Further development & updating of paper for Mystery of Fermat Number

Author: Debajit Das

ABSTRACT

With the help of N-equation property and Ns-Nd operation which was published in August edition 2013 an attempt was made to penetrate the mystery of Fermat Number and it was published in June & July edition 2015 of this journal. With further development of different properties of N-equation now it is felt necessary to update the papers of Fermat Number and to reestablish its proof that it is always composite for n > 4, on solid ground. Earlier proofs seem to be not so convincing as it is now. This paper contains some new properties of N-equation, rearrangement of points with little change and the proof of Fermat Number in a different angle.

Keywords

ab-consecutive & √ab-consecutive equation, Bondless pair, Degree of intensity for even number (DOI), Equidistant Gap (EG), Fermat odd factor (F_{of}),& Fermat even factor(F_{ef}),Natural constant (k), wing.

1. Introduction

N-equation $a^2 + b^2 = c^2$ (a, b, c can be said as its elements) is nothing but the systematic arrangement of all Pythagorean triplets. According to this arrangement the N-equation has been divided into two kinds i.e. 1st kind includes where $k = c - b$ is in the form of $1^2, 3^2, 5^2$,…….and 2nd kind includes where $k = c - b$ is in the form of $2.1^2, 2.2^2, 2.3^2$, …….. assuming $a < b < c$ in both the cases. Regarding prime numbers' distribution if we look its arrangement in 'Mixed zygote' form we will observe that all the prime numbers satisfy the left hand odd element of a N-equation for k = 1 only. All other left hand odd elements except k = 1 are composite i.e. the left hand element for k = 1 contains all odd numbers but for k ≠ 1, it contains only composite numbers. Moreover, for k = 1, the nature of conjugate zygote expression i.e. $(\alpha^2 \pm \beta^2)$ is such that α, β are always consecutive integers.
Furthermore, RH odd element ± LH odd element = 2(integer)^2 and RH odd element ± LH even element = (odd integer)^2
From N_s operation we can review the following property.
$\pi\,(e_i^2 + o_i^2) = (e_1^2 + o_1^2)\,(e_2^2 + o_2^2)(e_3^2 + o_3^2)$…….. $(e_n^2 + o_n^2) = E(v_j^2 + d_j^2) = (v_1^2 + d_1^2) = (v_2^2 + d_2^2) = (v_3^2 + d_3^2) = \ldots\ldots\ 2^{n-1}$ wings where the symbol π & E stand for continued product & equalities, e, v for even integers & o, d for odd integers. All $(e_i^2 + o_i^2)$ are prime numbers & $\gcd(e_i, o_i) = \gcd(v_j, d_j) = 1$. If any prime is repeated thrice or more then in all cases $\gcd(v_j, d_j) \neq 1$. Because when p is prime, p & p^2 both have single wing but p^3 has double wing, one where gcd of elements = 1 and other where gcd of elements exists as p e.g. $5 = 2^2 + 1^2$ & $5^2 = 3^2 + 4^2$ whereas $5^3 = 11^2 + 2^2 = 5^2 + 10^2$.
The product & division rules of N_s operation are as follows.
$(e_1^2 + o_1^2).(e_2^2 + o_2^2) = (\,|e_1e_2 \pm o_1o_2|\,)^2 + (\,|e_1o_2 -/+ o_1e_2|\,)^2$ &
$(e_1^2 + o_1^2)/(e_2^2 + o_2^2) = \{|e_1e_2 \pm o_1o_2|/(e_2^2 + o_2^2)\}^2 + \{|e_1o_2 -/+ o_1e_2|/(e_2^2 + o_2^2)\}^2$ consider only one wing which has integer elements.

Based on the above theoretical background we can further develop the property of N-equation with updating the published papers and reestablish the proof that the Fermat number ($F_n = 2^{2^{\wedge}n} + 1$) will always represent a composite number for n > 4

2. If an odd integer in the form of $e^2 + o^2$ where gcd(e, 0) = 1 is composite then there must exist at least one pair of integer $I_{1,2}$ (one is odd & other is even) so that $[\{\mathrm{Min}(e, o)\}^2 + I_{1,2}]$ & $[\{\mathrm{Max}(e, o)\}^2 - I_{1,2}]$ both are square integers while the existence of any one of $I_{1,2}$ confirms the number to be composite.

