Application of Generalized (G'/G)-expansion Method to Modified Regularized Long Wave (MRLW) Equation

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Abstract— In this paper, the Modified Regularized Long Wave (MRLW) equation is solved to find out exact traveling wave solutions based on the generalized G'/G - expansion method using the computation software Maple-17. Three types of general solutions such as trigonometric function, hyperbolic function and rational function are constructed with some parameters. When the parameters take specific values, we get exact solutions. The extracted solutions are checked by putting them back to the original equation with the Maple software. Several three dimensional graphs of some availed solutions are provided to exhibit the wave pattern of the considered equation.

Index Terms— Generalized Regularized Long Wave (GRLW) equation, Modified Regularized Long Wave (MRLW) equation, Traveling Wave, G'/G-expansion, Generalized G'/G-expansion, Nonlinear Evolution Equation (NLEE), Homogeneous balance.

1 Introduction

Most of the physical phenomena in the real world can be described by the Nonlinear Evolution Equations (NLEEs). Seeking the exact solutions of NLEEs has significant importance in different areas of Mathematical Physics such as Fluid Dynamics, Water Wave Mechanics, Plasma Physics, Solid State Physics, Optical Fibers and Quantum Mechanics as well as their applications. In the past few decades several effective methods such as Homotopy peturbation method [8], Sin-Cosine method [9], Tanh function method [10,11], Jaccob-elliptic function method [12,13], Exp-function method [14], Homogeneous balance method [15], Hirota bilinear method [16], Auxiliary equation method [17], F-expansion method [18,19] and so on have been developed to explore explicit traveling wave solutions of NLEEs.

Recently, a new method known as G'/G - expansion method is proposed by Wang et all. [1] to search exact traveling wave solutions of NLEEs. In some literatures, as for example Z. L. Li [2], Zhang [3], Zayed and Gepreel [4], Malik et al [5], Bekir [6], Rashedunnabi [7] and so on, this method is successfully applied to investigate the traveling wave solutions of some significant NLEEs. Most of the researchers have shown that the G'/G - expansion method is very effective to solve some NLEEs involving higher order nonlinear terms. Subsequently, further research has been carried out to expand its applicability. More recently, Zhang et al. [20], Zayed [21] have been proposed generalized G'/G -expansion method and introduced it as much more effective to construct further new traveling wave solutions of NLEEs . In some articles such as Zayed et all. [22], Liu et all. [23], Naher et all. [24] the generalized G'/G - expansion method is applied and verified that this method is more general, extended and effective.

The MRLW equation is a special case of the following GRLW equation.

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$$u_t + u_x + p(p+1)u^p u_x - \beta u_{xxt} = 0 \quad (A)$$

where p is a positive integer and β is a positive real constant. When $p = 2$, the equation (A) transformed into the MRLW equation. It is noticed from the literature point of view that the solutions of this equation are kinds of solitary waves having shapes not affected by collisions. Moreover, it describes the propagation of unidirectional weakly nonlinear and weakly dispersive water waves including nonlinear transverse waves in shallow water, ion-acoustic waves and magneto-hydrodynamic waves in plasma, longitudinal dispersive waves in elastic rods, pressure waves in liquid-gas bubble mixtures and rotating flow down a tube. Though many researchers have shown that the generalized G'/G -expansion method is more general and effective but as far as I know yet this method has not been applied to the MRLW equation in any previous research work.

In this work, the generalized G'/G -expansion method is applied to solve the MRLW equation and the obtained solutions are checked by MAPLE-17 software.

The rest of the paper is organized as follows: Section 2 describes the generalized G'/G -expansion method to find out exact traveling wave solutions of NLEEs. In section 3, application of this method to the MRLW equation is illustrated. Section 4 deals with the results and discussion with graphical representations. Finally, conclusions are given in section 5.

