Determination of Lame's Constants of Surface soils and Shallow Sediments from Seismic Wave Velocities

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Abstract

This paper presents the results of seismic compressional, P- and shear, S-wave measurements carried out on the unconsolidated top-soil at the different locations of the study area to determine Lame's constants (μ and λ). The seismic refraction method employing P-wave and S-wave sources were used to generate seismic energies that propagated through the subsurface. A 12-channel seismograph with signal stacking ability was used together with high frequency (100 Hz) geophones on the top-soil. The geophone intervals were set to 5 m at all the locations. The results of the findings based on these parameters were Lame's first constant, μ (1.3751 x 10^8 N/m^2 – 2.8989 x 10^8 N/m^2) for first layer and (2.9294 x 10^8 N/m^2 – 7.0209 x 10^8 N/m^2) for the second layer; Lame's second constant (-2.023 x 10^8 N/m^2 – 2.7290 x 10^8 N/m^2) for first layer and (-3.9762 x 10^8 N/m^2 - 3.000 x 10^8 N/m^2) for second layer. The low values of these constants are symptomatic of occurrence of ripable anisotropic materials in the locations where they occur. The average depth of

the top and weathered zone should be removed and refilled with geomaterials that may be resilient to carry engineering loads.

Keywords: Seismic refraction, Lame's constants, Eket, P-wave, S-wave

Introduction

In continuum mechanics, the lame parameters (also called the lame coefficients or lame constants) are two material-dependent quantities denoted by λ and μ that arise in strain-stress relationships [1]. In general, λ and μ are individually referred to as Lame's first parameter and Lame's second parameter, respectively. Other names are sometimes employed for one or both parameters, depending on context. The parameter μ is sometimes referred to in fluid dynamics as the dynamic viscosity of a fluid; whereas in the context of elasticity, μ is called the shear modulus [2].

The two parameters together constitute a parameterization of the elastic moduli for homogeneous isotropic media, popular in mathematical literature, and are thus related to the other elastic moduli. Although the shear modulus, μ, must be positive, the Lame's first parameter, λ, can be negative, in principle; however, for most materials it is also positive.The Lame's constants depend on the material and its temperature.

The constants λ and μ arise in strain- stress relationships. They are given in terms of other solid properties as

$$\lambda \equiv \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{1}$$

$$= K - \tfrac{2}{3}G \tag{2}$$

$$= \frac{2\nu G}{1-2\nu} \tag{3}$$

$$= 3K\frac{\nu}{1+\nu} \tag{4}$$

$$= \rho(v_p^2 - 2v_s^2) \tag{5}$$

$$\mu \equiv \frac{E}{2(1+\nu)} \tag{6}$$

$$= \tfrac{3}{2}(K-\lambda) \tag{7}$$

$$= \lambda\frac{1-2\nu}{2\nu} \tag{8}$$

$$= 3K\frac{1-2\nu}{2+2\nu} \tag{9}$$

$$= \rho v_s^2, \tag{10}$$

where *E* is Young's modulus, ν is the Poisson ratio, (*G*, μ) is the shear modulus, *K* is the bulk modulus, ρ is the density, v_p is P-wave speed, and v_s is the S-wave speed [3,1].

Location and Geology of the Study Area

The study area, Eket lies between latitudes 4˚37' and 4˚7'N and longitudes 7˚38' and 8˚00'E. It has an estimated area of 214 km^2

(Figure 1). The study area belongs to the low-lying coastal deltaic plains of southern Nigeria [4]. The terrain is virtually flat to gently undulating, sloping generally towards the Atlantic Ocean. Elevation varies from about 100 to 120 m at the northern part of the study area to near sea level at the southern part [5]. The surface drainage basin within the study area is mainly due to the Qua Iboe River which drains the western part [6]. Geologically, the study area falls within the Niger Delta area.

The geologic formation in the Niger Delta area is made up of the Akata Formation (shales, intercalated sands and sandstones), the Agbada Formation (sands and sandstones intercalated with shales) in the middle and the Benin Formation (Coarse grained, gravelly sands with minor intercalations with clay) at the top [7]. However, only the Benin Formation otherwise called the coastal plain sands is exposed in the study area where the investigation concentrates. The coastal plain sand covers 80 percent of the area and forms the major aquiferous and foundation zones of the study area. The area is generally porous and permeable and this is usually interrupted by clay-sand sequence at different depths [8] [9].