Let us consider a composite number $N = (e^2 + o^2) = (e_1^2 + o_1^2)\,(e_2^2 + o_2^2) = (e_1o_2 \pm e_2o_1)^2 + (e_1e_2 -/+ o_1o_2)^2$

International Journal of Scientific & Engineering Research, Volume 6, Issue 9, September-2015
ISSN 2229-5518

Here one wing is $(e_1o_2 + e_2o_1)^2 + (e_1e_2 - o_1o_2)^2$
$\Rightarrow$ even element of other wing = $(e_1o_2 - e_2o_1) = \sqrt{\{(e_1o_2 + e_2o_1)^2 - 4e_1e_2o_1o_2\}} = \sqrt{(e^2 - 4e_1e_2o_1o_2)} \Rightarrow (e^2 - 4e_1e_2o_1o_2)$ must be square integer. Similarly for odd element of other wing $(o^2 + 4e_1e_2o_1o_2)$ must be a square integer.
In this case $o < e$
For $e < o$, consider one wing is $(e_1o_2 - e_2o_1)^2 + (e_1e_2 + o_1o_2)^2$
$\Rightarrow$ even element of other wing = $\sqrt{(e^2 + 4e_1e_2o_1o_2)}$ & odd element of other wing is $\sqrt{(o^2 - 4e_1e_2o_1o_2)}$
$\Rightarrow$ both $(e^2 + 4e_1e_2o_1o_2)$ & $(o^2 - 4e_1e_2o_1o_2)$ must be square integer.
Hence, $\{Min(e, o)\}^2 + 4e_1e_2o_1o_2 = P^2$ & $\{Max(e, o)\}^2 - 4e_1e_2o_1o_2 = Q^2$ say, $\{Min(e, o)\}^2 + V = P^2$ & $\{Max(e, o)\}^2 - V = Q^2$
$\Rightarrow \{Min(e, o)\}^2 + (Q^2 - P^2 + V) = Q^2$ & $\{Max(e, o)\}^2 - (Q^2 - P^2 + V) = P^2$ say, $\{Min(e, o)\}^2 + D = Q^2$ & $\{Max(e, o)\}^2 - D = P^2$
$\Rightarrow$ If V exists D must exist or vice-versa.
$\Rightarrow$ Existence of D or V confirms the composite nature of the number N.
$\Rightarrow$ For a composite number, internal equidistant elements from both the extremities must be two square integers of even & odd at least for once and this 'Equidistance Gap' (E.G) is multiple of 16 otherwise it is prime.
$\Rightarrow$ $[\{Min(e, o)\}^2 + 16k] + [\{Max(e, o)\}^2 - 16k] = P^2 + Q^2$ for a composite number where k = 1, 2, 3, ………

3. For a composite number $N = e_1^2 + o_1^2 = e_2^2 + o_2^2$, $Min(e_2, o_2) < (e_1 + o_1)/2 < Max(e_2, o_2)$ & $Min(e_1, o_1) < (e_2 + o_2)/2 < Max(e_1, o_1)$

Let $N = (v_1^2 + d_1^2)(v_2^2 + d_2^2) = (v_1d_2 \pm v_2d_1)^2 + (v_1v_2 \mp d_1d_2)^2$ where v is even & d is odd like e & o respectively.
Say the wing $(v_1d_2 + v_2d_1)^2 + (v_1v_2 - d_1d_2)^2 = e_1^2 + o_1^2$ & other wing = $e_2^2 + o_2^2$
It is obvious, $(v_1d_2 + v_2d_1)^2 < [\{(v_1d_2 + v_2d_1) + (v_1v_2 - d_1d_2)\}/2]^2 < (v_1v_2 - d_1d_2)^2$
$\Rightarrow (e_2^2 + 4v_1v_2d_1d_2) < \{(e_1 + o_1)/2\}^2 < (o_2^2 - 4v_1v_2d_1d_2) \Rightarrow e_2^2 < \{(e_1 + o_1)/2\}^2 < o_2^2 \Rightarrow e_2 < \{(e_1 + o_1)/2\} < o_2$
$\Rightarrow Min(e_2, o_2) < (e_1 + o_1)/2 < Max(e_2, o_2)$. Similarly, $Min(e_1, o_1) < (e_2 + o_2)/2 < Max(e_1, o_1)$
Obviously, all $v_1, v_2, d_1, d_2 < Min[(e_1 + o_1)/2, (e_2 + o_2)/2]$

