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Error analysis of the FEM calculations depending on the mesh density

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Abstract— Paper presents analyses of the FEM modeling error in the function of the mesh density. Tests were conducted on open-source FEM software, which allows magnetodynamics modeling with the utilization of Whitney elements to solve Maxwell’s equations. Simulations was based on modeling of magnetic flux distribution around the Helmholtz coils setup, which allowed to compare modeling result with analytical solution of magnetic flux value in the midpoint of the setup. Modeling was conducted on a typical model of Helmholtz coils with different mesh densities. Results confirmed that generally denser mesh resulted with lower modeling error, but the correlation was non-linear. Also, utilization of a mesh with premade parameters resulted with similar error like high density meshes with fixed element size.

Index Terms— Elmer FEM, Error Analysis, Finite Element Method, Helmholtz Coil, Mesh Density, Magnetodynamics Solver

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1 INTRODUCTION

FINITE Element Method is usefull numerical technique for solving partiall differentiall equations. It can be utilized for mechanical calculations, fluid mechanics, thermal flow and many other engineering and scientific problems. Development of Whitney edge elements [5] allowed development of new FEM software [2], which utilizes Maxwell's equations in order to solve various electromagnetic models [1] [4].

Each Finite Element Method solver requires proper mesh, which transposes created continuous model of analyzed geometry to set of coordinates and nodes describing finite elements. Proper mesh has significant influence on the modeling results, due to the influence on a boundary conditions, as well as on the geometry of model. This paper analyses an influence of mesh density on the accuracy of solving magnetoelectrics problem during modeling of Helmholtz coils. Helmholtz coils were selected, due to the fact that value of magnetic flux in the center of coils setup (on the axis of both coils and halfway between them) has analytical solution [7] which be used as a reference value for simulation results.

2 HELMHOLTZ COIL MODEL

2.1 Review Stage

During modeling universal model of Helmholtz coil was developed. It consisted two coaxial rings with average 0.9 m radius and average distance between them 0.9 m. Coils are placed in a 10 m radius sphere with the midpoint between the coils. Sphere is utilized in order to apply proper Dirichlet conditions [2] on its external boundary, which is crucial for proper utilization of FEM. Also magnetoelectric properties of elements forming sphere are significantly different from material in coils. Thus magnetic flux distribution in air can be approximately solved. Exemplary clipped view of the coils in sphere is presented in Figure 1.

Fig.1 Clipped view of utilized model of Helmholtz coils

As mentioned before, Helmholtz coils model was selected for simulation, due to the fact, that the value of magnetic flux between the coils can be analytically solved [7] and equals:

$$B=((4/5)\textasciicircum 2)*\ u_0nI/R \qquad (1)$$

where B is for the magnetic flux in the center point, μ_0 is free space permeability, n is the number of the turns in wire, I is the current powering the coils and R is their radius and distance.

During simulations coils were formed by one wire turn and were powered by constant 0.04 A current. Thus reference value of analyzed magnetic flux in the middle of the coils equals 3,99 * 10^(-8) T.

3 MODELING DESCRIPTION

3.1 Analyzed mesh

In order to analyze an influence of the mesh density on simulations accuracy different meshes of described model were prepared. They varied with the number of finite elements

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forming each element of model. This variation was obtained by modifying the parameter describing maximal size of the elements in object ('maxh'). Minimal size of elements varies due to Delaunay algorithm [3][6]. Near the objects boundaries mesh is significantly denser is order to assure proper solving of equations with high values of local derivatives.

Created model contained two coils and air sphere around them. Mesh density was changed both in air model as well as in the coils. Values of maxh parameter in created models and number of finite elements forming each model are presented in Table 1.

Tab.1 Number of nodes forming each elements

Air Maxh	Coil Maxh	No. Of Elements in Coil	No. Of Elements in Air	Relative error[%]
1	0,5	81	21878	5,514
	0,2	168	28239	1,003
	0,1	1238	47538	2,506
	0,05	8217	115505	1,253
	0,02	106332	404722	1,253
0,5	0,5	81	184744	7,519
	0,2	168	188249	1,003
	0,1	1238	209323	2,506
	0,05	8225	275457	1,253
	0,02	106336	566715	1,003
0,2	0,5	168	3232381	0,752
	0,2	168	3232381	0,251
	0,1	1252	3247910	4,261
	0,05	8341	3324974	2,506
	0,02	106574	3602608	1,253

Tests were conducted also on the five meshes created without 'maxh' parameter. Their density was changed in mesh-generating software (Netgen 5.3), as a 'mesh granularity' parameters. Five available options are:

- -Very Coarse
- -Coarse
- -Moderate
- -Fine
- -Very Fine

Exemplary view of low density mesh is presented in Figure 2, and high density mesh is presented in Figure 3.

