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Comparison of BER for BPSK & QAM modulation with Alamouti STBC

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Abstract— On comparing the error performance of various space-time block codes(STBC) for the quasi-static Rayleigh fading channels and with different modulation techniques as BPSK,QAM, it is observed that the slope of BER is steepening as E_b/N_0 is increased. It is also observed that a higher-order diversity is obtained with a larger number of transmit antennas, that is, steepening the slope of BER curves as the number of transmit antenna increases. From this it is confirms that all space-time block codes achieve the maximum diversity order of N_T. The Space-Time Block Coding (STBC) is a MIMO(Multiple input Multiple output) transmit strategy which exploits transmit diversity and high reliability .

Index Terms— MIMO (Multiple input Multiple output) ,Space-time block codes (STBC), BPSK , QAM ,BER, Rayleigh fading channels and diversity.

1 Introduction

1 Antenna Diversity: Diversity techniques are used to mitigate degradation in the error performance due to unstable wireless fading channels, for example, subject to the multipath fading [1]. Diversity in data transmission is based on the following idea: The probability that multiple statistically independent fading channels simultaneously experience deep fading is very low. There are various ways of realizing diversity gain as space diversity, Polarization diversity, Time diversity, Frequency diversity and Angle diversity. The concept can be extended to various antenna configurations. Some examples of single input multiple output (SIMO), multiple input single output (MISO), and multiple input multiple output (MIMO) antenna configurations are illustrated in Figure (1).

2 Receive Diversity : Consider a receive diversity system with NR receiver antennas. Assuming a single transmit antenna as in the single input multiple output (SIMO) channel of Figure(1), the channel is expressed as

$$h=[h_1 h_2 \ldots\ldots\ldots h_{NR}]^T \quad (1)$$

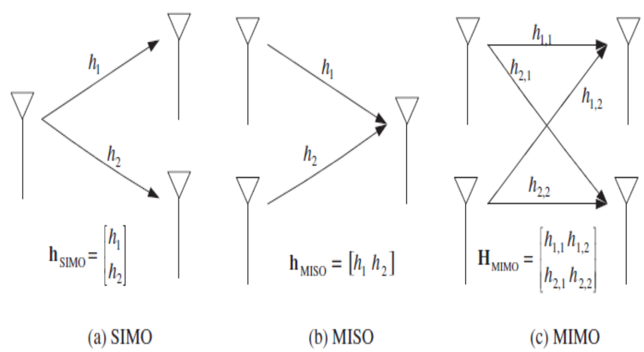

Figure (1) : Examples of various antenna configurations.

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for N_R independent Rayleigh fading channels [2]. Let x denote the transmitted signal with the unit variance in the SIMO channel. The received signal y $\in C^{NR \times 1}$ is expressed as

$$y=\sqrt{\frac{Ex}{No}}\; h_x+z \quad (2)$$

where z is zero-mean circular symmetric complex Gaussian(ZMCSCG) noise with $E\{zz^H\}=I_{NR}$. The received signals in the different antennas can be combined by various techniques. These combining techniques include selection combining (SC), maximal ratio combining (MRC), and equal gain combing (EGC). In SC, the received signal with the highest SNR among NR branches is selected for decoding. Let γ_i be the instantaneous SNR for the ith branch, which is given as

$$\gamma_i = \|h_i\|^2 \frac{Ex}{No}, \qquad i=1,2,\ldots\ldots\ldots N_R \quad (3)$$

Then the average SNR for SC is given as

$$\varrho_{sc} = E\{\max_i(\|h_i\|^2)\}\frac{Ex}{No} \qquad i=1,2,\ldots\ldots\ldots N_R \quad (4)$$