For a composite number $1 + e^2$ where lower element is one E.G cannot be more than e^2 hence its all other wings $P_i^2 + Q_i^2$ has the property $1 < P_i, Q_i < e$ i.e. $1 < Min(P_i, Q_i) < (e + 1)/2$ & $(e + 1)/2 < Max(P_i, Q_i) < e$ or physically it is quite understood.

4. Nature of $(b \pm a)$ for a N-equation $a^2 + b^2 = c^2$ $(a < b)$

The 1st kind N-equation is of the form $\{b + (2x - 1)^2 - 2y^2\}^2 + b^2 = \{b + (2x - 1)^2\}^2$ where $2y^2 > (2x - 1)^2$ & $k = (2x - 1)^2$ is constant whereas y is variable.
$\Rightarrow (b - a) = 2y_i^2 - k$ for i > an assigned value m.
The 2nd kind N-equation is of the form $\{b + 2x^2 - (2y - 1)^2\}^2 + b^2 = (b + 2x^2)^2$ where $(2y - 1)^2 > 2x^2$ & $k = 2x^2$ is constant whereas y is variable.
$\Rightarrow (b - a) = (2y_i - 1)^2 - 1$ for i > an assigned value m.
For both kinds of N-equation (b + a) follow the same sequence as (b – a) and for a particular value of k the ordered pair (b – a, b + a) follows the sequence $(\alpha_1,\alpha_2), (\alpha_2,\alpha_3), (\alpha_3,\alpha_4)$, …….

5. For a N-equation $a^2 + b^2 = c^2$ how a, b form consecutive integers.

If a is odd & b is even then from property of N-equation we can write c + a is of the form $2\beta^2$ & c + b is of the form α^2
$\Rightarrow a \sim b$ is of the form $\alpha^2 \sim 2\beta^2 \Rightarrow$ if $a \sim b = 1$ then $\alpha^2 \sim 2\beta^2 = 1$.

Case-I: When $\alpha^2 - 2\beta^2 = 1$.

Say, $\alpha^2 = (2x + 1)^2 \Rightarrow \beta^2 = 2x(x + 1)$ which is possible only for x = 1 i.e. $3^2 - 2.2^2 = 1$

It produces the only relation under 2nd kind N-equation i.e. $20^2 + 21^2 = 29^2$.

Case-II: When $2\beta^2 - \alpha^2 = 1$

Here, $\beta^2 = x^2 + (x + 1)^2$
$\Rightarrow (2x + 1)^2$ will be of the form 'sum of two consecutive integers' square' when
$x^2 + (x + 1)^2$ is a square integer.
We have $3^2 + 4^2 = 5^2$ where $x = 3$
Hence, next consecutive phenomenon will be observed for $(2.3 + 1)^2 < 50$ i.e. 2.5^2 i.e. for $k = 7^2$.
Say, $(b - 1)^2 + b^2 = (b + 7^2)^2 \Rightarrow b = 120$ that follows the relation $119^2 + 120^2 = 169^2$.
Next phenomenon will be observed for $(2.119 + 1)^2 < 2.169^2$ i.e. for $k = 239^2$.
Say, $(b - 1)^2 + b^2 = (b + 239^2)^2 \Rightarrow b = 137904$ that follows $137903^2 + 137904^2 = 195025^2$.
Similarly, next (a, b) – consecutive phenomenon will be observed as $183648021599^2 + 183648021600^2 = 259717522849^2$ & so on.
So, with the help of an (a, b)-consecutive equation (or can be said as abc-equation) we can produce next abc-equation. Hence, the existence of abc-relation is infinite.
abc-equation always falls under 1st kind except $20^2 + 21^2 = 29^2$ which is under 2nd kind. This is because of the fact that $2u^2 - 1 = I^2$ has infinitely many solutions but $2u^2 + 1 = I^2$ has the only solution i.e. $2.2^2 + 1 = 3^2$.
$\Rightarrow$ if abc-equation falls under $k = p^2$ then $p^2 + 1$ must be in the form of $2u^2$. Obviously, abc-eq. is the leading set of $k = p^2$.
For a N-eq. $a^2 + b^2 = c^2$ if c is free from any abc-relation then it cannot be under $k = 1$ and hence all the left hand odd elements (say a_i) produced by $(a_i)^2 + (b_i)^2 = c^2$ are composite.

