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Classification & Properties of Composite Numbers with respect to its wings

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Abstract

Consequent upon further development of generalized Fermat number with the help of N-equation published in December edition of this journal it is felt necessary to once again review the proof and to classify all the odd composite numbers that exist in our numbering system with respect to its positive or negative prime wings. This paper also includes few important properties of twin primes at a gap of 2λ where $\lambda = 1, 2, 3, \ldots\ldots.$

Keywords

Perfect & Imperfect composite odd number, Positive & negative prime wings, Intermediate wings

INTRODUCTION

So far prime wing of a number is concerned we know that number of wings does not depend upon the exponents of prime factors of the number but depends on the number of prime factors contained by the number. A number having n number of prime factors produces 2^{n-1} number of negative or positive prime wings whatever may be the number or distribution of exponents of prime factors. So while classifying a number with respect to its prime wings it does not matter if we ignore the exponents of all prime factors. Basically there exists two kinds of primes, one is of the nature $4x - 1$ known as 1st kind and other is of the nature $4x + 1$ known as 2nd kind. The difference in properties of these two types of primes with respect to formation of positive or negative prime wings plays the hidden roll of entire N-equation theory that have been developed as on today with several branches. Now It needs to invite further classification of numbers with respect to different angles of requirements.

1. Classification of a number with respect to Generalized Fermat Number

A number which is a product of all odd primes can be divided into two types.
Say, $N = P_1P_2P_3\ldots\ldots\ldots P_n$ in ascending order of magnitudes where $n \geq 3$.
Or, $N = \alpha_1^2 - \beta_1^2 = \alpha_2^2 - \beta_2^2 = \alpha_3^2 - \beta_3^2 = \ldots\ldots.. = \alpha_m^2 - \beta_m^2$ i.e. 2^{n-1} wings in ascending order of magnitude $(\alpha_i + \beta_i)$.
$\alpha_m^2 - \beta_m^2$ is a negative prime wing where sum of the elements is maximum & can be denoted by W_{max}, $\alpha_{m-1}^2 - \beta_{m-1}^2$ can be denoted by $W_{pre\text{-}max}$, $\alpha_1^2 - \beta_1^2$ can be denoted by W_{min} and all other wings are intermediate wings (W_{int}).

1.1 Perfect composite odd number (PCON): If $P_n < (P_1P_2P_3\ldots\ldots\ldots P_{n-1})$ the number N can be said as Perfect composite odd number where in all the produced negative prime wings sum of the elements are composite. With respect to any such wing say $\alpha^2 - \beta^2$, $\pi(\alpha^{2^n} + \beta^{2^n})$ where n = 0, 1, 2, 3,……. can be said as Fermat's product sequence of PCON.

1.2 Imperfect composite odd number (ICON): Here $P_n > (P_1P_2P_3\ldots\ldots\ldots P_{n-1})$ & in all the produced negative prime wings sum of the elements are composite except for $W_{pre\text{-}max}$ where $\alpha_{m-1} + \beta_{m-1} = P_n$.

With respect to this $W_{pre\text{-}mix}$ say $\alpha^2 - \beta^2$, $\pi(\alpha^{2^n} + \beta^{2^n})$ where n = 0, 1, 2, 3, ……. can be said as Fermat's product sequence of ICON.

In Fermat's product sequence of ICON every term is considered as prime and it is fully dependent on its preceding term i.e. on unique values of (α, β). If any term, say F_x is found to be composite then the ascending order arrangement of all primes is changed with a new introduction of greatest prime with respect to which all F_{x+j}, j = 1, 2, 3, …… cannot stand as a prime. It is bound to be composite.

Finally we can conclude that if $(\alpha + \beta)$ is composite $(\alpha^2 + \beta^2)$ is also composite and if $(\alpha^2 + \beta^2)$ is prime $(\alpha + \beta)$ is also prime when α, β are non-consecutive and in case of consecutive $(\alpha + \beta)$ is a prime or square of a prime. In both the cases reverse is not true. Here lies the mystery of all Fermat Numbers. Once it is composite for exponent n = m it will be always composite for $n \geq m$.

If the prime factors of a number contain exponents then to classify PCON or ICON the ascending order is followed according to the magnitude of P^x. Because whatever may be the value of x, P^x has a single unique negative prime wing i.e. $\{(P^x + 1)/2\}^2 - \{(P^x - 1)/2\}^2$

The above mentioned each type of number can be again divided into following two categories.

2.1 PCON(–): It contains at least one prime factor of 1st kind i.e. of the nature 4x – 1.
It can produce only negative wings.