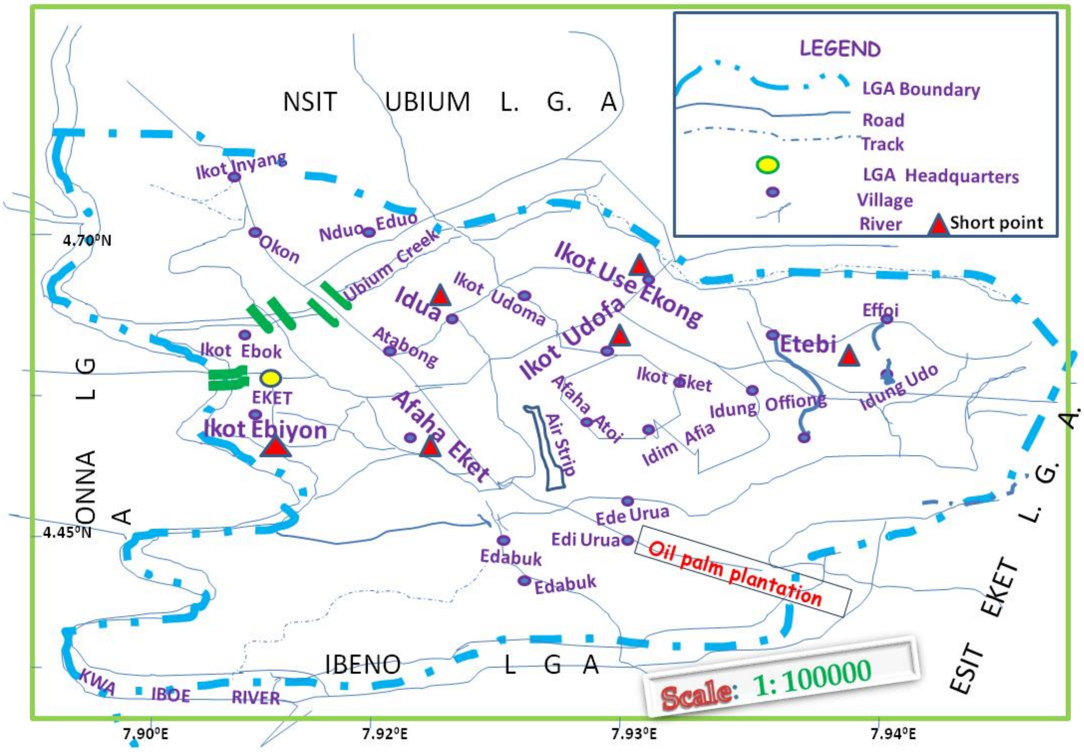

Figure 1: Location map of study area

Methodology

The seismic study was based on recording the travel time of an elastic wave travelled through the ground, refracted from a subsurface, and received via geophones on the surface. Seismic P- and S-waves were recorded using vertical and horizontal components geophones (100 Hz), laid along a line with 5 m spacing, and connected to a 12-channel digital recorder. The required energy for P-wave measurement was created by a sledge hammer hitting the wooden cone at the shot points. The S-wave was created by hitting the ends of a flat-lying wooden timber loaded by a person, the weight of which increases the friction and the contact area of the timber with the ground surface [10].

The data obtained from the seismograph (seismogram) were processed using Pickwin software to obtain the arrival time for P-wave and S-wave. With the geophone separations of 5 m interval, T-X graph were plotted for the different locations using IX Refrax software and the inverse of the slope were obtained as velocity for each of the layer penetrated [11]. The Pickwin software helped in picking the arrival times while the IX Refrax directly converted the slope into velocity for the different layers. The IX Refrax also gave the depth of each of the layers penetrated.

Result and Discussion

To actually map the Lame's constants which depend on lithology, the V_P/V_S ratios usually viewed as lithology discriminator were determined for each location (table 1). For compressional and shear waves, the velocity increases with depth. The underlain velocities were all greater than the overlain velocities at all the locations. In all the locations the velocities ranged between 297.0 and 383.0 m/s for the top layer and the mean value of 355.1 m/s in P-wave. The range of the second layer P-wave velocity was 462.0 to 861.0 m/s and the average was 597.8 m/s. The range of S-wave velocity for the first layer was 250.0 to 363.0 m/s and the mean value was 309.6 m/s. For the second layer, the velocity ranged from 325.0 to 552.5 m/s and the mean value for S-wave velocity in this layer was 424.6 m/s. In agreement with the theoretical findings, the P-wave velocity was greater than the S-wave velocity in all the locations.

