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Gravity and Density Relationship (Forward Modeling)

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Abstract— The Gravimetry technique is mainly used to measure variation in gravitational field at specific locations on the earth's surface to determine subsurface densities. The variation in gravitational field is clear in zones that having a greater or lesser density than the surrounding material. However, the problem is the ambiguity in gravity anomaly interpretation However; a unique solution can be obtained by incorporating some a-priori information such as assigning a simple geometry to the causative source.

Direct gravity problem is called forward modeling and it means calculating the gravity anomaly from known density of simple bodies through integration. This integration is computed over the volume of anomalous mass. The gravity anomaly (the vertical component of the gravity attraction) is directly related to the density contrast.

This paper presents basic formulas for the use of relative gravimetry techniques for gravity anomaly computations in geophysical exploration in general and voids detection in specific. The direct techniques explored in this paper extensively through different mathematical bodies that are corresponding to expected voids that will be under investigation with real case study in the future. The mathematical shaped bodies are picked up and will be tested to help overcoming the ambiguity of gravity anomaly interpretation.

Index Terms— Gravity anomaly, voids detection, Density, Gravity interpretation and forward modeling.

1 Introduction

Exploration applications always involve measurement of differences in vertical component of the gravity fields, which are depended on buried geologic features. In fact, the attraction of the earth does not only affected by buried geologic features, but there are other sources that cause variation in attraction value, such as latitude variation. If all other sources of attraction considered constant over the surveyed area, the contrast density (between certain body and surrounding material), which is known by anomalous density is the main source for vertical component of attraction. Consequently, it can be measured by any type of gravimeters, which are designed for measuring vertical force alone [1].

For indirect interpretation approach, the causative body of a gravity anomaly is simulated by a model whose theoretical anomaly can be computed, and the shape of the model is altered until the computed anomaly closely matches the observed anomaly [2], Although the inverse problem will not be a unique interpretation, but the ambiguity can be decreased by using other constraints on the nature and form of the anomalous body. Indirect methods are either based on the application of simple analytic formulas for elementary source bodies like vertical and dipping faults, spheres, cuboids, horizontal and vertical cylinders. Although, that indirect methods are used extensively, but the inverse problem is not robust and its stability is an issue and needs extensive information to help with the regularization of the inverse mathematical problem. Consequently, in this research paper, the main objective will be on building a model and procedure for detecting different voids with different geometrical shapes for void detection using simulated environment will be used.

2 Methodology

The computation carried out by integrating process for the vertical component of attraction force from three-dimensional bodies below the computational surface by the following [3]:

$$g_z = \int^{v} G * \Delta\rho * \frac{z}{r^3} dv \qquad (1)$$

Where $\Delta\rho$ is the density contrast, G is the Newton's gravitational constant, z is the depth to the mass point and r is the distance between the mass point and the observation point.

In general, it's preferable in the vertical component of attraction computation to deal with bodies with simplest geometry or that are symmetrical with respect to vertical axis passing through the center of gravity like (sphere, rectangular prism, horizontal cylinder…)[4].

3 Forward Gravity Modeling of Simple-Shaped Bodies

3.1 The sphere

The attraction at any external point of a homogeneous solid sphere to the attraction of a point mass located at its center and can be computed from simple form [5].

$$g_z\,(mgal) = \frac{G * \Delta\rho * V * z}{(x^2 + z^2)^{\frac{3}{2}}} \qquad (2)$$

Where:

g_z : is the vertical component of attraction force.

G: is universal gravity constant = 6.67 * $10^{-11} \mathrm{m^3 Kg^{-1} s^{-2}}$

$\Delta\rho(kg/m^3)$: is the density contrast of the sphere

$V(m^3)$: is the volume of the sphere

$z(m)$: is the depth of the center of sphere

$x(m)$: is the location of calculation point

For example let: a sphere with radius a= 50 m and z=100 m and $\Delta\rho$ = 2000 $\mathrm{Kg.m^{-3}}$

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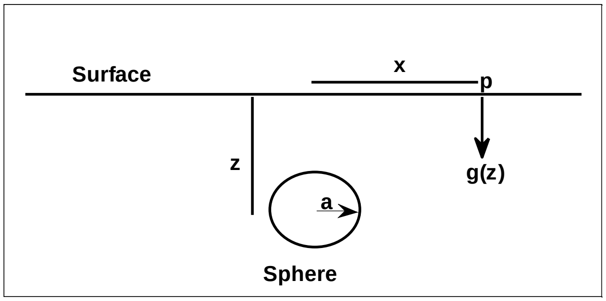

Fig. 1. subsurface sphere with radius (a) and depth (z).

