Critical Path in a Project Network using TOPSIS Method and Linguistic Trapezoidal Fuzzy Numbers

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Abstract:- The intent of this paper, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is implemented for project management to get to the foundation of the critical path in the network of a fuzzy project. Linguistic Trapezoidal fuzzy numbers are appended to establish the final evaluation value of fuzzy activity times for each path in the fuzzy project network. A numerical paradigm is furnished to illuminate the procedure of the TOPSIS method insinuates and regulating the critical path with distinctive yardsticks.

Key words:- Trapezoidal fuzzy numbers, TOPSIS, Project network, Critical path.

1 Introduction

Fuzzy TOPSIS is deemed a standout at the core of most ascertained techniques so as to embark upon Multi Media Decision Making (MCDM). Hwang and Yoon [11] commenced fuzzy TOPSIS. It converges on the postulation that the elective picked up should be at the longest distance from the deleterious perfect outcome, thence the end result that extends the cost criteria and lessens the benefit criteria alongside the momentary division from the positive flawless outcome, where the outcome that expands the criteria of benefit and trims down the criteria of expenditure. The encumbrances and reconnaissance of the criteria are recognized precisely in the settled TOPSIS. Conversely, it was stated by Hwang and Yoon[11] that, although under the genuine set of circumstances, crisp information is insufficient to make plain the regular circumstance since human intercessions are tentative and they cannot be appraised with befitting numeric attributes. So as to envision the incongruity which sprouts up customarily in information from human implications, fuzzy set speculation has been integrated in inestimable MCDM procedures that encompass TOPSIS. In fuzzy TOPSIS, all the weights and evaluation are described by the technique for semantic variables. Numbers of fuzzy TOPSIS methods and procurements have been stimulated of late. Chen and Hwang [4] had united fuzzy numbers in their work for the first time to coin TOPSIS. Triantaphyllou, E. and Lin, C.L [23], originated a technique in fuzzy TOPSIS by which the relative proximity of every locum is weighed and concentrated upon juggling operations of fuzzy numbers. Liang [18] was in favour of fuzzy MCDM focused around ideal and anti-ideal perceptions. Chen [2] owned triangular fuzzy numbers and typified crisp Euclidean disjunction between two fuzzy numbers to stretch the TOPSIS strategy to fuzzy GDM environment. Chu [7] and Chu and Lin [8] once again fostered the technique made known by Chen [2]. Chen and Tsao[6] strove to amplify the stratagem aimed at interval esteemed fuzzy sets in the analysis of decision. Jahanshahloo etal. [12], alongside Chu and Lin [9], gave an enhancement to the fuzzy TOPSIS technique which has been cantered on alpha level sets with interim number-crunching. . The theory has been enlarged by Chen and Lee [5] coming to grips with sort-2 fuzzy TOPSIS system, bearing in mind the end goal to offer extra level of opportunity to advocate for the susceptibilities and fuzziness of this reality of the present day. Fuzzy TOPSIS has been presented for a range of multi-characteristic issued to the choices of production. Yong [25] had put to use fuzzy TOPSIS to fix the plant area and the very same year, Chen and others employed it to regulate the suppliers. Kahraman and etal[13]. had made use of fuzzy TOPSIS for picking up the mechanical automated framework. An interim-esteemed fuzzy TOPSIS criterion that targets at taking care of MCDM issues in which the weights of the criteria are asymmetrical and that uses ideas of interim-esteemed fuzzy sets, has been employed by Asthiani [1]. Ekmekcioglu etal. [10] had put forth a modified fuzzy TOPSIS for selecting a strong waste transfer and site. Kutlu and Ekmekcioglu[16],had endeavoured to coordinate fuzzy TOPSIS and fuzzy AHP to propose another FMEA disappointed modes and impacts dissection that permits the conquering of the deficiencies of classical FMEA. Once again in Kaya and Kahraman [14] had proposed of an altered fuzzy TOPSIS for selecting the best vitality elective of engineering. By using Fuzzy TOPSIS by Kim etal.[15].

In this section, an algorithm based on fuzzy TOPSIS method is projected, to prefer the critical path under the four criteria. In section-2, some fundamental definition of trapezoidal fuzzy number and its Arithmetic operations like addition, subtraction, multiplication and division were discussed. In section 3, concentrate on linguistic variables and its arithmetic operation between trapezoidal fuzzy numbers. Section 4, presented a new distance between the trapezoidal fuzzy numbers using centroid of centroids. In section 5 an algorithm to deal with the critical path selection problem in the project network method. In section 6 exemplify the projected algorithmic method.

