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Energy of Fuzzy Labeling Graph [EF_l(G)]- Part I

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Abstract—In this paper, we introduced energy of fuzzy labeling graph and its denoted by [EF_l(G)]. We extend the concept of fuzzy labeling graph to the energy of fuzzy labeling graph [EF_l(G)]. This result tried for some fuzzy labeling graphs such as butterfly graph, book graph, wheel graph, caterpillar graph, theta graph, Hamiltonian circuit graph , $K_2\ J\ \overline{K_2}$ graph, $K_3\ J\ \overline{K_3}$ graph and studied the characters.

Keyword: Labeling, fuzzy labeling graph, energy graph, energy of fuzzy labeling graph.

1 INTRODUCTION

Most graph labeling methods trace their origin to one introduced by Rosa [10] in 1967,and one given by Graham and Sloane [5] in 1980.Pradhan and Kumar [10] proved that graphs obtained by adding a pendent vertex of hair cycle C_n $\Theta K_1$are graceful if n≡ 0(mod4m).They further provide a rule for determining the missing numbers in the graceful labeling of $C_n \Theta K_1$and of the graph obtained by adding pendent edges to each pendent vertex of C_n ΘK_1. Abhyanker[2] proved that the graph obtained by deleting the branch of length 1 from an olive tree with 2n branches and identifying the root of the edge deleted tree with a vertex of a cycle of the form C_{2n+3} is graceful.In 1985 Koh and Yap[7] generalized this by defining a cycle with a P_k -chord to be a cycle with the path P_k joining two nonconsecutive vertices of the cycle.

Fuzzy relation on a set was defined by Zadeh in 1965. Based on fuzzy relation the first definition of a fuzzy graph was introduced by Rosenfeld and Kaufmann in 1973. Fuzzy graph have many more applications in modeling real time system where the level of information inherent in the system varies with different levels of precision.

The concept of energy graph was defined by I. Gutman in 1978. The energy, E(G), of a graph G is defined to be the sum of the absolute values of its eigen values. Hence if A(G) is the adjacency matrix of G, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of A(G), then E(G) = $\sum_{i=1}^{n} |\lambda_i|$. The set {$\lambda_1, \ldots, \lambda_n$ } is the spectrum of G and denoted by Spec G. The upper and lower bound for energy was introduced by R. Balakrishnan[3]. The totally disconnected graph K_n^c has zero energy while complete graph K_n with the maximum possible number of edges has energy 2(n-1). It was therefore conjectured in P.Prabhan and A. Kumar [9] that all graphs have energy at most 2(n-1). But then this was disproved in A. Nagoorgani, and D. Rajalaxmi (a) Subahashini, [8].

We generalise the energy of fuzzy labeling graph EF_l(G) for butterfly graph (13, 21) book graph (8,10), wheel graph(6, 10), (7,12), caterpillar graph(11,10) ,theta graph(6,7), Hamiltonian circuit graph(12 17), $K_2\ J\ \overline{K_2}$ (4,5) graphs, $K_3\ J\ \overline{K_3}$ (6,12)graph. And also find upper bounds and lower bounds for energy of fuzzy labeling graphs.

2 PRELIMINARIES

2.1 Labeling

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

2.2 Vertex Labeling

Given a graph G, an injective function f :V(G)→ N has been called a vertex labeling of G.

2.3 Edge Labeling

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An edge labeling of a graph is a bijection from E(G) to the set {1, 2. … , ∣E(G∣ }.

2.4 Fuzzy Graph

A fuzzy graph G =(σ,μ) is a pair of function σ: V →[0,1] and μ : V×V → [0,1], where for all u,v ∈ V,we have μ (u,v) ≤ σ (u) ∧ σ (v).

2.5 Fuzzy Labeling Graph

A graph G = (σ,μ) is said to be a fuzzy labeling graph if σ :V →[0,1] and u : V×V [0,1] is bijective such that the membership value of edges and vertices are district and u (u,v) < σ (u) ∧ σ(V) for all u,v ∈ V , [7], [6]

2.6 Energy Graph

Energy of a simple graph G = (V,E) with adjacency matrix A is defined as the sum of absolute values of eigenvalues of A. It is denoted by E (G). E(G) =$\sum_{i=1}^{n}|\lambda_i|$ where λ_i is an eigenvalues of A, i =1, 2, . . . , n.