Table 1: Summary of P-wave, S-wave and V_p/V_s ratio and Lame's constants (μ and λ)

Location/ name	Lat^0	Long^0	Layer	V_p(m/s) mean	V_s(m/s) mean	V_p/V_s	μ(N/m^2)	λ(N/m^2)
Idua	4.6760	7.9256	L_1	357.5	303.0	1.1709	2.0198	-1.2252
			L_2	563.5	325.0	1.5492	2.9294	1.0137
Afaha Eket	4.6800	7.9145	L_1	350.0	333.5	1.0495	2.4469	2.1986
			L_2	650.0	410.5	1.5834	3.8757	1.9665
Ikot Ebiyon	4.6667	7.9147	L_1	369.5	338.0	1.0932	2.5134	-2.0230
			L_2	462.0	439.5	1.0512	4.4427	-3.9762
Ikot Use Ekong	4.6636	7.9156	L_1	383.0	270.0	1.4185	1.6038	0.0373
			L_2	509.0	430.5	1.1823	4.2626	2.5667
Ikot Udofia	4.6206	7.9325	L_1	373.5	363.0	1.0289	2.8989	2.7290
			L_2	861.0	552.5	1.5584	7.0209	3.0090
Etebi	4.6125	7.9419	L_1	297.0	250.0	1.1880	1.3751	0.9635
			L_2	541.0	389.5	1.3890	3.4893	-0.2465

The ratio of $\frac{V_p}{V_s}$, which is a lithology discriminator, was determined for each of the locations and the values seem to reflect the porous and air-filled environment. The lame constants are quite sensitive to the $\frac{V_p}{V_s}$ ratio and the variations in its values reflect a high degree of anisotropy which is glaringly manifested in the negativity of the Lame's constants in some locations. Using the mean layer velocities, the V_p/V_s ratios were estimated and the Lame's constants (Shear modulus, (μ) and (λ) were computed.

For shear modulus, 3-D plots have been used to show the distribution in the first and second layers in figures 2 and 3 respectively. For layer 1, the shear modulus is higher in the Southern region whereas in the North and Northeastern region, lower values were obtained. In layer two, the same pattern of increment is also observed for shear modulus.

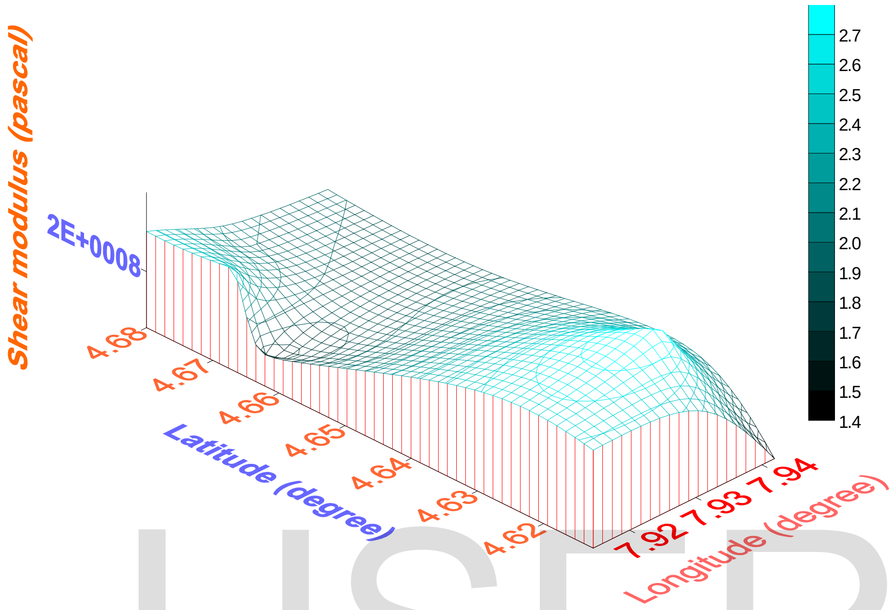

Figure 2: 3-D contour map of layer one first lame's constant (Shear modulus, μ) in the study area