2.2 PCON(+/–): It contains all prime factors of 2nd kind i.e. of the nature 4x + 1.
It can produce positive/negative both wings.

2.3 Similarly **ICON(–) & ICON(+/–)**

Irrespective of PCON or ICON these two type of composite odd numbers can be simply denoted by N(–) & N(+/–).

If N(–) has n nos. of prime factors it will produce 2^{n-1}(= w) nos. of negative prime wings and equating any two of them we can get ${}^wc_2$ nos. of dual relation of positive prime wings i.e. $N_i = a_i^2 + b_i^2 = c_i^2 + d_i^2$ where i = 1, 2, 3, …….,w.

Similarly, N(+/–) will also produce 2^{n-1}(= w) nos. of negative prime wings and equating any two of them we can get ${}^wc_2$ nos. of dual relation of positive prime wings i.e. $N_i = a_i^2 + b_i^2 = c_i^2 + d_i^2$ where i = 1, 2, 3, …….,w and 2^{n-1}(= w) nos. of positive prime wings and equating any two of them we can get ${}^wc_2$ nos. of dual relation of negative prime wings i.e. $N_i = a_i^2 - b_i^2 = c_i^2 - d_i^2$ where i = 1, 2, 3, …….w and equating any two of opposite wings we can get w^2 nos. of diagonal-relation of parallelepiped i.e. $d^2 = a^2 + b^2 + c^2$ type.

3. Important properties regarding primes.

3.1 If $N = e_1^2 + o_1^2 = e_2^2 + o_2^2$ is a product of two primes then its negative image $N^/ = o_1^2 - e_2^2 = o_2^2 - e_1^2$ must contain two or more prime factors.

3.2 If $(2x^2)^2 + 1$ is a product of two primes then $(2x)^2 - 1$ must be product of twin primes including the fact that it may contain two consecutive odd integers one of which is a square of a prime.

3.3 Only acceptable integers of x is $x = u_{0,5}$ [u_y denotes an integer of unit digit y]

3.4 In general if $(2x^2)^2 + \lambda^2$ is a product of two primes then $(2x)^2 - \lambda^2$ must be product of twin primes at a gap of 2λ.

Say, $N = (\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2) = e_1^2 + o_1^2 = e_2^2 + o_2^2$ where $(\alpha_1^2 + \beta_1^2)$ & $(\alpha_2^2 + \beta_2^2)$ both are primes i.e. $(\alpha + \beta)$ in both cases is prime. When α, β are consecutive $(\alpha + \beta)$ may be the square of a prime.

So, $N^/ = (\alpha_1^2 - \beta_1^2)(\alpha_2^2 - \beta_2^2) = o_1^2 - e_2^2 = o_2^2 - e_1^2 \Rightarrow N^/$ contains minimum two prime factors when α, β are

consecutive i.e. $(\alpha_1^2 + \beta_1^2)$ & $(\alpha_2^2 + \beta_2^2)$ both satisfy c of a N-equation for k = 1. Say, both are functionally consecutive i.e. $N = \{(x)^2 + (x - 1)^2\}\{(x)^2 + (x + 1)^2\}$ & by N_s-operation $N = (2x^2)^2 + 1 = (2x)^2 + (2x^2 - 1)^2$
$\Rightarrow N^/ = (2x)^2 - 1$ must be product of twin primes including the fact that it may contain two consecutive odd integers one of which is a square of a prime but both cannot be square of primes.
Now N is always of the form u_5 except for $x = u_{0,5}$. So for $x \neq u_{0,5}$, if N is to be product of two functional consecutive primes under c for k = 1 of a N-equation it is possible only when $N = 5.13 = (1^2 + 2^2)(2^2 + 3^2)$ i.e. x = 2 & $N^/ = 3.5$. So, apart from x = 2 all acceptable integers of x are of the nature $u_{0,5}$.
Examples: for x = 5, N = 2501 = 41.61 & $N^/ = 9.11 = 3^2.11$. For x = 10, N = 40001 = 13.17.181 i.e. product of more than two primes & hence not applicable. For x = 30, N = 3240001 = 1861.1741 i.e. product of two primes. Hence, $N^/ = 59.61$ i.e. product of two primes. Here nature of all produced twin primes is $u_{9,1}$ but all twin primes of such nature are not arrested by it.
In general by the product of $N = \{(x)^2 + (x - \lambda)^2\}\{(x)^2 + (x + \lambda)^2\}$ we can say, if $(2x^2)^2 + (\lambda^2)^2$ is a product of two primes then $(2x)^2 - \lambda^2$ must be product of twin primes at a gap of 2λ. Here, for $\lambda \neq 1$ square of a prime in $N^/$ will not come into picture. It should be further noted that $(2x^2)^2 + (\lambda^2)^2$, $\lambda \neq 1$ cannot be product of two consecutive functional values of c of a N-equation $a^2 + b^2 = c^2$ for a particular value of k.