In MRC, all NR branches are combined by the following weighted sum:

$$\mathbf{y_{MRC}=[w_1^{(MRC)}\, w_2^{(MRC)}\ldots\ldots\ldots W_{NR}^{(MRC)}]y}$$

$$= \sum_{i=1}^{NR} w_i^{(MRC)} y_i \quad (5)$$

where y is the received signal in Equation (2) and w_{MRC} is the weight vector. As

$$y_i = \sqrt{\frac{Ex}{No}}\, h_i x + z_i$$

from Equation (2), the combined signal can be decomposed into the signal and noise parts, that is,

$$y_{MRC}=w^T{}_{MRC}{}^T\left(\sqrt{\frac{Ex}{No}} h_x + z\right)$$

$$= \sqrt{\frac{Ex}{No}}\; w^T{}_{MRC}\, h_x + w^T{}_{MRC}\, z \quad (6)$$

Average power of the instantaneous signal part and that of the noise part in Equation (6) are respectively given as

$$P_s = E\{\| \sqrt{\frac{Ex}{No}}\ w^T{}_{MRC}\, h_x \|^2\}$$
$$= \sqrt{\frac{Ex}{No}}\ E\{\| w^T{}_{MRC}\, h_x \|^2\}$$
$$= \sqrt{\frac{Ex}{No}}\ \| w^T{}_{MRC}\, h_x \|^2 \quad (7)$$

And

$$P_z = E\{\| w^T{}_{MRC}\, h_x \|^2\} = \| w^T{}_{MRC} \|^2_2 \quad (8)$$

From Equations (7) and (8), the average SNR for the MRC is given as

$$\rho_{MRC} = \frac{Ps}{Pz} = \frac{Ex}{No}\frac{\| wT\, MRC\, h \| 2}{\| wT\, MRC \| 22} \quad (9)$$

Invoking the Cauchy-Schwartz inequality,

$$\| w^T{}_{MRC}\, h \|^2 \le \| w^T{}_{MRC} \|^2_2 \| h \|^2_2 \quad (10)$$

Equation (9) is upper-bounded as

$$\rho_{MRC} = \frac{Ex}{No}\frac{\| wT\, MRC\, h \| 2}{\| wT\, MRC \| 22}$$
$$\le \frac{Ex}{No}\frac{\| wT\, MRC \| 22 \| h \| 22}{\| wT\, MRC \| 22} = \sqrt{\frac{Ex}{No}}\ \| h \|^2_2 \quad \mathbf{(11)}$$

Note that the SNR in Equation (11) is maximized at $w_{MRC} = h^*$, which yields $\rho_{MRC} = E_x \| h \|^2_2 / N_o$. In other words, the weight factor of each branch in Equation (5) must be matched to the corresponding channel for maximal ratio combining (MRC). Equal gain combining (EGC) is a special case of MRC in the sense that all signals from multiple branches are combined with equal weights. In fact, MRC achieves the best performance, maximizing the post-combining SNR.

3 Transmit Diversity : A critical drawback of receive diversity is that most of computational burden is on the receiver side, which may incur high power consumption for mobile units in the case of downlink. Diversity gain can also be achieved by space-time coding (STC) at the transmit side, which requires only simple linear processing in the receiver side for decoding. In order to further reduce the computational complexity in mobile units, differential space-time codes can be used, which do not require CSI estimation at the receiver side .

4 Space-Time Block Code (STBC): The very first and well-known STBC is the Alamouti code, which is a complex orthogonal space-time code specialized for the case of two transmit antennas [3]. Consider the Alamouti space-time coding technique .

5 Alamouti Space-Time Code: A complex orthogonal space-time block code for two transmit antennas was developed by Alamouti . In the Alamouti encoder, two consecutive symbols x_1 and x_2 are encoded with the following space-time codeword matrix:

$$X = \begin{bmatrix} x1 & -x2^* \\ x2 & x1^* \end{bmatrix}$$

As shown in Figure (2) Alamouti encoded signal is transmitted from the two transmit antennas over two symbol periods. During the first symbol period, two symbols x1 and x2 are simultaneously transmitted from the two transmit antennas. During the second symbol period, these symbols are transmitted again, where $-x_2^*$ is transmitted from the first transmit antenna and x_1^* transmitted from the second transmit antenna.

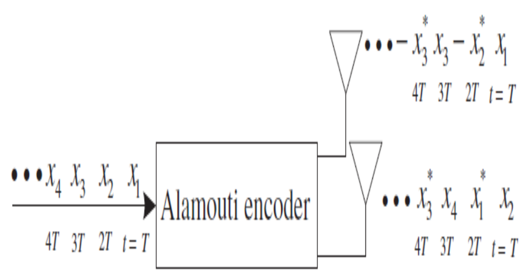

Figure (2): Alamouti encoder.