2 Basic definitions

In this segment, some basic definitions of fuzzy sets, fuzzy numbers are reviewed from Rao, P.B etal. [20], S.H,Chen [22].

Fuzzy set

Let X is the space of positive real values associated with variable and X is a generic element of X. A fuzzy set $\tilde{A}$ in X defined as the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$ such that $\mu_{\tilde{A}} : X \to [0,1]$

Fuzzy number

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics

(i) $\tilde{A}$ is convex i.e.,

$$\tilde{A}(\lambda x_1 + (1-\lambda) x_2) \geq \min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$$

for all $x_1, x_2 \in R$

(ii) $\tilde{A}$ is normal i.e.,

$\exists\, x_0 \in R$ Such that $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}(x)$ is piece wise continuous

A fuzzy number $\tilde{A}$ is called non negative number if $\mu_{\tilde{A}}(x) = 0 \ \forall x < 0$

Trapezoidal fuzzy number

A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be Trapezoidal fuzzy number, if it is a convex set which is defined as $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a, \\ \dfrac{w(x-a)}{b-a}, & a < x \le b, \\ w & b < x \le c, \\ \dfrac{w(x-d)}{c-d}, & c < x \le d, \\ 0, & \text{otherwise}, \quad \text{where } 0 \le w \le 1 \end{cases}$$

Generalized trapezoidal fuzzy number

A fuzzy set $\tilde{A}$ defined on the universal set of real number R, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : X \to [0, w]$ is continuous

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a] \cup [d, \infty]$

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$

(iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [b,c]$, $0 \le w \le 1$.

Normal trapezoidal fuzzy numbers

If $w = 1$ then $\tilde{A} = (a, b, c, d; 1)$ is a normalized fuzzy number.

Arithmetic operation between trapezoidal fuzzy numbers

Addition and subtraction of any two trapezoidal fuzzy numbers is a trapezoidal fuzzy numbers but the multiplication of any two Trapezoidal fuzzy number is only an approximate trapezoidal fuzzy number. Two, positive trapezoidal fuzzy numbers, $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$,and a positive real number k ,the operation between the trapezoidal fuzzy numbers $\tilde{A}$ and can be as follows:

$$\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \quad (1)$$

$$-\tilde{B} = (-d_2, -c_2, -b_2, -a_2) \quad (2)$$

$$\tilde{A} - \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \quad (3)$$

$$k(.)\tilde{A} = (ka_1, kb_1, kc_1, kd_1) \quad (4)$$

$$\tilde{A} \times \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2) \quad (5)$$

$$\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}\right) \quad (6)$$

Fuzzy matrix

A matrix $\tilde{D}$ is a fuzzy matrix if at least one element in a matrix is a fuzzy number.

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3. Linguistic variables and its arithmetic operations

A linguistic variable is a variable values of which are expressed in linguistic terms. The concept of using a linguistic variable comes very handy in dealing with situations that are very complex or ill-defined to be reasonable described in conventional quantitative. For example, "weight" is a linguistic variable whose values are very low, low, medium, high, very high as presented in the Table-1 respectively. Wel and Chen [21] restricted the generalized Trapezoidal fuzzy numbers to linguistic Trapezoidal fuzzy number represented by $(a,b,c,d;w)$ where $(a \le b \le c \le d \le 1 : 0 < w < 1)$, Madhuri, K. Usha etal. [17] presented a new arithmetic operation of the linguistic Trapezoidal fuzzy numbers $(a,b,c,d;w)$ $(a \le b \le c \le d \le 1 : 0 \le w \le 1)$

$\tilde{A}=(a_1,b_1,c_1,d_1 : w_1)$ $\tilde{B}=(a_2,b_2,c_2,d_2 : w_2)$ two linguistic Trapezoidal fuzzy numbers

$$\tilde{A} \oplus \tilde{B}=(a_3,b_3,c_3,d_3 : w_3) \qquad (7)$$

$$a_3 = a_1 + a_2 - a_1 a_2$$

$$b_3 = a_1 + a_2 + (b_1 - a_1)\frac{w_3}{w_1} + (b_2 - a_2)\frac{w_3}{w_2} - b_1 b_2$$

$$c_3 = d_1 + d_2 - (d_1 - c_1)\frac{w_3}{w_1} - (d_2 - c_2)\frac{w_3}{w_2} - c_1 c_2$$