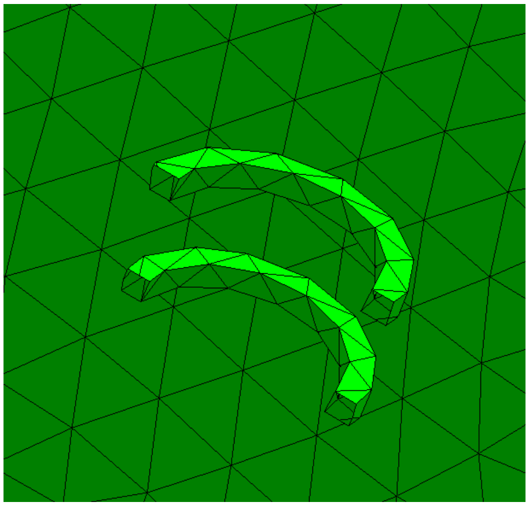

Fig.2 Clipped view of coarse mesh

3.2 Modeling results

Obtained magnetic flux distribution is consisted with data presented in literature [7]. Exemplary results of magnetic field modeling for average dense mesh are presented in Figure 4.

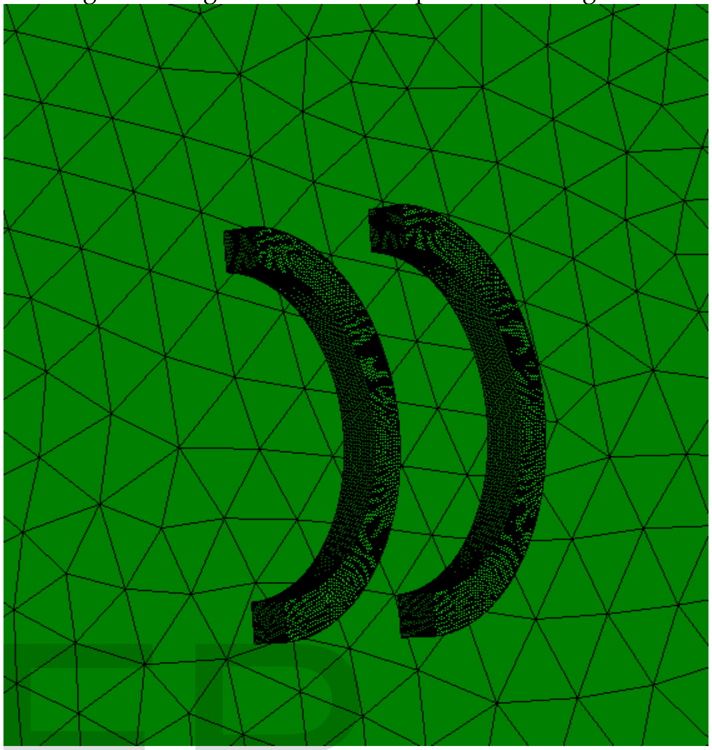

Fig.3 Clipped view of high density mesh

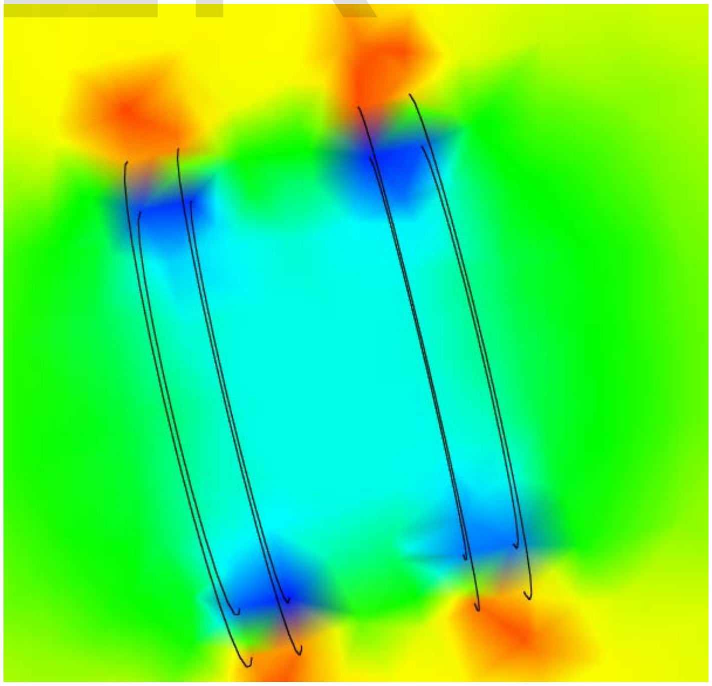

Fig.4 Magnetic flux distribution around the coils

4 RESULT ANALYZE

Results of simulations, compared with the number of elements in coil and in air are presented in Table 1. In order to visualize the influence of mesh density, two bubble graphs are presented in figure 5 and 6. In both graphs X axis represents the average number of elements forming each coil, where Y axis represents the number of elements in each sphere. The size of the bubble placed on (x,y) coordinates represents the relative error of simulation.