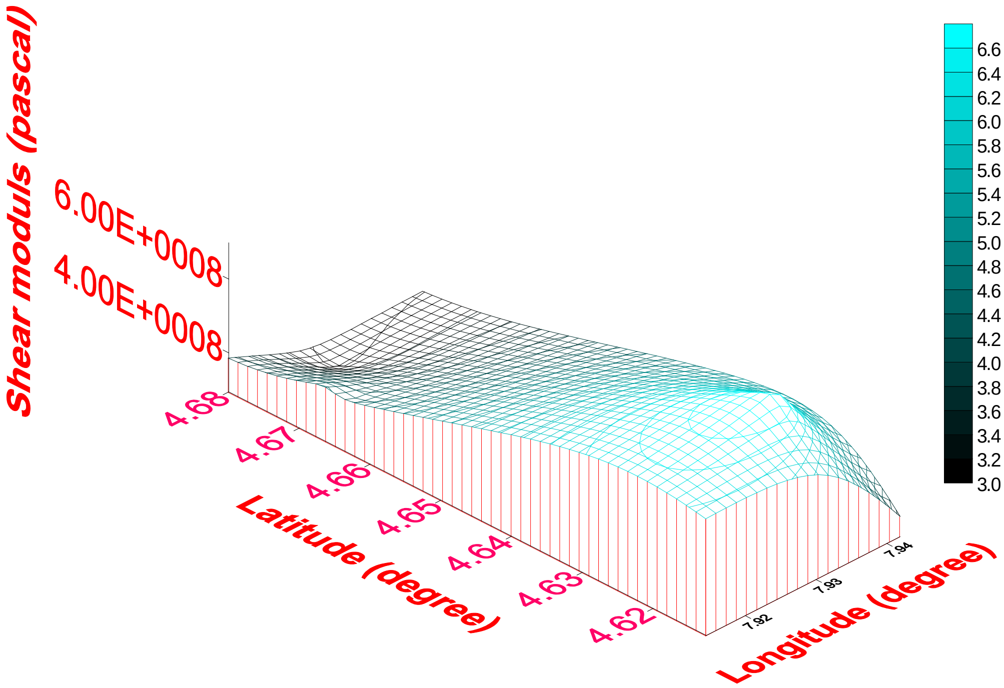

Figure 3: 3-D contour map of layer two first lame's constant (Shear modulus, μ) in the study area

Lame's second constant (λ) in layer (figure 4) one seems higher except in the northeastern region where anisotropy has affected the value heavily. In the second layer (figure 5), the Lame's constant (λ) is also high at the Southern zone but lower at the Northern zone, mostly due to high anisotropy.