$$d_3 = d_1 + d_2 - d_1 d_2 \text{ , } w_3 = \min(w_1, w_2).$$

$$\tilde{A} \otimes \tilde{B}=(a_1,b_1,c_1,d_1 : w_1) \otimes (a_2,b_2,c_2,d_2 : w_2)$$

$$=(a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2 : w_4) \; w_4 = \min(w_1, w_2)$$

$$\tilde{A} \varnothing \tilde{B}=(a_1,b_1,c_1,d_1 : w_1) \varnothing (a_2,b_2,c_2,d_2 : w_2)$$

$$=(a_4,b_4,c_4,d_4 : w_4) \qquad (8)$$

Where

$$a_4 = \frac{a_4}{d_2}, b_4 = \frac{b_1}{c_2}, c_4 = \frac{c_1}{b_2}, d_4 = \frac{d_1}{a_2},$$

$$w_4 = \min(w_1, w_2) \text{ , } a \varnothing b = \begin{cases} a/b, a<b \\ 1, else \end{cases}.$$

Table-1 Linguistic Variables

Linguistic Variables	Trapezoidal Fuzzy Number
Very low(V.L)	(0,0.1,0.2,0.3)
Low(L)	(0,0.1,0.3,0.5)
Medium low(M.L)	(0.1,0.3,0.5,0.7)
Medium(M)	(0.3,0.5,0.7,0.9)
Mediumhigh(M.H)	(0.5,0.7,0.8,0.9)
High(H)	(0.6,0.7,0.9,1)
Very high(V.H)	(0.7,0.8,0.9,1)

4. Proposed distance between trapezoidal fuzzy numbers

Let $\tilde{A}$ to be a given trapezoidal fuzzy number such that $\tilde{A} = (a_1, a_2, a_3, a_4)$ then the centroid of centriods point of $\tilde{A}$ is obtained from Shankar, N. Ravi etal. [19].

$$G_{\tilde{A}} = \left(\frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, \frac{4w}{9} \right) = \left(\varepsilon_{\tilde{A}}, \varepsilon'_{\tilde{A}} \right) \quad (9)$$

Where $\varepsilon_{\tilde{A}} = \dfrac{a_1 + 2a_2 + 5a_3 + a_4}{9}$, $\varepsilon'_{\tilde{A}} = \dfrac{4w}{9}$.

Hence, for any triangular fuzzy number its centroid of centroids can be written as

$$G_{\tilde{A}} = \left(\frac{a_1 + 7a_2 + a_4}{9}, \frac{4w}{9} \right) = \left(\varepsilon_{\tilde{A}}, \varepsilon'_{\tilde{A}} \right) \qquad (10)$$

where $\varepsilon_{\tilde{A}} = \dfrac{a_1 + 7a_2 + a_4}{9}$ and $\varepsilon'_{\tilde{A}} = \dfrac{4w}{9}$

A briefly review about the relation between trapezoidal fuzzy number and its centroid formula is given. Let us consider the left and right spreads, $(l_{\tilde{A}}, r_{\tilde{A}})$ where $l_{\tilde{A}} = a_2 - a_1$ and $r_{\tilde{A}} = a_4 - a_3$ and centroid of centroids point $(cc_{\tilde{A}}, c'c'_{\tilde{A}})$, where

$$cc_{\tilde{A}} = \frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, c'c'_{\tilde{A}} = \frac{4w}{9}. \quad (11)$$

A new distance measure for trapezoidal fuzzy numbers using their centroid of centroids point and left - right spread were proposed .Let us consider trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ with centroid of centroids points $(cc_{\tilde{A}}, c'c'_{\tilde{A}})$ and $(cc_{\tilde{B}}, c'c'_{\tilde{B}})$, left and right spreads $(l_{\tilde{A}}, r_{\tilde{A}})$ and $(l_{\tilde{B}}, r_{\tilde{B}})$ respectively. The distance measure of Trapezoidal fuzzy numbers

$$f_d\left(\tilde{A},\tilde{B}\right)=\max\left\{\left|cc_{\tilde{A}}-cc_{\tilde{B}}\right|,\left|l_{\tilde{A}}-l_{\tilde{B}}\right|,\left|r_{\tilde{A}}-r_{\tilde{B}}\right|\right\} \quad (12)$$