2.7 Energy of Fuzzy Graph

Let G= (V,σ,μ) be a fuzzy graph and A be its adjacency matrix. The eigenvalues of A are called eigenvalues of G. The spectrum of A is called the spectrum of G. It is denoted by Spec G. Let G = (V, σ,μ) be a fuzzy graph and A be its adjacency matrix. Energy of G is defined as the sum of absolute values of eigenvalues of G[1].

3 MAIN RESULTS

3.1 Energy of Fuzzy Labeling Graph

The following three conditions are true if the graph is called a energy of fuzzy labeling graph. We denote $EF_l(G)$ be a energy of fuzzy labeling graph.

(i) $EF_l(G) =\sum_{i=1}^{n}|\lambda_i|$

(ii) μ (u, v) > 0

(iii) μ (u, v) < σ(u) ∧ σ(v)

(iv) Let F_l (G) be a fuzzy labeling graph with ∣V∣ = n vertices and μ = {e_1,…..e_m }. If m_i = μ(e_i) , then $\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$.

Theorem 1. *Let F_l (G) be a fuzzy labeling graph with ∣V∣ = n vertices and μ = {e_1,…..e_m }.If m_i = μ(e_i) , then*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$$

Proof. **Upper bound** for energy of fuzzy graph is same as an energy of fuzzy labeling graph. [1]

ie ., $EF_l(G) \leq \sqrt{2(\sum_{i=1}^{m} m_i^2)n}$

Lower bound

$$[EF_l(G)]^2 = F_l\ (\sum_{i=1}^{n}|\lambda_i|)^2$$

$$=F_l\ \sum_{i=1}^{n}|\lambda_i|^2 + 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|$$

$$= F_l\ 2\sum_{i=1}^{m} m_i^2 + 2\,\frac{n\,(n-1)}{2}\,AM\,\{|\lambda_i\lambda_j|\}$$

AM {$|\lambda_i\lambda_j|$} ≤ GM {$|\lambda_i\lambda_j|$} if F_l for $1\le i<j\le n$

$$EF_l(G) \le \sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)GM\,\{|\lambda_i\lambda_j|\}}$$

$$GM\{|\lambda_i\lambda_j|\} = (\prod_{1\le i<j\le n}|\lambda_i\lambda_j|)^{2/n(n-1)}$$

$$=(\prod_{i=1}^{n}|\lambda_i\lambda_j|^{n-1})^{2/n(n-1)}$$

$$=(\prod_{i=1}^{n}|\lambda_i|)^{2/n} = |A|^{2/n}$$

$$EF_l(G) \le \sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}}$$

Therefore, $\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$

Here, we found energy of fuzzy labeling graphs $EF_l(G)$, upper bound, and lower bounds of some graphs like Hamiltonian Circuit graph, Book Graph, Caterpillar Graph, Wheel Graph, Theta Graphand etc., and also made between them.

Theorem 2. *Fuzzy Labeling of $\boldsymbol{K_2 J K_2^-}$ satisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G)$$

for 4 vertices and 5 edges.

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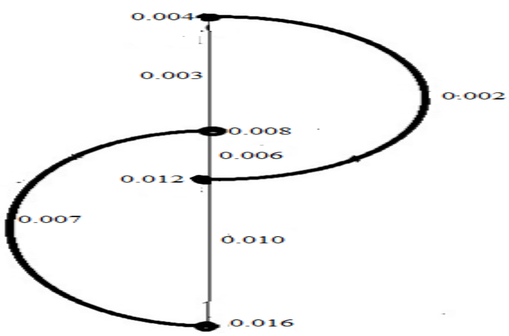


Adjacency matrix of the fuzzy labeling graph of (above) is

$$A = \begin{bmatrix} 0 & 0.003 & 0.002 & 0 \\ 0.003 & 0 & 0.006 & 0.007 \\ 0.002 & 0.006 & 0 & 0.010 \\ 0 & 0.007 & 0.010 & 0 \end{bmatrix}$$

The eigen values are = -0.0104, -0.0058, 0.003, 0.0159

The energy of the graph $EF_1(G)$ is = 0.0324

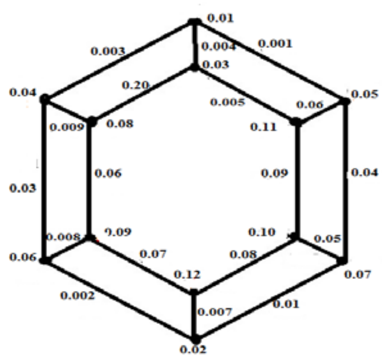
The upper bound of the graph is $EF_1(G)$ = 0.03979

The lower bound of the graph is $EF_1(G)$ = 9.6439

Theorem 3. *Fuzzy Labeling of Hamiltonian Circuit graph statisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_1(G)$$

for 12 vertices and 18 edges.