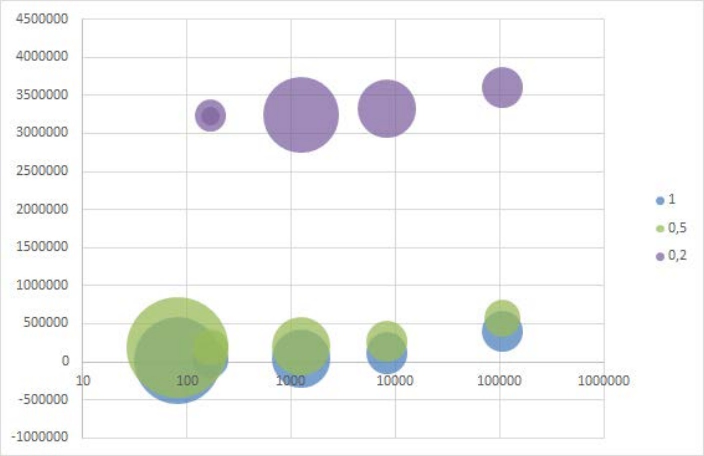

Fig.5 Error value of simulation error in the function of number of elements in coils and sphere for fixed size of elements.

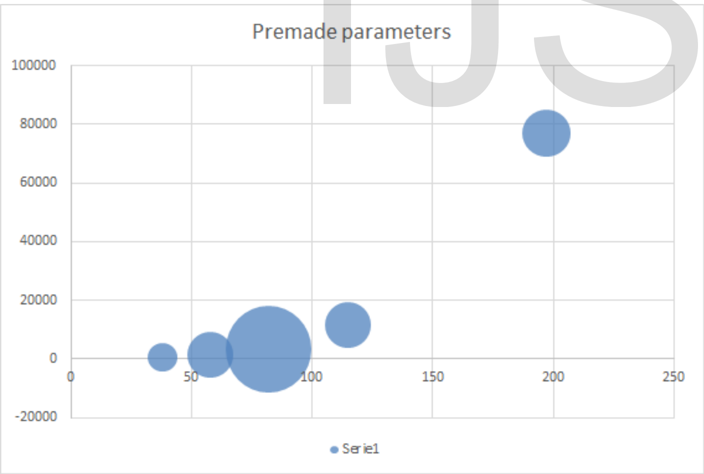

Fig.6 Error value of simulation error in the function of number of elements in coils and sphere for premade mesh parameters

As one can see, increase of the mesh density does not unequivocally improve the quality of the simulation. This is clearly visible for simulations conducted for model, in which 'maxh' parameter of coil was set to 0.1. All three simulations result with significantly bigger error than those conducted for more coarse meshes (with 'maxh' set to 0.2).
Utilization of too coarse mesh with premade elements size (meshes with coil 'maxh' set to 0.5) results in the greatest errors. This is caused by not accurate modeling of current distribution in the coils, which significantly influences the simulations. On the other hand, utilization of premade meshes parameters results with low errors, despite utilization of the lowest number of mesh elements (even the 'very fine' predefined mesh utilizes less elements than the coarse meshes with fixed element height). This is due to the meshing algorithm, which fluently modifies the size of the elements. Thus on the object boundaries mesh is significantly denser, than in more homogeneous space. This results with lower number of elements in mesh, without significant influence on modeling accuracy.

5 CONCLUSION

Presented simulations provided data, about influence of mesh density on electromagnetic simulations. Simulations were conducted in open-source FEM software, on typical model of Helmholtz coil. Obtained results were not unambiguous - simulation error varied in the function of the mesh density, but despite previously described situation (with coil 'maxh' set to 0.1), typically denser mesh returned more accurate results.
Also, the density of coils mesh had more influence on the modeling error. Denser air mesh results with the lower average error for analogously dense coils models.
On the other hand, paper focused on single value of magnetic flux in specific mesh point and did not consider the homogeneously of the results. For coarse mesh, even when the magnetic flux value in the midpoint of the coil setup was correct, the shape of magnetic was jerky. This issue will be analyzed in future work.

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