2 The Generalized G'/G -expansion method

We assume that the nonlinear evolution equation in two variables, namely x and t is given by

$$F(u, u_x, u_{xt}, u_{tt}, u_{xx}, ...) = 0 \quad (1)$$

Where u is an unknown function of two variables x and t, F is a polynomial in u and subscripts are indicating partial derivatives involving the highest order derivatives and nonlinear terms. The main steps of the generalized G'/G -expansion method are described as follows:

Step 1:

First, we use the following traveling wave transformation to substitute the independent variables x and t by the variable ξ.

$$u(x,t) = u(\xi), \; \xi = x - ct \quad (2)$$

where ξ is a traveling wave variable and c is the wave velocity. Using equation (2), the equation (1) transformed into the following Ordinary Differential Equation (ODE) of the form:

$$F(u, u', u'', ...) = 0 \quad (3)$$

Step 2:

Now we consider that the solution of equation (3) can be expressed by a polynomial in $\frac{G'}{G}$ as follows:

$$u(\xi) = \sum_{j=-n}^{n} a_j \left(\frac{G'}{G}\right)^j$$

(4)

where a_j's are constants to be determined such that a_n cannot be zero at the same time and $G = G(\xi)$ satisfies the following second order nonlinear ODE

$$GG'' - \lambda GG' - \mu G^2 - \nu (G')^2 = 0 \quad (5)$$

where λ, μ and ν are real constants and

$$G' = \frac{dG}{d\xi}, G'' = \frac{d^2G}{d\xi^2}.$$

Step 3:

Substituting the equation (4) in the equation (3) and making a homogeneous balance between the highest order derivative and highest order nonlinear term yields the value of the positive integer n appearing in equation (4).

Step 4:

Placing n into the equation (4) and then equation (4) to the equation (3) provides a polynomial in $\frac{G'}{G}$ .Equating the coefficients of this polynomial to zero, we obtain a set of algebraic equations in a_j, c, λ, μ and ν. Solving the system by algebraic computation, values of a_j, c, λ, μ and ν can be found.

Step 5:

Finally, putting back the general solution of equation (5), values of $a_j, c, \lambda, \mu, \nu$ and equation (2) into the equation (4) we avail more traveling wave solutions of equation (1).

3 Application of the method

We employ the generalized $\frac{G'}{G}$- expansion method to the following Modified Regularized Long Wave Equation.

$$u_t + u_x + 6u^2 u_x - \beta u_{xxt} = 0 \quad (6)$$

where β is a positive real constant. Using the traveling wave transformation (2), the equation (6) converted into the nonlinear ODE

$$-cu'(\xi) + u'(\xi) + 6u(\xi)^2 u'(\xi) + c\beta u'''(\xi) = 0 \quad (7)$$

Integrating equation (7) with respect to ξ reduces to

$$-cu(\xi) + u(\xi) + 3u^3(\xi) + c\beta u''(\xi) + K = 0 \quad (8)$$

where K is an arbitrary constant and prime denotes the derivative with respect to ξ.

Substituting equation (4) into equation (8) and considering the homogeneous balance between u'' and u^3 we find $n = 1$. Therefore the solution of the equation (6) can be expressed as:

$$u(\xi) = a_{-1}\left(\frac{G'}{G}\right)^{-1} + a_0 + a_1\left(\frac{G'}{G}\right) \quad (9)$$

where a_{-1}, a_0 and a_1 are non-zero arbitrary constants.

Replacing equation (9) in equation (8) with the help of equation (5) yields a polynomial in $\frac{G'}{G}$.

Now collecting the coefficients of $\left(\frac{G'}{G}\right)^j$ $(j = 0,1)$ equal to zero, we obtain a system of algebraic equations in $a_0, a_1, a_2, \lambda, \mu$ and K as follows:

$$2c\nu^2\beta a_1 - 4c\nu\beta a_1 + 2c\beta a_1 + 2a_1^3 = 0$$
$$3c\nu\beta\lambda a_1 - 3c\beta\lambda a_1 + 6a_0 a_1^2 = 0$$
$$2c\mu\nu\beta a_1 + c\beta\lambda^2 a_1 - 2c\mu\beta a_1 + 6a_{-1}a_1^2 + 6a_0^2 a_1 - ca_1 + a_1 = 0$$
$$6\beta c\lambda\mu a_2 + c\beta\lambda^2 a_1 + 2c\mu\beta a_1 + \alpha a_0 a_1 - ca_1 + a_1 = 0$$
$$c\beta\lambda a_1 + c\nu\beta\lambda a_{-1} - c\beta\lambda a_{-1} + 12a_{-1}a_0 a_1 + 2a_0^3 - ca_0 + K + a_0 = 0$$
$$2c\mu\nu\beta a_1 + c\beta\lambda^2 a_{-1} - 2c\mu\beta a_{-1} + 6a_{-1}^2 a_1 + 6a_{-1}a_0^2 - ca_{-1} + a_{-1} = 0$$
$$3c\mu\beta\lambda a_{-1} + 6a_{-1}^2 a_0 = 0$$

$$2c\mu^2\beta a_{-1} + 2a_{-1}^3 = 0$$

Solving the above system of equations for a_{-1}, a_0, a_1, c and K by Maple -17 software we have the following solutions:

Solution Set: 1

$$a_{-1}=\pm\frac{2\mu\beta}{\sqrt{\sigma}},\quad a_0=\pm\frac{\beta\lambda}{\sqrt{\sigma}},\quad a_1=\pm\frac{\nu-1}{\sqrt{\sigma}},\quad c=\pm\frac{2}{\sqrt{\sigma}}$$

and

$$K=\pm\frac{8\lambda\beta^2(\nu-1)\mu}{\sqrt{\sigma}}. \tag{10}$$

where $\sigma=8\mu\beta-8\mu\nu\beta-\beta\lambda^2-2>0$, $\beta\neq 0$, λ,μ and ν are free parameters.

Solution Set: 2

$$a_{-1}=\pm\frac{2\beta\mu}{\sqrt{\delta}},\ a_0=\pm\frac{\beta\lambda}{\sqrt{\delta}},\ a_1=0,\ \ c=-\frac{2}{\delta}\text{ and } K=0 \tag{11}$$

where $\delta=4\mu\nu\beta-\beta\lambda^2-4\mu\beta-2>0$, $\beta\neq 0$, λ,μ and ν are free parameters.

Solution Set: 3

$$a_{-1}=0,\ a_0=\pm\frac{\beta\lambda}{\sqrt{\rho}},\ a_1=\pm\frac{2\beta(\nu-1)}{\sqrt{\rho}},\ c=-\frac{2}{\rho}\text{ and } K=0 \tag{12}$$

where $\rho=4\mu\nu\beta-\beta\lambda^2-4\mu\beta-2>0$, $\beta\neq 0$, λ,μ and ν are free parameters.

Replacing the general solution of equation (5) in equation (9) yields the following results:

I Hyperbolic function solution

When $\lambda^2-4\mu\nu+4\mu>0$ we obtain

$$u_1(\xi)=\frac{a_{-1}}{-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}}+a_0+a_1\left(-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}\right) \tag{13}$$

where $\Omega=\lambda^2-4\mu\nu+4\mu,\ \psi=\nu-1, U=C_1-C_2,\ V=C_1+C_2$ and $\xi=x-ct$

When $\lambda=0$ and $-4\mu\nu+4\mu>0$ we obtain

$$u_2(\xi) = -a_{-1}\frac{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)}{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)} + a_0 - a_1\frac{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)}{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)} \tag{14}$$

where $\Delta = -4\mu\nu + 4\mu,\ \psi = \nu - 1, U = C_1 + C_2,\ V = C_1 - C_2$ and $\xi = x - ct$

II Trigonometric function solution

When $\lambda^2 - 4\mu\nu + 4\mu < 0$ we obtain

$$u_3(\xi) = \frac{a_{-1}}{-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}+U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}+U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}} + a_0 + a_1\left(-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}+U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}+U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}\right) \tag{15}$$

where $\Omega = \lambda^2 - 4\mu\nu + 4\mu,\ \psi = \nu - 1,\ U = C_1 - C_2,\ V = C_1 + C_2$ and $\xi = x - ct$