In reverse way it can be said that if $(2x)^2 - \lambda^2$ has more than two prime factors then $(2x^2)^2 + (\lambda^2)^2$ has also more than two prime factors.

4. $(4^p + 1)/5$ where p > 3 is an odd prime, always represents a composite number.

For $\lambda = 1$ put $x = 2^{n-1} \Rightarrow (2x + 1)(2x - 1) = (2^n + 1)(2^n - 1)$ which has more than two prime factors.
Hence, $(2x^2)^2 + 1 = (2^{2n-1})^2 + 1$ has also more than two prime factors & it is also true if 2n – 1 is a prime. Say 2n – 1 = p. Hence $4^p + 1$ has more than two prime factors. $\Rightarrow (4^p + 1)/5$ has at least two prime factors i.e. $(4^p + 1)/5$ is always composite. Why p > 3 has already been explained above.

5.1 If $(4x^2 + 4x + \alpha^2)^2 + (2\alpha)^2$ where α is odd and $2x > (\sqrt{2} + 1)\alpha$, is a product of two primes then $(2x + \alpha)$ & $(2x + \alpha + 2)$ are twin primes
5.2 If $(4x^2 + 8x + 3 + \alpha^2)^2 + (2\alpha)^2$ where α is even & $(2x + 1) > (\sqrt{2} + 1)\alpha$, is a product of two primes then $(2x + \alpha + 1)$ & $(2x + \alpha + 3)$ are twin primes

Let us consider the product of two consecutive c of a 2nd kind N-equation $a^2 + b^2 = c^2$ having $k = 2\alpha^2$ where α is odd i.e. $(c_i)(c_{i+1}) = \{(2x)^2 + \alpha^2\}\{(2x + 2)^2 + \alpha^2\}$ where one wing is $(4x^2 + 4x + \alpha^2)^2 + (2\alpha)^2$ by N_s operation. So, obviously if it is product of two primes then sum of the elements of product wings i.e. $(2x + \alpha)$ and $(2x + \alpha + 2)$ are twin primes. For a particular case if $\alpha = 1$, $(c_i)(c_{i+1}) = (2x + 1)^4 + 2^2$ and it is quite obvious that 2x + 1 is of the nature u_5 for product of two primes excepting the case 2x + 1 = 3 i.e. x = 1. So $u_5^4 + 2^2$ always produce twin primes when it is product of two primes.
Now, as $u_5^4 + 2^2 = (u_5^2 + 2 + 2u_5)(u_5^2 + 2 - 2u_5)$ nature of this product of two primes is obviously $u_7.u_7$ and that of twin primes is (u_7, u_9). But all twin primes of such nature are not arrested by it.
Similarly, for α is even from the product of $\{(2x + 1)^2 + \alpha^2\}\{(2x + 3)^2 + \alpha^2\}$ we can have one wing as $(4x^2 + 8x + 3 + \alpha^2)^2 + (2\alpha)^2$ which follows the same logic for the generation of twin primes.
For k = 1, when two consecutive functional values of c are primes it produces twin primes of the nature (u_9, u_1) & for k = 2 nature of twin prime is (u_7, u_9). This product of two functional primes for k = 2 can be of four combinations with respect to their digits at 10th place i.e. (…17, …37), (…17, …57), (…57, …97) and (…77, …77) which can be easily established considering the facts a) 10th digits of both the primes must be odd as they are of 2nd kind i.e. 4x + 1 nature & b) (product of these two primes – 4) must contain last 3 digits' number 625 i.e. 10th digit is 2.

CONCLUSION

This little theorem regarding product of two primes seems to be a very powerful tools that can penetrate the mysteries of so many conjectures in Number theory. Still we are not in a position to claim a sound proof of infinite existence of twin primes. It needs further investigations.

References

[1] Academic text books of Algebra.

[2] In August edition, Vol-4 2013 of IJSER, Author: self

[3] In April edition, 2014 of IJSER, Author: self

[4] In May edition, 2014 of IJSER, Author: self

[5] In June edition, 2014 of IJSER, Author: self

[6] In Oct edition 2014 of IJSER, Author: self

[7] In June edition 2015 of IJSER, Author: self

[8] In July edition, 2015 of IJSER, Author: self

[9] In Sept edition 2015 of IJSER, Author: self

[10] In Nov edition 2015 of IJSER, Author: self

[11] In Dec edition, 2015 of IJSER, Author: self

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