6. For a N-equation $a^2 + b^2 = c^2$ (a < b), c^4 will produce an abc-equation having $k = (b - a)^2$ if $(b - a)^2$ is of the form $2u^2 - 1$.

$a^2 + b^2 = c^2$ may be of any kind and similarly, $(b^2 - a^2)^2 + (2ab)^2 = c^4$ can be of any kind. But c^4 will produce abc-eq. when it is of 1st kind i.e. $(b^2 - a^2) < (2ab)$
Let us consider a N-eq. of 1st kind $\{b + (2x - 1)^2 - 2y^2\}^2 + b^2 = \{b + (2x - 1)^2\}^2 = c^2$ where $2y^2 > (2x - 1)^2$
$\Rightarrow [b^2 - \{b + (2x - 1)^2 - 2y^2\}^2]^2 + [2b\{b + (2x - 1)^2 - 2y^2\}]^2 = c^4$.
$\Rightarrow 2b\{b + (2x - 1)^2 - 2y^2\} - [b^2 - \{b + (2x - 1)^2 - 2y^2\}^2] = 1$
$\Rightarrow 2b^2 - 4bp + p^2 - 1 = 0$ where $p = 2y^2 - (2x - 1)^2$ & $D = 8(p^2 + 1)$
If D is a square integer p^2 must be in the form of $2u^2 - 1$ i.e. $(b - a)^2$ is in the form of $2u^2 - 1$.
Similarly, if we consider a N-eq. of 2nd kind we will get the same result i.e. $(b - a)^2$ is in the form of $2u^2 - 1$.
As $(b \pm a)$ both exist in a particular value of k with equal integer in consecutive two sets we can write the condition $\{(b \pm a)_k\}^2 = (b \pm a)_{k=1}$.

7. There exists only one relation under k = 1 where c^4 produces abc-equation.

Functional form of N-equation for $k = 1$ is $(2x + 1)^2 + \{2x(x + 1)\}^2 = c^2$
$\Rightarrow [\{2x(x + 1)\}^2 - (2x + 1)^2]^2 + \{2.2x(x + 1)(2x + 1)\}^2 = c^4$ where $\{2.2x(x + 1)(2x + 1)\} - [\{2x(x + 1)\}^2 - (2x + 1)^2] = 1$
On simplification, $x^3 - 3x - 2 = 0 \Rightarrow x = 2$
Hence, $5^2 + 12^2 = 13^2$ will produce abc-relation i.e. $(12^2 - 5^2)^2 + (2.12.5)^2 = 13^4$ or, $119^2 + 120^2 = 13^4$.

8. For a sequence of N-equation $(a_i)^2 + (b_i)^2 = (c_i)^2$ of a particular value of k, c^4 will remain under 1st kind so long $2a^2 - (b - a)^2 > 0$. If $2a^2 - (b - a)^2 = 1$, c^4 will produce abc-relation where c^2 can be said as square root of ab-consecutive equation or simply √abc-equation.