When $\lambda = 0$ and $-4\mu\nu + 4\mu < 0$ we obtain

$$u_4(\xi) = -a_{-1}\frac{\psi\left(V\cos\left(\xi\sqrt{-\Delta}\right)+U\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right)-V\sin\left(\xi\sqrt{-\Delta}\right)\right)} + a_0 - a_1\frac{\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right)-V\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\psi\left(V\cos\left(\xi\sqrt{-\Delta}\right)+U\sin\left(\xi\sqrt{-\Delta}\right)\right)} \tag{16}$$

where $\Delta = -4\mu\nu + 4\mu,\ \psi = \nu - 1, U = C_1 + C_2,\ V = C_1 - C_2$ and $\xi = x - ct$

II Rational function solution

When $\lambda^2 - 4\mu\nu + 4\mu = 0$ we obtain

$$u_5(\xi) = \frac{a_{-1}}{-\frac{1}{2}\frac{\lambda}{\psi} + \frac{V}{\psi(-V\xi + U)}} + a_0 + a_1\left(-\frac{1}{2}\frac{\lambda}{\psi} + \frac{V}{\psi(-V\xi + U)}\right) \tag{17}$$

When $\psi = \nu - 1, U = C_1,\ V = C_2$ and $\xi = x - ct$

Substituting the solution sets (10), (11) and (12) in the above three general solutions yields the following family of solutions:

Family1. (Hyperbolic function solutions)

Case 1. $(U = 0, V \neq 0)$
From (13) we have

$$u_{1,1}(\xi)=\pm\frac{\left(\Omega\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)^2+2\lambda\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda^2\right)\pm 2\beta\lambda\left(\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda\right)\pm 8\psi\mu\beta}{2\sqrt{\sigma}\left(\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda\right)} \tag{18}$$

where $\xi=x\pm\frac{2}{\sqrt{\sigma}}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\sigma=8\mu\beta-8\mu\nu\beta-\beta\lambda^2-2>0$

$$u_{1,2}(\xi)=\pm\frac{\beta\lambda\left(\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda\right)\pm 4\mu\beta\psi}{\sqrt{\delta}\left(\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda\right)} \tag{19}$$

where $\xi=x+\frac{2}{\delta}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\delta=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

$$u_{1,3}(\xi)=\pm\frac{\beta\left(\sqrt{\Omega}\coth\left(\frac{\xi}{2}\sqrt{\Omega}\right)+\lambda\right)\pm\beta\lambda}{\sqrt{\rho}} \tag{20}$$

where $\xi=x+\frac{2}{\rho}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\rho=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

From equation (14) we have

$$u_{1,4}(\xi)=\pm\frac{2\mu\beta\psi\tanh\left(\xi\sqrt{\Delta}\right)^2\pm\beta\lambda\sqrt{\Delta}\tanh\left(\xi\sqrt{\Delta}\right)\pm\Delta}{\sqrt{\sigma}\sqrt{\Delta}\tanh\left(\xi\sqrt{\Delta}\right)} \tag{21}$$

where $\xi=x\pm\frac{2}{\sqrt{\sigma}}t,\ \Delta=-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\sigma=8\mu\beta-8\mu\nu\beta-\beta\lambda^2-2>0$

$$u_{1,5}(\xi)=\pm\frac{2\mu\beta\psi\tanh\left(\xi\sqrt{\Delta}\right)\pm\beta\lambda\sqrt{\Delta}}{\sqrt{\delta}\sqrt{\Delta}} \tag{22}$$

where $\xi=x+\frac{2}{\delta}t,\ \Delta=-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\delta=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

$$u_{1,6}(\xi)=\pm\frac{2\beta\sqrt{\Delta}\coth\left(\xi\sqrt{\Delta}\right)\pm\beta\lambda}{\sqrt{\rho}} \tag{23}$$

where $\xi=x+\frac{2}{\rho}t,\ \Delta=-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\rho=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