As the Alamouti codeword X in Equation (12) is a complex-orthogonal matrix, that is,

$$XX^H = \begin{bmatrix} \|x_1\|^2 + \|x2\|^2 & 0 \\ 0 & \|x_1\|^2 + \|x2\|^2 \end{bmatrix}$$

$$= (\|x_1\|^2 + \|x2\|^2) I_2 \quad (13)$$

where I_2 denotes the 2 X 2 identity matrix. Since N = 2 and T = 2, the transmission rate of Alamouti code is shown to be unity. Consider two different Alamouti codes,

$$X_p = \begin{bmatrix} x_{1,p} & -x^*_{2,p} \\ x_{2,p} & x^*_{1,p} \end{bmatrix} \text{ and } X_q = \begin{bmatrix} x_{1,q} & -x^*_{2,q} \\ x_{2,q} & x^*_{1,q} \end{bmatrix} \quad (14)$$

Where $[\mathbf{x}_{1,p}\ \ x_{2,p}]^T \neq [\mathbf{x}_{1,q}\ \ x_{2,q}]^T$. Then the minimum rank is evaluated as

$$\mathbf{V = \min rank_{p \neq q} \begin{bmatrix} x_{1,p} - x_{1,q} & -x^*_{2,p} + x^*_{2,q} \\ x_{2,p} - x_{2,q} & x^*_{1,p} - x^*_{1,q} \end{bmatrix}\begin{bmatrix} x_{1,p} - x_{1,q} & -x^*_{2,p} + x^*_{2,q} \\ x_{2,p} - x_{2,q} & x^*_{1,p} - x^*_{1,q} \end{bmatrix}^H}$$

$$\mathbf{= \min rank_{p \neq q} \left\{\begin{bmatrix} e_1 & -e_2^* \\ e_2 & e_1^* \end{bmatrix}\begin{bmatrix} e_1^* & e_2^* \\ -e_2 & e_1 \end{bmatrix}\right\}}$$

$$\mathbf{= \min rank_{p \neq q} \{(\| e_1 \|^2 + \| e_2 \|^2) I_2} \quad \mathbf{(15)}$$

$$\mathbf{= 2}$$

where $e_1 = \mathbf{x}_{1,p} - \mathbf{x}_{1,q}$ and $e_2 = \mathbf{x}_{2,p} - \mathbf{x}_{2,q}$. Note that e_1 and e_2 cannot be zeros simultaneously. From Equation (14), the

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Alamouti code has been shown to have a diversity gain of 2. Note that the diversity analysis is based on ML signal detection at the receiver side.Now discuss ML signal detection for Alamouti space-time coding scheme. Here, assume that two channel gains, h_1 (t) and h_2 (t), are time-invariant over two consecutive symbol periods, that is,

$$h_1(t) = h_1(t+T_s) = h_1 = \| h_1 \| e^{j\theta_1}$$

And $$h_2(t) = h_2(t+T_s) = h_2 = \| h_2 \| e^{j\theta_2} \quad (16)$$

Where $\| h_i \|$ and θ_i denote the amplitude gain and phase rotation over the two symbol periods, i = 1,2. Let y_1 and y_2 denote the received signals at time t and $t+T_s$, respectively, then

$$y_1 = h_1 x_1 + h_2 x_2 + z_1 \quad (17)$$
$$y_2 = -h_1 x_2^* + h_2 x_1^* + z_2$$

where z_1 and z_2 are the additive noise at time t and $t+T_s$, respectively. Taking complex conjugation of the second received signal, we have the following matrix vector equation:

$$\begin{bmatrix} y1 \\ y2^* \end{bmatrix} = \begin{bmatrix} h1 & h2 \\ h2^* & -h1^* \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} z1 \\ z2^* \end{bmatrix} \quad (18)$$