The eigen values are = -0.2721,0.2721,-0.0880,0.0900,0.0696,0.0361,0.0211,0,0,-0.0360, -0.0234,-0.0194

The energy of the graph $EF_1(G)$ is = 0.9278

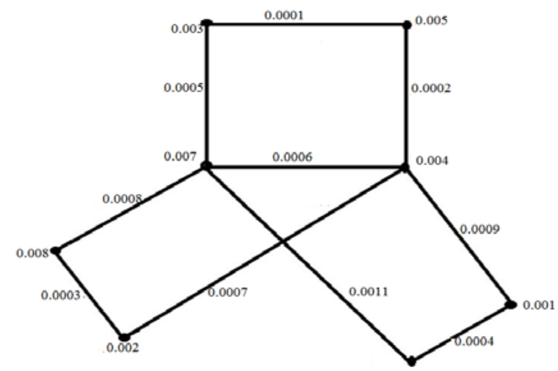
The upper bound of the graph is $EF_1(G)$ = 0.8811

The lower bound of the graph is $EF_1(G)$ =19.74273

Theorem 4. *Fuzzy Labeling of Book Graph statisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_1(G)$$

for 8 vertices and 10 edges.



The eigen values are = -0.0018,-0.0008,-0.0003,-0.0001,0.0001,0.0003,0.0008,0.0018

The energy of the graph $EF_1(G)$ is = 0.006

The upper bound of the graph is $EF_1(G)$ = 0.0080

The lower bound of the graph is $EF_1(G)$ = 20.5869

Theorem 5. *Fuzzy Labeling of Caterpillar Graph satisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_1(G)$$

for 11 vertices and 10 edges.

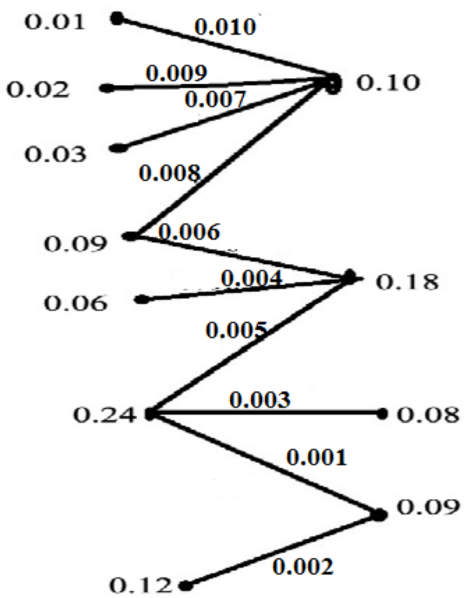
Solution of Caterpillar graph generalized. Below values verified for the same.

The eigen values are = 0.0374,-0.0374,0,0,-0.1827,0.1827,-0.0918,0.0918,0.0372, -0.0372,0

The energy of the graph $EF_1(G)$ is = 0.6982

The upper bound of the graph is $EF_1(G)$ = 0.9322

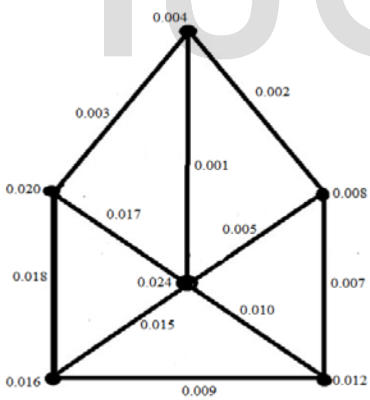
The lower bound of the graph is $EF_1(G)$ = 11.023



Theorem 6. *Fuzzy Labeling of Wheel Graph satisfy $EF_1(G)$ <*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$$

vertices and 10 edges.