Let us consider a N-equation of any kind $a^2 + b^2 = c^2$ where $b > a$
$\Rightarrow (b^2 - a^2)^2 + (2ab)^2 = c^4$ which will remain under 1st kind when $2ab > (b^2 - a^2)$ i.e. $(b/a)^2 - 2(b/a) - 1 < 0$

$\Rightarrow b/a \in (0, \sqrt{2} + 1)$. As $(b/a) > 1$, $b/a \in (1, \sqrt{2} + 1) \Rightarrow b/a < \sqrt{2} + 1 \Rightarrow 2a^2 - (b - a)^2 > 0$
Now say , $2a^2 - (b - a)^2 = 1$ i.e. $a^2 + 2ab - b^2 - 1$
Considering it as a quadratic equation of a, $a = -b + \sqrt{(2b^2 + 1)}$ where $(2b^2 + 1)$ is a square integer only for $b = 2$
$\Rightarrow$ for $b = 2$, $a = 1$ & √abc-equation is $1^2 + 2^2 = 5$ against abc-equation $(2^2 - 1^2)^2 + (2.1.2)^2 = 5^2$ i.e. $3^2 + 4^2 = 5^2$.
It cannot be further continued as $(2b^2 + 1)$ fails to produce further square integer.
But considering the quadratic equation with respect to b we have $b = a + \sqrt{(2a^2 - 1)}$ where there exists infinite nos. of square integers against $(2a^2 - 1)$. First one is obviously 5 & for $a = 5$, $b = 12$ i.e. $5^2 + 12^2 = 13^2$ is the √abc-eq. of abc-eq. $(12^2 - 5^2)^2 + (2.12.5)^2 = (13^2)^2$ i.e. $119^2 + 120^2 = 169^2$
Next square integer of $(2a^2 - 1)$ is for $a = 169$ & for $a = 169$, $b = 169 + 239 = 408$
$\Rightarrow 169^2 + 408^2 = 195025$ is the √abc-eq. of abc-eq. $(408^2 - 169^2)^2 + (2.408.169)^2 = (195025)^2$ i.e. $137903^2 + 137904^2 = 195025^2$.
Next $a = 195025$ & $b = 195025 + 275807 = 470832$
$\Rightarrow 195025^2 + 470832^2 = 259717522849$ is the √abc-eq. of abc-eq. $(470832^2 - 195025^2)^2 + (2.\ 470832.\ 195025)^2 = (259717522849)^2$ i.e. $183648021599^2 + 183648021600^2 = 259717522849^2$ & so on.
Sequences of √abc-equations are as follows:
$5^2 + \{5 + \sqrt{(2.5^2 - 1)}\}^2$ i.e. $5^2 + 12^2 = 13^2 = 169$
$169^2 + \{169 + \sqrt{(2.169^2 - 1)}\}^2$ i.e. $169^2 + 408^2 = 195025$
$195025^2 + \{195025 + \sqrt{(2.\ 195025^2 - 1)}\}^2$ i.e. $195025^2 + 470832^2 = 259717522849$ & so on.
We have observed that every abc-relation has a definite √abc-relation which is obtained considering the sequence $2a^2 - (b - a)^2 = 1$
$\Rightarrow$ The condition $\{(b \pm a)_k\}^2 = (b \pm a)_{k=1}$ as stated in Point-6 is true only for $k = 1$ against p.

9. Left hand odd element of a N-equation like $a^2 + b^2 = c^2$ is always composite if c is composite having one wing in the form of $(\alpha^2 + 1)$ but the reverse is not true.