In the course of time, from time t to t+T, the estimates for channels, $\widetilde{h1}$ and $\widetilde{h2}$ are provided by the channel estimator. However, assume an ideal situation in which the channel gains, h_1 and h_2, are exactly known to the receiver. Then the transmit symbols are now two unknown variables in the matrix of Equation (18). Multiplying both sides of Equation (18) by the Hermitian transpose of the channel matrix, that is

$$\begin{bmatrix} h1^* & h2 \\ h2^* & -h1 \end{bmatrix} \begin{bmatrix} y1 \\ y2^* \end{bmatrix} = \begin{bmatrix} h1^* & h2 \\ h2^* & -h1 \end{bmatrix}$$
$$\begin{bmatrix} h1 & h2 \\ h2^* & -h1^* \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} h1^* & h2 \\ h2^* & -h1 \end{bmatrix} \begin{bmatrix} z1 \\ z2^* \end{bmatrix}$$

$$= (\| h1 \|^2 + \| h2 \|^2) \begin{bmatrix} x1 \\ x2 \end{bmatrix} \quad \begin{matrix} h1^* z1 & h2 z1^* \\ h2^* z1 & -h1 z1^* \end{matrix}$$
(19)

obtain the following input-output relations as :

$$\begin{bmatrix} \widetilde{y1} \\ \widetilde{y2} \end{bmatrix} = (\| h1 \|^2 + \| h2 \|^2) \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} \widetilde{z1} \\ \widetilde{z2} \end{bmatrix} \quad (20)$$

In Equation (20), note that other antenna interference does not exist anymore, that is, the unwanted symbol x_2 dropped out of y_1, while the unwanted symbol x_1 dropped out of y_2. This is attributed to complex orthogonality of the Alamouti code in Equation (12). This particular feature allows for simplification of the ML receiver structure as follows:

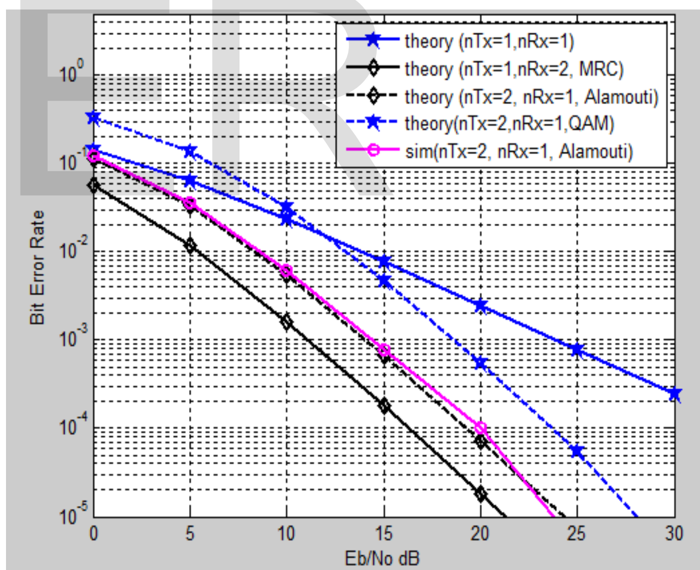
$$\widehat{x1}_{ML} = Q\left(\frac{\widetilde{y_i}}{\| h1 \|^2 + \| h2 \|^2}\right), \ i=1,2. \quad (21)$$

where Q(.) denotes a slicing function that determines a transmit symbol for the given constellation set. The above equation implies that x_1 and x_2 can be decided separately, which reduces the decoding complexity of original ML-decoding algorithm from $\| C \|^2$ to $2\| C \|$ where C represents a constellation for the modulation symbols, x_1 and x_2. Furthermore, the scaling factor ($\| h1 \|^2 + \| h2 \|^2$) in Equation (20) warrants the second-order spatial diversity, which is one of the main features of the Alamouti code.

6.Comparison of different BER for BPSK , QAM modulation techniques with Alamouti STBC:

On comparing the Alamouti coding and MRC in terms of BER performance that is obtained.Assume the independent Rayleigh fading channels and perfect channel estimation at the receiver. Note that the Alamouti coding achieves the same diversity order as 1 X 2MRC technique (implied by the same slope of the BER curves). Due to a total transmit power constraint (i.e., total transmit power split into each antenna by one half in the Alamouti coding), however, MRC technique outperforms Alamouti technique in providing a power combining gain in the receiver.