Case 2. $(U\neq 0, V=0)$

From (13) we get

$$u_{1,7}(\xi) = \pm \frac{\left(\Omega \tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right)^2 + 2\lambda\sqrt{\Omega}\tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda^2\right) \pm 2\beta\lambda\left(\sqrt{\Omega}\tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right) \pm 8\psi\mu\beta}{2\sqrt{\sigma}\left(\sqrt{\Omega}\tan\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right)} \tag{24}$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{1,8}(\xi) = \pm \frac{\beta\lambda\left(\sqrt{\Omega}\tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right) \pm 4\mu\beta\psi}{\sqrt{\delta}\left(\sqrt{\Omega}\tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right)} \tag{25}$$

where $\xi = x + \frac{2}{\delta}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{1,9}(\xi) = \pm \frac{\beta\left(\sqrt{\Omega}\tanh\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right) \pm \beta\lambda}{\sqrt{\rho}} \tag{26}$$

where $\xi = x + \frac{2}{\rho}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

From equation (14) we have

$$u_{1,10}(\xi) = \pm \frac{2\mu\beta\psi\coth\left(\xi\sqrt{\Delta}\right)^2 \pm \beta\lambda\sqrt{\Delta}\coth\left(\xi\sqrt{\Delta}\right) \pm \Delta}{\sqrt{\sigma}\sqrt{\Delta}\coth\left(\xi\sqrt{\Delta}\right)} \tag{27}$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Delta = -4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{1,11}(\xi) = \pm \frac{2\mu\beta\psi\coth\left(\xi\sqrt{\Delta}\right) \pm \beta\lambda\sqrt{\Delta}}{\sqrt{\delta}\sqrt{\Delta}} \tag{28}$$

where $\xi = x + \frac{2}{\delta}t,\ \Delta = -4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{1,12}(\xi) = \pm \frac{2\beta\sqrt{\Delta}\tanh\left(\xi\sqrt{\Delta}\right) \pm \beta\lambda}{\sqrt{\rho}} \tag{29}$$

where $\xi = x + \frac{2}{\rho}t,\ \Delta = -4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Case 3. $(U \neq 0, V \neq 0)$

From equation (13) we get

$$u_{1,13}(\xi)=\frac{\pm\frac{2\mu\beta}{\sqrt{\sigma}}}{-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}}\pm\frac{\beta\lambda}{\sqrt{\sigma}}\pm\frac{\psi}{\sqrt{\sigma}}\left(-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}\right) \tag{30}$$

where $\xi=x\pm\frac{2}{\sqrt{\sigma}}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\sigma=8\mu\beta-8\mu\nu\beta-\beta\lambda^2-2>0$

$$u_{1,14}(\xi)=\frac{\pm\frac{2\mu\beta}{\sqrt{\delta}}}{-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}}\pm\frac{\beta\lambda}{\sqrt{\delta}} \tag{31}$$

where $\xi=x+\frac{2}{\delta}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\delta=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

$$u_{1,15}(\xi)=\pm\frac{\beta\lambda}{\sqrt{\rho}}\pm\frac{2\beta\psi}{\sqrt{\rho}}\left(-\frac{1}{2}\frac{\lambda}{\psi}-\frac{1}{2}\frac{U\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}}{\psi\left(U\cosh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}+V\sinh\left(\frac{1}{2}\xi\sqrt{\Omega}\right)\sqrt{\Omega}\right)}\right) \tag{32}$$

where $\xi=x+\frac{2}{\rho}t,\ \Omega=\lambda^2-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\rho=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