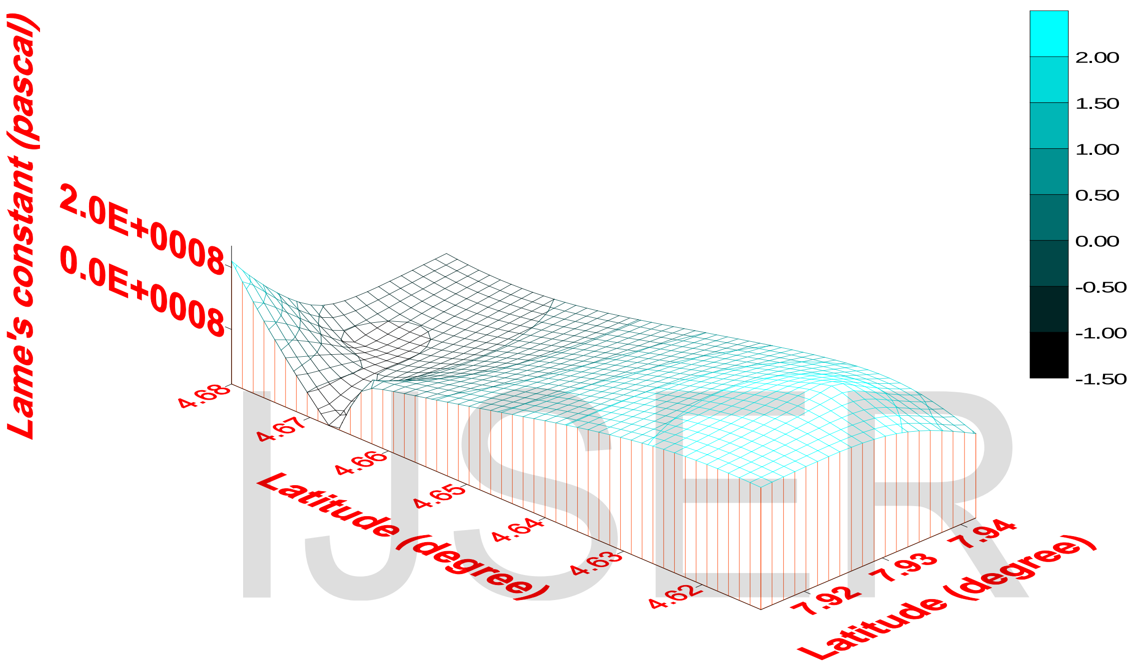

Figure 4: 3-D contour map of layer one second Lame's constant (λ) in the study area

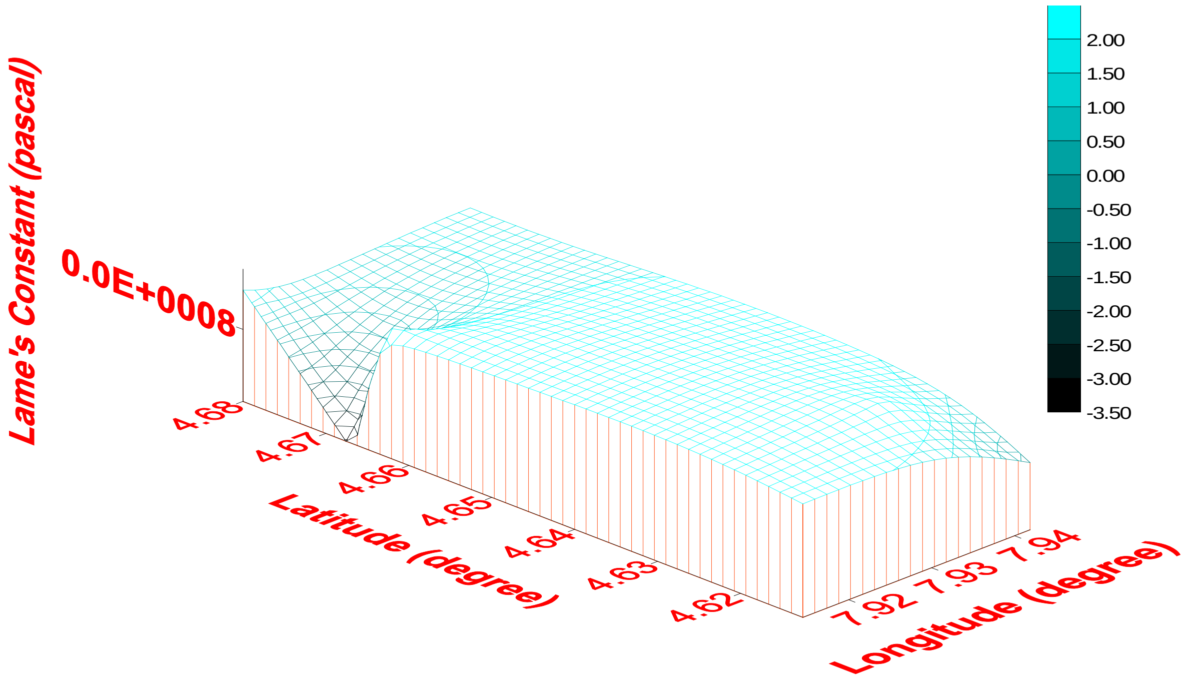

Figure 5: 3-D contour map of layer two second Lame's constant (λ) in the study area

Conclusion

Geophysical testing has increasingly been useful and economical since its results can represent the true geological conditions of a geological province. In this study, P and S wave velocities determined for the top soil of the weathered zone by carrying out seismic refraction survey were used to determine the first and second Lame's constants. The results obtained shows that a reasonable thickness of the top layer is porous, swampy, air-filled and weak according to the determined velocities. The $\frac{V_p}{V_s}$ ratios were generally less than $\sqrt{2}$ in layer one and some locations in layer two were slightly greater than $\sqrt{2}$. Due to the possibility of lithologic changes, wildcat engineering use of the top and weathered soil for construction (road and building) should be discouraged. Rather, geophysical and geological information should be the guideline for engineering construction in the study area.

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