There cannot exist a relation like $\alpha^2 + 1 = \beta^2 + (\beta + 1)^2$
As per theory explained in point no.3, if the said relation exists then $\beta = (\alpha + 1)/2 - ½$ & $\beta + 1 = (\alpha + 1)/2 + ½$
i.e. $\beta = \alpha/2$ & $\beta + 1 = (\alpha + 2)/2 \Rightarrow \alpha^2 + 1 = \alpha^2/4 + (\alpha + 2)^2/4$ or, $\alpha^2 - 2\alpha = 0$ i.e. $\alpha = 2$
$\Rightarrow 1^2 + 2^2 = 2^2 + 1^2$ & this is not accepted.
So, in a N-equation like $a^2 + b^2 = (\alpha^2 + 1)^2 = c^2$, c cannot be expressed as abc-consecutive relation**.**
$\Rightarrow$ it cannot fall under $k = 1$. Hence, the left hand odd element is always composite.
If c is prime then it has a single wing in the form of $\alpha^2 + 1$. Then also LH odd element is $\alpha^2 - 1$ which is composite excepting the case $3^2 + 4^2 = (2^2 + 1)^2$.
The same conclusion can be drawn in case of $a^2 + b^2 = (F_n)^2$ where F_n denotes a Fermat Number.
As $1 + b^2 = c^2$ does not exist Fermat Number cannot be a square integer.
Furthermore, for $n > 0$, if unit digit (UD) of L H odd element $\neq 5$ then F_n is bound to be composite.
Because, if F_n is prime it has only one relation $[\{2^{2^\wedge(n-1)}\}^2 - 1^2]^2 + b^2 = [\{2^{2^\wedge(n-1)}\}^2 - 1^2]^2$ where UD of a = 5.
If UD of a $\neq 5$ then definitely F_n has other wing. $\Rightarrow F_n$ is composite.
For F_n is composite the possibility of UD of 'a' other than 5 is 3 only. Because, UD of $(F_n)^2$ is 9 which attracts two combinations only i.e. (UD of a = 5)^2 + (UD of b = 2)^2 & (UD of a = 3)^2 + (UD of b = 0)^2.

10. How a relation $u_1^2 + v_1^2 = u_2^2 + v_2^2$ is formed where $u_{1,2}$ & $v_{1,2}$ are respectively odd & even and u_1, u_2 both satisfy c of a N-equation $a^2 + b^2 = c^2$ by replacement method.

Let us consider a N-equation $a^2 + b^2 = c^2$ where the odd element a possesses at least two prime factors.
Then obviously there must exist at least one relation obtained by N_d operation of a^2 like $a^2 + b_1^2 = c_1^2$
$\Rightarrow c_1^2 + b^2 = c^2 + b_1^2$. Here a must be composite.
All such relations with respect to b & c that exist are obtained by this method. These types of set of double wings can be said as bondless pair.

11. If $u^2 + 1$ is composite then $u^4 + 1$ is also composite that follows the proof that Fermat Number ($F_n = 2^{2^{\wedge}n} + 1$) always represents a composite number for n > 4.

If $u^2 + 1$ is composite it must have at least another wing.
Say, $u^2 + 1 = a^2 + b^2 \Rightarrow (u^4 + 1)^2 = \{(a^2 + b^2)^2 - 2u^2\}^2$ or, $(u^4 + 1)^2 + \{2u(a^2 + b^2)\}^2 = (a^2 + b^2)^4 + (2u^2)^2$……. (A)
According to theorem 10, there must exist another relation i.e. $p^2 + (2u^2)^2 = (u^4 + 1)^2$ ……… (B) where p is composite and it is the common relation and really exists irrespective of the fact whether $(u^4 + 1)$ is prime or composite i.e. (A) is a bondless pair.
Now the logic stands that if $(u^2 + 1)$ is composite then only (B) exists.
Similarly, if $(u^4 + 1)$ is composite then only $p^2 + (2u^4)^2 = (u^8 + 1)^2$ will exist & so on.
So, for $F_n = (u^2 + 1) = \{2^{2^{\wedge}(n-1)}\}^2 + 1$ it will also be true.
By physical verification we have observed that F_5 is composite. Hence, F_5, F_6, F_7, …… all are composite.