The following Figure.1 represents the Trapezoidal f fuzzy number having centroid of centroids

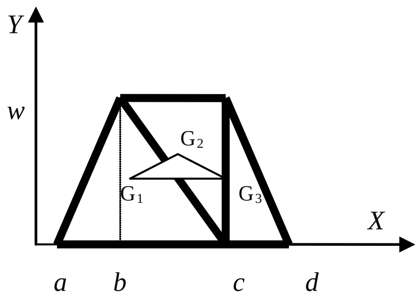

Fig 1. Trapezoidal fuzzy number

5 Fuzzy TOPSIS method

The protocol is addressed to as Technique for Order Preference by Similarity of ideal Solutions pronounced as fuzzy TOPSIS. This procedure is used to weigh up multiple alternatives against the selected criteria. One of the alternatives which is nearest to the Fuzzy positive ideal solution (FPIS) A^+ and farthest from the Fuzzy negative ideal solution (FNIS) A^- have a preference as the most propitious. An all-embracing explanation and the behaviour of TOPSIS is discoursed by Chen and Hwang [4], and in reference to Hwang and Yoon [17]. A protracted version of TOPSIS has been proposed by Chen etal. [3]. This method of fuzzy TOPSIS is capable of dealing with the grading of both quantitative as well as qualitative criteria and can meritoriously pick out an opposite alternative. Therefore the TOPSIS method is very adaptable. As per the proximity of the coefficient, not just the order of ranking but the status of appraisal can be resolute as well. The fuzzy TOPSIS method Chen etal [3] is put forward in order to discover the best alternative. This procedure is furnished as in the following steps;

The decision group has K members, the k^{th} decision maker's ratings and imperative weights are the i^{th} alternative on j^{th} criterion epitomizes as

$\tilde{x}_{ij}^k=\left(a_{ij}^k,b_{ij}^k,c_{ij}^k,d_{ij}^k\right)$ and

$\tilde{w}_j^k=\left(w_{j1}^k,w_{j2}^k,w_{j3}^k,w_{j4}^k\right)$ respectively.

Where $i=1,2,...,m$, $j=1,2,...,n$ .The aggregated fuzzy rating $\tilde{x}_{ij}$ of alternatives (i) with respect to each criterion(j) are given by

$\tilde{x}_{ij}=\left(a_{ij},b_{ij},c_{ij},d_{ij}\right)$ Such that $a_{ij}=\min\limits_k\left\{a_{ij}^k\right\}$

$b_{ij}=\frac{1}{K}\sum\limits_{k=1}^{K}b_{ij}^k$, $c_{ij}=\frac{1}{K}\sum\limits_{k=1}^{K}c_{ij}^k$,

$d_{ij}=\max\limits_k\left\{d_{ij}^k\right\}$ .The aggregated fuzzy weights $\tilde{w}_{ij}$ of each criterion are calculated as

$\tilde{w}_j^k=\left(w_{j1},w_{j2},w_{j3},w_{j4}\right)$ where

$w_{j1}=\min\limits_k\left\{w_{jk1}\right\}$, $w_{j2}=\frac{1}{K}\sum\limits_{k=1}^{K}w_{jk2}$

$w_{j3}=\frac{1}{K}\sum\limits_{k=1}^{K}w_{jk3}$, $w_{j4}=\max\limits_k\left\{w_{jk4}\right\}$

Fuzzy multi criteria group decision making problems can be expressed in matrix form

$$\tilde{D}=\begin{matrix} \\ A_1 \\ A_2 \\ \cdots \\ A_m\end{matrix}\begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix}\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \cdots & \cdots & \tilde{x}_{ij} & \cdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn}\end{matrix}\right]\end{matrix}$$

$$\tilde{W}=\left(\tilde{w}_1,\tilde{w}_2,...,\tilde{w}_n\right)$$

where $\tilde{x}_{ij}$ and $\tilde{w}_j$, $i=1,2,...,m$, $j=1,2,...,n$ Are the linguist variables which can be represented by trapezoidal fuzzy numbers as of the form $\tilde{x}_{ij}=\left(a_{ij},b_{ij},c_{ij},d_{ij}\right)$ and

$\tilde{w}_j=\left(w_{j1},w_{j2},w_{j3},w_{j4}\right)$

Step 1

Let a normalized fuzzy decision matrix be constructed:

