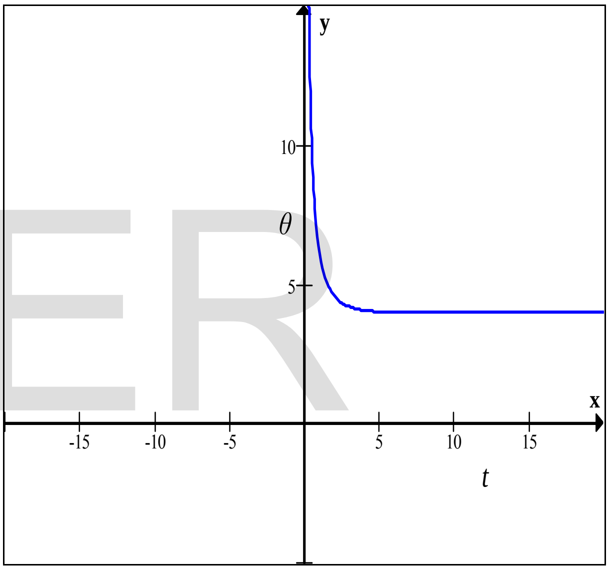

Fig. (iv) Expansion scalar θ vs. cosmic time t *for* $m = \alpha = 1$.

In this paper we have studied Kaluza-Klein cosmological model with anisotropic dark energy and special form of deceleration parameter. The physical and geometrical aspects of the model are also studied and analyze in details. Thus, even if we observe an isotropic expansion in the present universe we still cannot rule out possibility of DE with an anisotropic EoS. We can also conclude that an anisotropic DE does not necessarily distort the symmetry of the space, and consequently even if it turns out that spherical symmetry of the universe that achieved during inflation has not distorted in the later times of the universe, we can not rule out the possibility of an anisotropic nature of the DE at least in Kaluza- Klein framework.

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