12. How Fermat Number ($F_n = 2^{2^{\wedge}n} + 1$) initially represents five prime numbers.

It is quite obvious from the product rule of N_s operation that $F_n = 2^{2^{\wedge}n} + 1$ will represent a composite number when it is possible to divide $2^{2^{\wedge}n-1}$ i.e. $(F_{n-1} - 1)$ into two even numbers say e_1o_1 & e_2o_2 where $|e_1e_2 - o_1o_2| = 1$. If not possible it is a prime. The term $|e_1e_2 - o_1o_2|$ can be said as Fermat odd factor (F_{of}) & the term $(e_1o_1 + e_2o_2)$ can be said as Fermat even factor (F_{ef}) of Fermat no. F_n
Here, obviously both the even nos. e_1 & e_2 have same degree of intensity i.e. both contain the factors 2^p. DOI of F_{ef} means DOI of e_1 or e_2 i.e. p but actual DOI of $F_{ef} > p$.
Say, $e_1 = (2^pd_1)$ & $e_2 = (2^pd_2) \Rightarrow (F_{n-1} - 1) = (2^pd_1)o_1 + (2^pd_2)o_2 = 2^p(o_1d_1 + o_2d_2)$ and obviously $(o_1d_1 + o_2d_2)$ is in the form of 2^m where $m + p = 2^{n-1} \Rightarrow$ there cannot exist any common factor between any two among d_1, d_2, o_1 & o_2
Hence we cannot have any factor of F_n in the form of $(ke)^2 + (ko)^2 \Rightarrow F_n$ is the product of prime numbers without any exponent to any prime.

Example:

$F_0 = 2^{2^{\wedge}0} + 1 = 3$. As there does not exist any number before F_0 it is prime.
$F_1 = 2^{2^{\wedge}1} + 1 = 5$. Here $F_0 - 1 = 2$ & 2 cannot be divided into two even numbers. Hence, $F_1 = 5$ is a prime.
$F_2 = 2^{2^{\wedge}2} + 1 = 17$. Here $F_1 - 1 = 4$ & $4 = 2.1 + 2.1$ where $F_f = 2.2 - 1.1 \neq 1$. Hence $F_2 = 17$ is a prime.
$F_3 = 2^{2^{\wedge}3} + 1 = 257$. Here $F_2 - 1 = 16$ and 16 can be written as,
$16 = 2.1 + 2.7$ for which $F_f = 1.7 - 2.2 \neq 1$
$16 = 2^2.1 + 2^2.3$ & $F_f \neq 1$ and lastly $16 = 2.3 + 2.5$ for which $F_f \neq 1$
For equal division obviously $F_f \neq 1$ Hence, $F_3 = 257$ is a prime.
$F_4 = 2^{2^{\wedge}4} + 1 = 65537$. Here $F_3 - 1 = 256$ and 256 can be written as,
2 + 2.127, 4 + 4.63, 2.3 + 2.125, ……. where in all cases $F_{of} \neq 1$ Hence, $F_4 = 65537$ is a prime.
But for $F_5 = 2^{2^{\wedge}5} + 1$ where $F_4 - 1 = 65536$ it is found that $65536 = 4.409 + 25.2556$ where $F_{of} = (409.25 - 4.2556) = 1$
Hence, F_5 is composite & $F_5 = (25^2 + 4^2)(409^2 + 2556^2) = 4294967297$

Conclusion:

Now I believe that the proof of this composite nature of all Fermat numbers for n > 4 will be accepted by all mathematical communities. In general it will not be wrong to conclude that any number in the form of $F_n = (odd)^{2^{\wedge}n} + (even)^{2^{\wedge}n}$ where gcd(odd, even) = 1 produces constantly composite numbers after certain operations of n for prime numbers. So at initial stage if $(odd)^2 + (even)^2$ is found to be composite it will never produce prime numbers. The possibility of constant generation of prime numbers cannot be ruled out also.
I also believe that with the help of this N_s operation and N_d operation as defined in my earlier papers published in 'IJSER, Houston, USA' so many problems in Number Theory are possible to be solved.

IJSER

References

Books

[1] Academic text books of Algebra.

[2] In August edition, Vol-4 2013 of IJSER, Author: self

[2] In June & July edition, Vol-6 2015 of IJSER, Author: self

Author: Debajit Das (dasdebjit@indianoil.in)
(Company: Indian Oil Corporation Ltd, Country: INDIA)

I have already introduced myself in my earlier publications. By profession I am a civil Engineer working in a Public Sector Oil Company as a Senior Project Manager. But to play with mathematics particularly in the field of Number theory is my passion. I am Indian, born and brought up at Kolkata, West Bengal. My date of birth is 12th July, 1958.
If anybody wants may contact at my e-mail given above.

IJSER