The process used is of linear-scale in order to avert the impediment of mathematical operations and to transfigure a range of scales of yardsticks into analogous scales. These benchmark sets can be categorized into benefit criteria wherein the preference would be increasing with the increase in the rating as well as cost criteria wherein the rating and the preference are inversely proportional. Hence, the normalized fuzzy decision matrix can be characterized as

$\tilde{R}=\left[\tilde{r}_{ij}\right]_{m\times n}$ where

$$\tilde{r}_{ij}=\left(\frac{a_{ij}}{d_j^+},\frac{b_{ij}}{d_j^+},\frac{c_{ij}}{d_j^+},\frac{d_{ij}}{d_j^+}\right),\ j\in B \quad (12)$$

$$\tilde{r}_{ij}=\left(\frac{a_j^-}{d_{ij}},\frac{a_j^-}{c_{ij}},\frac{a_j^-}{b_{ij}},\frac{a_j^-}{a_{ij}}\right)\ j\in C \quad (13)$$

where B in Eq.8 and C in Eq.9 are the sets of benefit criteria and cost criteria, respectively.

$$d_j^+ = \max_i \left(c_{ij}\right) \quad (14)$$

$$a_j^- = \min_i \left(a_{ij}\right) \quad (15)$$

The method of standardization alluded to above is fabricated to preserve the property in which the rudiments

$\tilde{r}_{ij}$ are normalized trapezoidal fuzzy numbers.

Step 2

Let us construct a weighed up normalized fuzzy-decision matrix:

Taking into consideration the variety of the importance of each yardstick, the fuzzy decision matrix that is contemplated and standardized is built up as

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m\times n} \quad i = 1,2,3,\ldots,m \text{ and } j = 1,2,3,\ldots,n \quad (16)$$

where in $\tilde{v}_{ij} = w_j(.)\tilde{r}_{ij}$ which is also considered as the fuzzy weight of each criterion.

Step 3

Determine FPIS and FNIS: in proportion to the weighed up normalized fuzzy decision matrix, the stabilized positive triangular fuzzy numbers are also able to approximate the components $\tilde{v}_{ij}$. Then, the fuzzy positive ideal solution, FPIS $\left(A^+\right)$ and fuzzy negative ideal Solution, FNIS $\left(A^-\right)$ can be defined as

$$A^+ = \left(\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+\right) \quad (17)$$

$$A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right) \quad (18)$$

Where in $\tilde{v}_j^- = \min_i\left(v_{ij1}\right), \tilde{v}_j^+ = \max_i\left(v_{ij4}\right)$, $i = 1,2,3,\ldots,m$ & $j = 1,2,3,\ldots,n$.

The index v_{ij1} and v_{ij4} ,1 and 4 stipulate the first and fourth elements in a Trapezoidal fuzzy number respectively.

Step 4

Compute the distance of each replacement from FPIS and FNIS correspondingly: The distance of each substitute from A^+ and A^- can be presently evaluated as

$$d_i^+ = \sum_{j=1}^{n} fd_v\left(\tilde{v}_{ij}, \tilde{v}_j^+\right), i = 1,2,3,\ldots,m \quad (19)$$

$$d_i^- = \sum_{j=1}^{n} fd_v\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) \; i = 1,2,3,\ldots,m \quad (20)$$

Step 5

Reckon the coefficient of imminence of each substitution:

A coefficient of proximity is delineated to demarcate the order of ranking of all conceivable substitutes once d_i^+ and d_i^- wherein for each replacement, A_i has been evaluated. The closeness coefficient speaks for the distances to the fuzzy positive ideal solution A^+ and the fuzzy negative ideal solution A^- concurrently by picking up the virtual proximity to the fuzzy positive ideal solution. The coefficient of proximity A^- of each replacement is assessed as

$$cc_i = \frac{d_i^-}{d_i^- + d_i^+} \quad (21)$$

Step 6

In relation to the closeness of the contiguity, it can be comprehended that the status of assessment of each substitute and ascertain the order of ranking of all alternatives (each benchmark has a larger closeness coefficient and a sophisticated level in the ranking order of all replacements).

In the subsequent section, we propose the method to opt for the critical path in fuzzy environment.