On comparing error performance of the Alamouti coding with different modulation technique then it is observed that the slope of BER is steepening as E_b/N_0 is increased .The error performance of the Alamouti coding with different modulation technique with QAM modulation order = 4 is as shown below :



Figure(3): Error performance of Alamouti encoding scheme and QAM modulation order = 4

The error performance of the Alamouti coding with different modulation technique with QAM modulation order = 16 is as shown below :

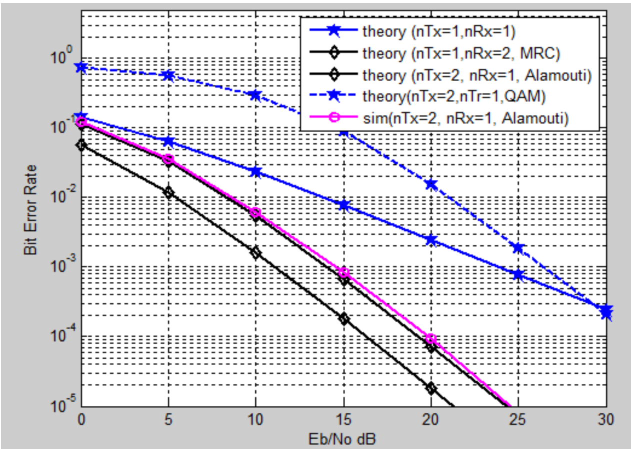


Figure (4) : Error performance of Alamouti encoding scheme and QAM modulation order = 16

The error performance of the Alamouti coding with different modulation technique with QAM modulation order = 64 is as shown below :

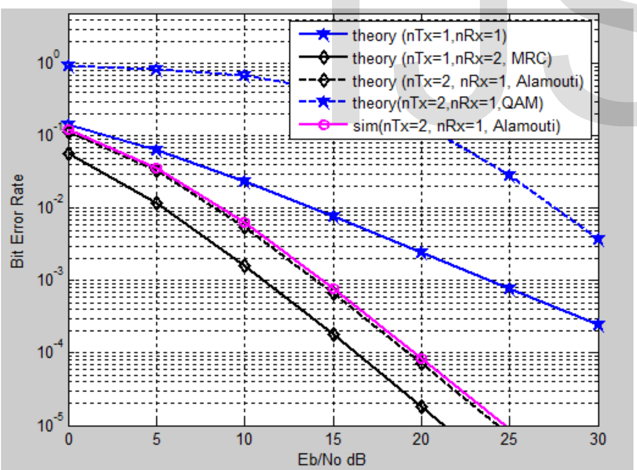


Figure (5) : Error performance of Alamouti encoding scheme and QAM modulation order =64

7 Conclusion

The different Figure shows that on comparing the Alamouti coding and MRC in terms of BER performance with different modulation technique , it is observed that the slope of BER is steepening as E_b/N_0 is increased. Here it is assumed that the independent Rayleigh fading channels and perfect channel estimation at the receiver. Note that the Alamouti coding achieves the same diversity order as 1 X 2 MRC technique (implied by the same slope of the BER curves). Due to a total transmit power constraint (i.e., total transmit power split into each antenna by one half in the Alamouti coding), however, MRC technique outperforms Alamouti technique in providing a power combining gain in the receiver.

It is also observed that a higher-order diversity is obtained with a larger number of transmit antennas, that is, steepening the slope of BER curves as the number of transmit antenna increases.

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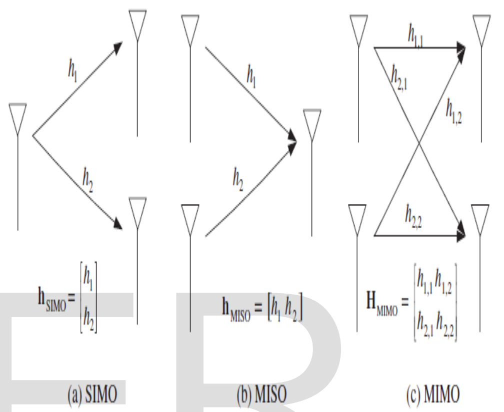

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