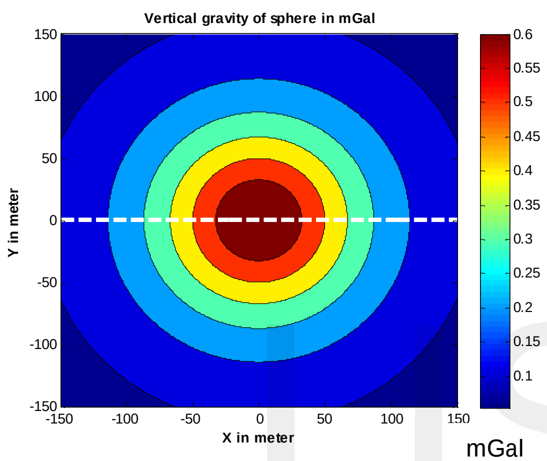

Fig. 2. contour map for vertical gravity attraction of subsurface sphere.

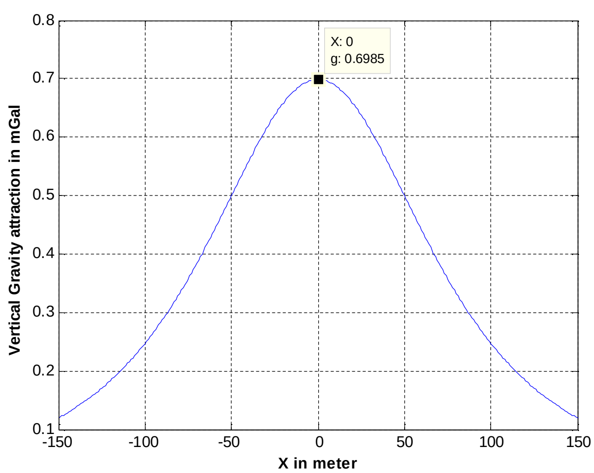

Fig. 3. profile for vertical gravity attraction of subsurface sphere

g_z has its maximum value directly above the sphere where x = 0, the maximum attraction is

$$g_z^{max} = \frac{G * \Delta\rho * V}{z^2} \qquad (3)$$

3.2 **The horizontal cylinder**

The attraction at any external point of a homogeneous solid horizontal cylinder can be calculated from the following equation [1]:

$$g_z\ (mgal) = \frac{2 * G * \pi * a^2 * z * \Delta\rho}{(x^2 + z^2)} \qquad (4)$$

Where:

g_z : is the vertical component of attraction force.
G: is universal gravity constant = 6.67 * $10^{-11} m^3 Kg^{-1} s^{-2}$
$\Delta\rho(kg/m^3)$: is the density contrast of the horizontal cylinder
$a(m)$: is the radius of cylinder
$z(m)$: is the depth of the center of cylinder
$x(m)$: is the location of calculation point

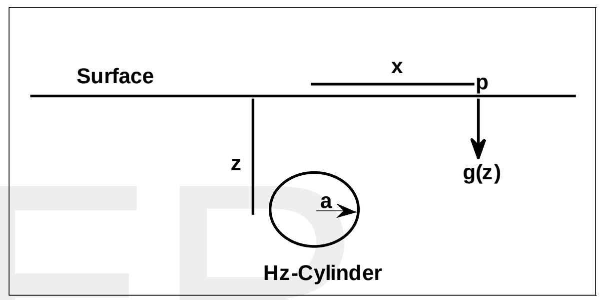

Fig. 4. subsurface Hz-cylinder with radius (a) and depth (z)

For example let: a = 50 m and z=100 m and $\Delta\rho$ = 2000 $Kg.m^{-3}$

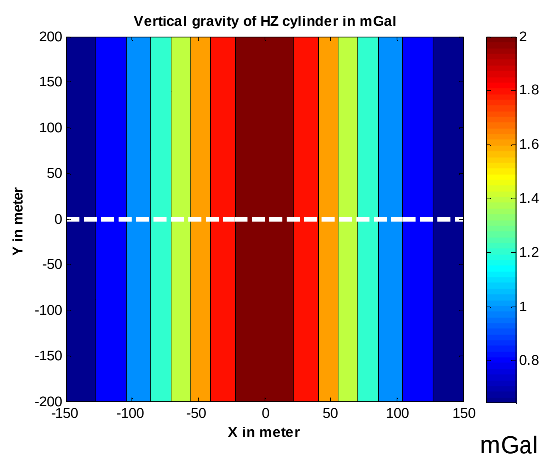

Fig. 5. contour map for vertical gravity attraction of subsurface Hz-cylinder.