6 Proposed method for the critical path selection

A methodical approach to find the critical problem has been ventured in this section. Different criteria such as weights and qualitative methods are supposed as linguistic variables that are represented as positive trapezoidal fuzzy numbers are discussed in this paper. Right now, a comprehensive explanation is offered for the proposed method. A massive project could be divided into scores of activities. Ascertain the duration and preference relations of these activities. The preference relationship pertaining to these activities may be envisaged in the fuzzy project network. Hence drawing the precedence project network about which arc denotes activities also stipulates all the criteria which are very important to select the critical node to the ending node. Categorize all the paths in the fuzzy project network that begins with a starting event and ends with an ending event. Each path is considered to be chosen as a critical path. Very suitable linguistic variables are to be picked for qualitative criteria and trapezoidal fuzzy numbers for quantitative criteria thus the fuzzy evaluation of activity under each criterion is achieved. Then all linguistic evaluations are transfigured into apposite trapezoidal fuzzy numbers. The length of a path is the sum of durations of activities on the path. Therefore add up trapezoidal fuzzy number so that the final assessment is established under way beginning with the starting event and concluding with the ending event. In succession, create a fuzzy decision matrix as illustrated in Table-4. The standardized fuzzy decision matrix is cited in Table-5. The regularized fuzzy decision matrix is built up as described in Table-6. Then FPIS and FNIS have to be finalized as

$$A^{+} = \{(0.9, 0.9, 0.9, 0.9)\}$$

$$A^{-} = \{(0.357, 0.357, 0.357, 0.357)\}$$

Assess the distance of each path from FPIS and FNIS in regard to each criterion as exhibited in the Tables 7 and 8 respectively. Ascertain any five conceivable paths and subsequently calculate the proximity of coefficient of each path as shown in the Table-9. In proportion to the closeness coefficient of the five paths, it can be derived that the second path (1-3-6-10) is the critical path under the time, cost, risk, and quality criteria. A noteworthy characteristic is that this example is deciphered only with time being the yardstick. In this case, the trapezoidal fuzzy value of each criterion for paths is assimilated. The length of the longest path of the entire project network is the duration of the project. And the longest path of the project network is christened as critical path. Hence, make the fuzzy-decision matrix stand up in which its alternative is the path that starts with the starting point and ends with the ending event. We need the concluding critical path in the project network under various yardsticks in order to calculate the completion time of the project. In order to prefer a suitable, alternative critical path under different criteria, fuzzy TOPSIS method can be applied which deals with the ratings of both qualitative as well as quantitative criteria. Now draw the priority of the network as displayed in the Fig.2. Linguistic weighing variable as depicted in the Table-1 are used by the decision makers so that the significance of the criteria is evaluated. All the important heights path in the project network under these criteria. A path is one of the routes from the starting of the criteria are determined by the decision makers are exhibited in the Table-1. Let all the activities in the paths that commence with the starting event and conclude with the ending event be regulated. Decision makers use the linguistic rating variable – as described in the Table-1 – at this point alongside the trapezoidal fuzzy numbers so as to appraise the ratings of these activities in connection with each criterion. Ratings of the activities by the decision makers under various benchmarks are revealed in Table-4. Thenceforth the linguistic calculations as illustrated in Tables 1 and 2 are transfigured into trapezoidal fuzzy numbers as displayed in Table-3.The distances between paths and A^{+}, A^{-} produced in Table-8, Table-9 respectively. Encapsulate all the trapezoidal fuzzy numbers in order to ascertain the values of the final evaluation of each criterion for paths which get the stipulated yardstick such as the activity times of each path thus the value of fuzzy activity which commences with the starting event and concludes with the ending event could be calculated. In accordance with this, we possessed five alternatives (paths) with trapezoidal fuzzy numbers assigned. Hence Yager's [24]fuzzy ranking method is employed so as to classify these five fuzzy numbers and pick out the alternative which is the first rank is obtained by the first path. In relation to the largest amount of ranking function among other paths and therefore it is the critical path. Thus it is very essential to take into account different criteria in ascertaining a critical path.

Table-2 Rating of the activity by decision makers under various criteria

Activity	Time
1-2	Very low
1-3	Medium
1-4	High
2-5	Medium Low
3-5	Medium
4-6	Low
8-9	Very High
3-6	Very low
5-7	Medium High
4-8	High
6-10	Medium
7-10	Medium
9-10	High

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Table-3 Converted linguistic evaluation in to Trapezoidal fuzzy numbers.