The eigen values are = -0.0218,-0.0158,-0.0069,0.0008,0.0063,0.0374

The energy of the graph $EF_1(G)$ is = 0.089

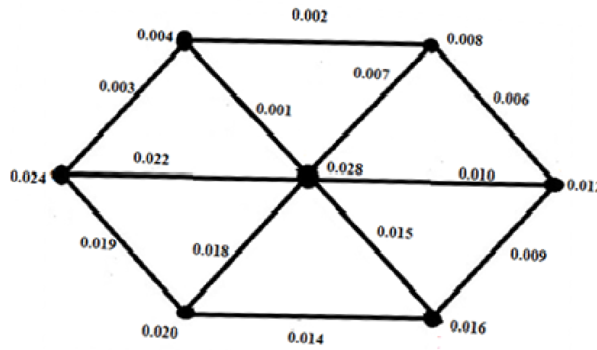
The upper bound of the graph is $EF_1(G)$ =0.5396

The lower bound of the graph is $EF_1(G)$ =11.131

Theorem 7. *Fuzzy Labeling of Wheel Graph satisfy $EF_1(G)$ <*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$$

vertices and 12 edges.



The eigen values are =-0.0286,-0.0208,-0.0089,-0.0021,0.0029,0.0094,0.0481

The energy of the graph $EF_1(G)$ is = 0.1208

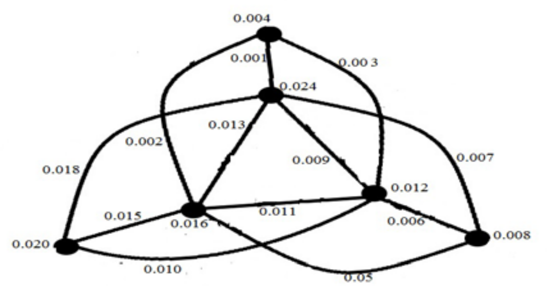
The upper bound of the graph is $EF_1(G)$ = 0.1069

The lower bound of the graph is $EF_1(G)$ =10.16180

Theorem 8. *Fuzzy Labeling of* $\boldsymbol{K_3 J K_3^-}$ *satisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$$

for 6 vertices and 12 edges.



The eigen values are = -0.0523,-0.0177,-0.0079,0.0005,0.0148,0.0625

The energy of the graph $EF_1(G)$ is = 0.1557

The upper bound of the graph is $EF_1(G)$= 0.5396

The lower bound of the graph is $EF_1(G)$= 11.3149

Theorem 9. *Fuzzy Labeling of Theta Graph satisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$$

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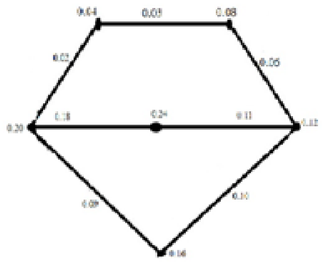
$EF_1(G)$ for 5 vertices and 7 edges.



The eigen values are = 0.2532,-0.2493,-0.0658,0.0515,0,0

The energy of the graph $EF_1(G)$ is = 0.6198

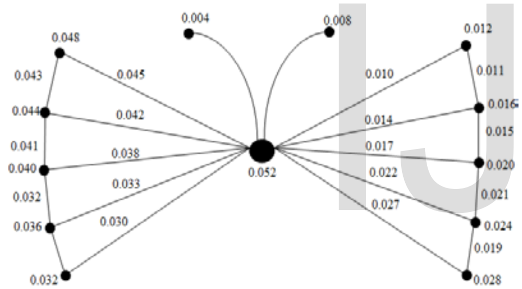
The upper bound of the graph is $EF_1(G)$ = 0.9

The lower bound of tha graph is $EF_1(G)$ =11.32

Theorem 10. *Fuzzy Labeling of Butterfly Graph satisfy*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$$

for 13 vertices and 20 edges.



The eigen values are = -0.0793,-0.0612,-0.0373,-0.0319,

-0.0179,0,0,0,0.0028,0.0190,0.0374, 0.0402,0.0183

The energy of the graph $EF_1(G)$ is = 0.4553

The upper bound of the graph is $EF_1(G)$= 0.41870

The lower bound of the graph is $EF_1(G)$ =20.5869

4 CONCLUSION

In this paper, we find some energy of fuzzy labeling graphs and to the energy level is $\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} > \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n} > EF_1(G)$. Our future work are s to apply π electron in fuzzy labeling and also discuss with other labeling graphs .

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