g_z has its maximum value directly above the cylinder where x = 0, the maximum attraction is

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$$g_z^{max} = \frac{2 * G * \pi * a^2 * \Delta\rho}{z} \quad (5)$$

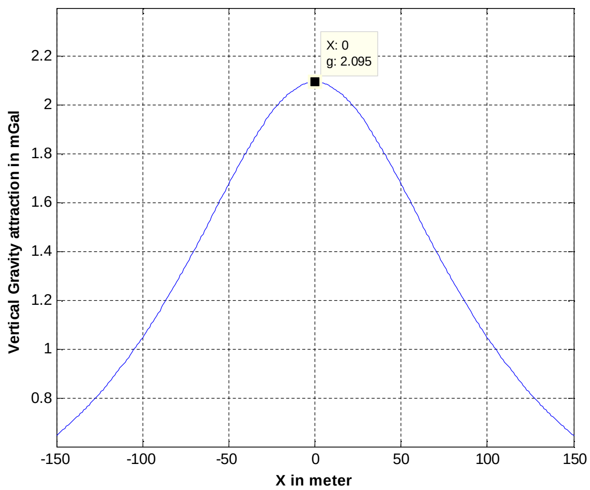

Fig. 6. profile for vertical gravity attraction of subsurface Hz cylinder.

The attraction because of the Hz-cylinder is not as sharp as that from a sphere at the same depth. Since the term involving the horizontal distance**x**, which is in the denominator and taken to a lower power for the cylinder. It is seen by comparing the both equations 2 and 4 that a horizontal cylinder will have a maximum gravitational attraction about (1.5 **z**/ **a**) times as great as a sphere of the same radius, depth, and density. This should be expected in view of the much greater mass contained in the cylinder.

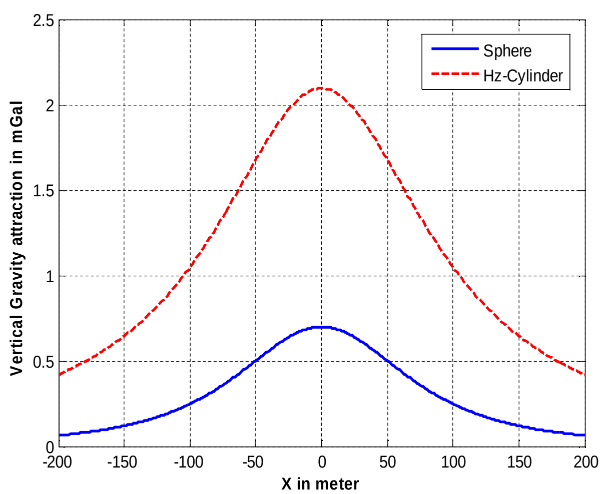

Fig. 7. comparison between vertical gravity attraction of sphere and Hz-cylinder with the same depth and radius.

3.3 **The right rectangular prism**

The attraction at any external point of a homogeneous solid right rectangular prism can be calculated from the following equation [6]:

$$g_z(mgal) = G * \Delta\rho * \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\mu_{ijk} * \Big[\Delta z_k * arctan\left(\frac{\Delta x_i * \Delta y_j}{\Delta z_k * R_{ijk}}\right) - \Delta x_i * log\left(R_{ijk} + \Delta y_j\right) - \Delta y_j * log\left(R_{ijk} + \Delta x_i\right)\Big] \quad (6)$$

Where:

g_z: is vertical component of attraction force.

G: is the universal gravity constant = 6.67 * $10^{-11} m^3 Kg^{-1} s^{-2}$

$\Delta\rho(kg/m^3)$: is the density contrast of the rectangular prism

$\mu_{ijk} = (-1)^i(-1)^j(-1)^k$

$\Delta x_i = (x_i - x_p), \Delta y_i = (y_j - y_p), and\ \Delta z_k = (z_k - z_p)$The distances from each corner to calculation point (p).

$$R_{ijk} = \sqrt{\Delta x_i^{\ 2} + \Delta y_j^{\ 2} + \Delta z_k^{\ 2}} \quad (7)$$

For example using the following parameters:

x_1= -100 m, x_2= 100 m, y_1= -100 m, y_2= 100 m, z_1= -100 m, z_2= -200 m, $\Delta\rho$ = 2000 $Kg.m^{-3}$