Activity	Time
1-2	(0,0.1,0.2,0.3)
1-3	(0.3,0.5,0.7,0.9)
1-4	(0.6,0.7,0.9,1)
2-5	(0.1,0.3,0.5,0.7)
3-5	(0.3,0.5,0.7,0.9)
4-6	(0,0.1,0.3,0.5)
8-9	(0.7,0.8,0.9,1)
3-6	(0,0.1,0.2,0.3)
5-7	(0.5,0.7,0.8,0.9)
4-8	(0.6,0.7,0.9,1)
6-10	(0.3,0.5,0.7,0.9)
7-10	(0.3,0.5,0.7,0.9)
9-10	(0.6,0.7,0.9,1.0)

Table-4 Fuzzy-decision matrix, Fuzzy weight of criteria

Criteria Activity	Time (0.7,0.9,0.9,0.9)
1-2-5-7-10	(0.69,0.91,0.97,0.99)
1-3-6-10	(0.51,0.78,0.93,0.99)
1-3-5-7-10	(0.83,0.97,0.994,0.999)
1-4-6-10	(0.72,0.87,0.98,1)
1-4-8-9-10	(0.98,0.998,0.99,1)

Table-5 Normalized Fuzzy –decision matrix

Criteria Activity	Time (0.7,0.9,0.9,0.9)
1-2-5-7-10	(0.52,0.53,0.56,0.74)
1-3-6-10	(0.52,0.55,0.65,1)
1-3-5-7-10	(0.51,0.513,0.53,0.61)
1-4-6-10	(0.51,0.52,0.58,0.71)
1-4-8-9-10	(0.51,0.515,0.52,0.54)

Table-6 Weighted Normalized Fuzzy –decision matrix

Criteria Activity	Time
1-2-5-7-10	(0.364,0.477,0.504,0.66)
1-3-6-10	(0.364,0495,0.585,0.9)
1-3-5-7-10	(0.357,0.461,0.477,0.549)
1-4-6-10	(0.357,0.468,0.522,0.639)
1-4-8-9-10	(0.357,0.463,0.468,0.486)

Table-7 Distance between paths and A^+ with respect to each criterion

Criteria Activity	Time
$d^+\left(A_1, A^+\right)$	0.399
$d^+\left(A_2, A^+\right)$	0.325
$d^+\left(A_3, A^+\right)$	0.432
$d^+\left(A_4, A^+\right)$	0.394
$d^+\left(A_5, A^+\right)$	0.443

Table-8 Distance between paths and A^- with respect to each criterion

Criteria Activity	Time
$d^-\left(A_1, A^-\right)$	0.162
$d^-\left(A_2, A^-\right)$	0.315
$d^-\left(A_3, A^-\right)$	0.105
$d^-\left(A_4, A^-\right)$	0.148
$d^-\left(A_5, A^-\right)$	0.11

Table-9 Distances d_i^+, d_i^- and cc_i

Criteria Activity	d_i^+	d_i^-	$d_i^+ + d_i^-$	$cc_i = \frac{d_i^-}{d_i^- + d_i^+}$
1-2-5-7-10	0.399	0.162	0.561	0.288
1-3-6-10	0.325	0.315	0.639	**0.493**
1-3-5-7-10	0.472	0.105	0.577	0.182
1-4-6-10	0.394	0.148	0.542	0.273
1-4-8-9-10	0.443	0.11	0.553	0.19

The following Figure 2 represents by the Fuzzy project network:

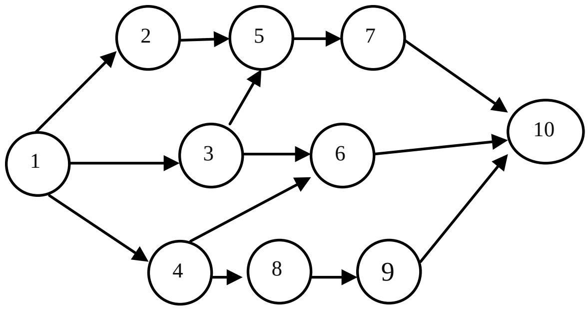

Fig 2: Fuzzy Project network

From the table-*669, Critical path of the fuzzy project network as per criteria using TOPSIS method is $1-3-6-10$.

6 Conclusions

In this paper TOPSIS method has been applied to fuzzy project network to determine the critical path using several criteria. Trapezoidal fuzzy numbers have been used as fuzzy activity times, to find criticality using linguistic terms. A new fuzzy distance measure has been proposed to select critical path in new TOPSIS method using linguistic trapezoidal fuzzy numbers as activity times. A numerical example related to this problem has provided to explain the procedure of proposed TOPSIS method in determining critical path with different criteria..

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