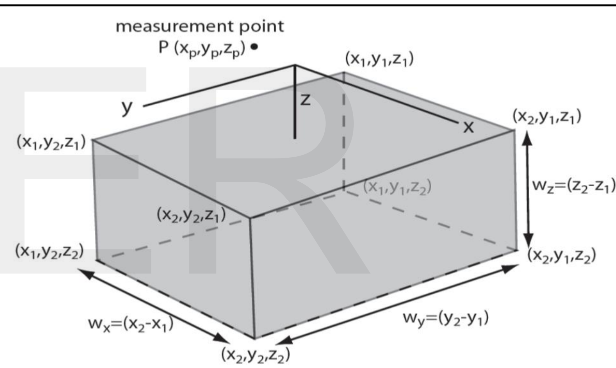

Fig. 8. right rectangular prism with dimensions$(x, y, z)_{1:2}$.

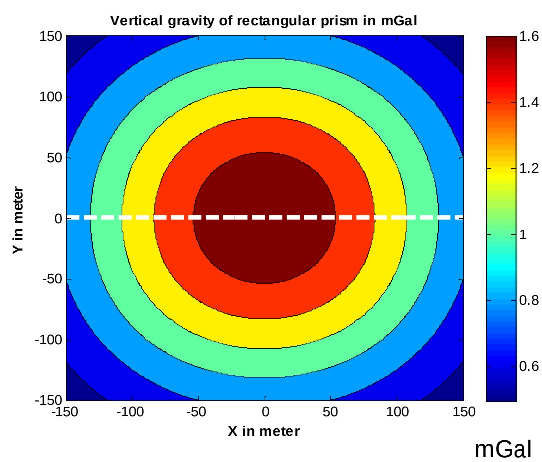

Fig. 9. contour map for vertical gravity attraction of subsurface prism.

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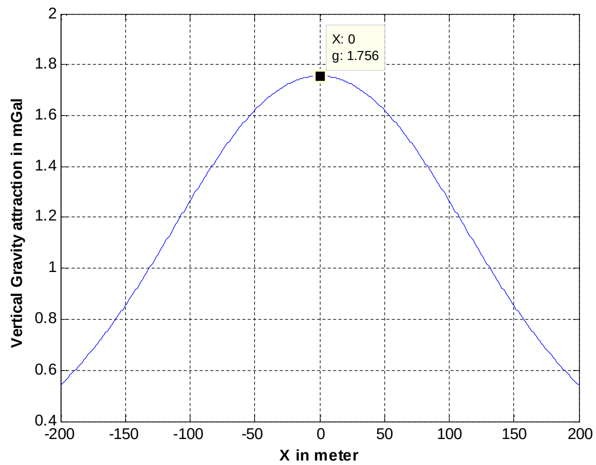

Fig. 10. profile for vertical gravity attraction of subsurface prism.

3.4 The dipping thin sheet with finite length

The vertical gravitational attraction $g_z$of a dipping thin sheet with an inclination (α) equal (90+dipping angle) with horizontal plan and has finite length (l) and thickness (T) can be computed from the following form [7]:

$$g_z(mgal) = 2 * G * \Delta\rho * T * \left[\sin \propto * \ln\left(\frac{r_2}{r_1}\right) - (\theta_1 + \theta_2) * \cos\alpha\right] \quad (8)$$

Where:
g_z: is the vertical component of attraction force.
G: is the universal gravity constant = 6.67 * $10^{-11} m^3 Kg^{-1} s^{-2}$
$\Delta\rho(kg/m^3)$: is the density contrast
$T(m)$: is the thickness

$$r_1 = \sqrt{x^2 + h^2} \quad (9)$$

$$r_2 = \sqrt{(x + l * \cos \propto)^2 + (h + l * \sin \propto)^2} \quad (10)$$

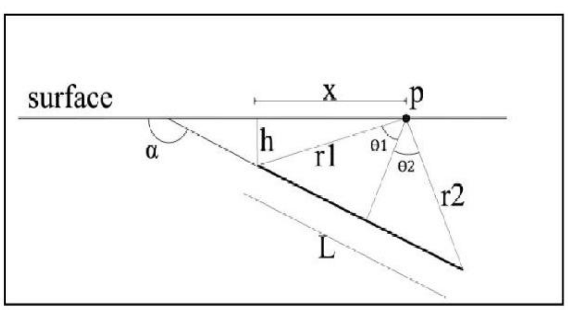

Fig. 11. thin Sheet with finite length (l) and (α = 90+dip angle).

When the dipping angle equals 0 degree, the thin sheet will be vertical and the vertical gravity equation is simplified to [7]:

$$g_z(mgal) = 2 * G * \Delta\rho * T * \ln\left[\frac{(h+l)^2 + x^2}{h^2 + x^2}\right] \quad (11)$$

The sheet will be horizontal when the dipping angle equals 90 degree and the vertical gravity attraction will be [7]:

$$g_z(mgal) = 2 * G * \Delta\rho * T * \left[\tan^{-1}\left(\frac{l-x}{h}\right) + \tan^{-1}\left(\frac{x}{h}\right)\right] \quad (12)$$

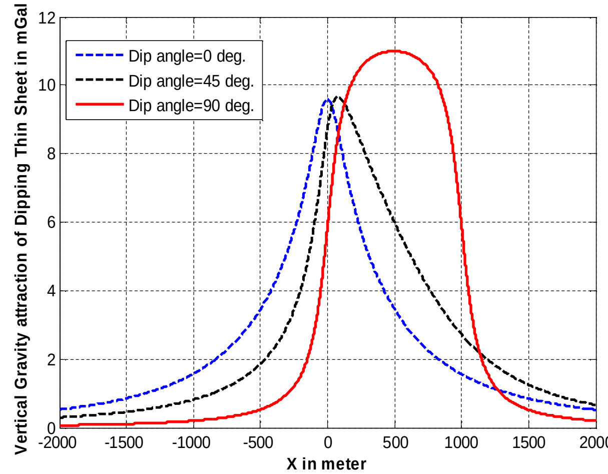

Fig. 12. profile for vertical gravity attraction of three dipping thin sheet with variable dipping angle (0, 45, 90) degrees.

3.5 The semi-infinite horizontal sheet

If the horizontal sheet has infinity in one direction, the vertical gravity attraction g_z will be as follows [1]:

$$g_z(mgal) = 2 * G * \Delta\rho * T * \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{x}{z}\right)\right] \quad (13)$$

Where:
g_z: is vertical component of attraction force.
G: is the universal gravity constant = 6.67 * $10^{-11} m^3 Kg^{-1} s^{-2}$
$\Delta\rho(kg/m^3)$: is the density contrast
$T(m)$: is the thickness

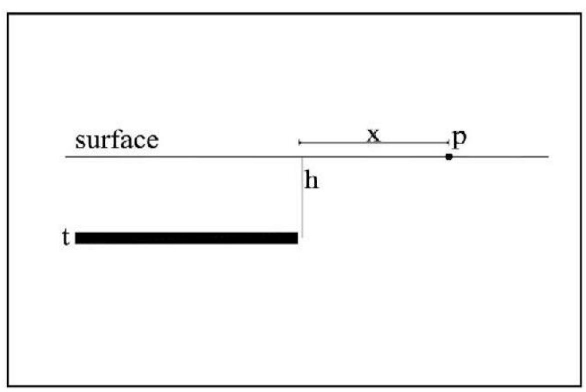

Fig. 13. semi-infinite horizontal sheet with thickness (T).

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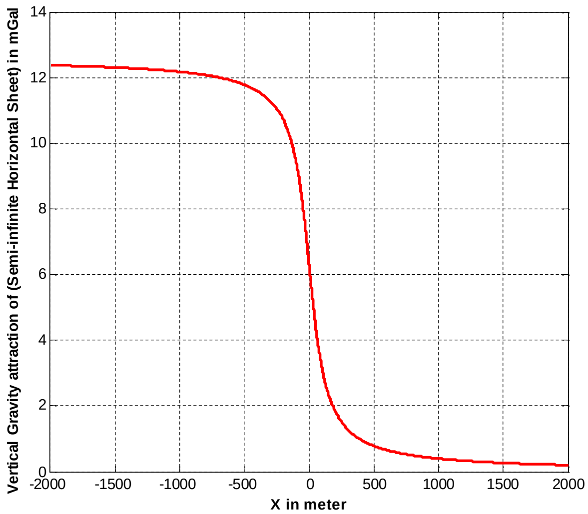

Fig. 14. profile for vertical gravity attraction of semi- infinite horizontal sheet.

4 Conclusion

Building a library of identified shapes for direct gravimetry void detection technique for geophysical exploration in small areas. It is clear the differentiation between the shape in case of fixing the size, depth, and density. A complete geometrical simulation was done and in the future a complete library of shapes will be tested for direct techniques in archeological applications with constrained environment for void detection.

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