From equation (14) we get

$$u_{1,16}(\xi)=\pm\frac{2\mu\beta}{\sqrt{\sigma}}\frac{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)}{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)}\pm\frac{\beta\lambda}{\sqrt{\sigma}}\pm\frac{\psi}{\sqrt{\sigma}}\frac{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)}{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)} \tag{33}$$

where $\xi=x\pm\frac{2}{\sqrt{\sigma}}t,\ \Delta=-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\sigma=8\mu\beta-8\mu\nu\beta-\beta\lambda^2-2>0$

$$u_{1,17}(\xi)=\pm\frac{2\mu\beta}{\sqrt{\delta}}\frac{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)}{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)}\pm\frac{\beta\lambda}{\sqrt{\delta}} \tag{34}$$

where $\xi=x+\frac{2}{\delta}t,\ \Delta=-4\mu\nu+4\mu>0,\ \psi=\nu-1$ and $\delta=4\mu\nu\beta-4\mu\beta-\beta\lambda^2-2>0$

$$u_{1,18}(\xi)=\pm\frac{\beta\lambda}{\sqrt{\rho}}\pm\frac{2\beta\psi}{\sqrt{\rho}}\frac{\sqrt{\Delta}\left(V\cosh\left(\xi\sqrt{\Delta}\right)+U\sin\left(\xi\sqrt{\Delta}\right)\right)}{\psi\left(U\cosh\left(\xi\sqrt{\Delta}\right)+V\sin\left(\xi\sqrt{\Delta}\right)\right)} \tag{35}$$

where $\xi = x + \frac{2}{\rho}t,\ \Delta = -4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Family2. (Trigonometric function solutions)

Case 1. $(U = 0, V \neq 0)$

$$u_{2,1} = \pm \frac{\frac{2\mu\beta}{\sqrt{\sigma}}}{-\frac{\lambda}{2\psi} + \frac{\sqrt{-\Omega}\tan\left(\frac{\xi}{2}\sqrt{-\Omega}\right)}{\psi}} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{\psi}{\sqrt{\sigma}}\left(-\frac{\lambda}{2\psi} + \frac{\sqrt{-\Omega}\tan\left(\frac{\xi}{2}\sqrt{-\Omega}\right)}{\psi}\right)$$

(36)

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,2} = \pm \frac{\frac{2\mu\beta}{\sqrt{\delta}}}{-\frac{\lambda}{2\psi} + \frac{\sqrt{-\Omega}\tan\left(\frac{\xi}{2}\sqrt{-\Omega}\right)}{\psi}} \pm \frac{\beta\lambda}{\sqrt{\delta}} \quad (37)$$

where $\xi = x + \frac{2}{\delta}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,3} = \pm \frac{\beta\lambda}{\sqrt{\rho}} \pm \frac{2\beta\psi}{\sqrt{\rho}}\left(-\frac{\lambda}{2\psi} + \frac{\sqrt{-\Omega}\tan\left(\frac{\xi}{2}\sqrt{-\Omega}\right)}{\psi}\right) \quad (38)$$

where $\xi = x + \frac{2}{\rho}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,4}(\xi) = \frac{2\mu\beta\psi\cot\left(\xi\sqrt{-\Delta}\right)^2 \pm \beta\lambda\sqrt{-\Delta}\cot\left(\xi\sqrt{-\Delta}\right) \pm \Delta}{\sqrt{\sigma}\sqrt{-\Delta}\cot\left(\xi\sqrt{-\Delta}\right)} \quad (39)$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,5}(\xi) = \pm \frac{2\mu\beta\psi\cot\left(\xi\sqrt{-\Delta}\right) \pm \beta\lambda\sqrt{-\Delta}}{\sqrt{\delta}\sqrt{-\Delta}} \quad (40)$$

where $\xi = x + \frac{2}{\delta}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,6}(\xi) = \pm \frac{2\beta\sqrt{-\Delta}\tan\left(\xi\sqrt{-\Delta}\right) \pm \beta\lambda}{\sqrt{\rho}} \quad (41)$$

where $\xi = x + \frac{2}{\rho}t,\ \Delta = -4\mu\nu + 4\mu > 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Case 2. $(U \neq 0, V = 0)$

From (15) we get

$$u_{2,7}(\xi) = \pm \frac{\left(\Omega\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right)^2 - 2\lambda\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right) - \lambda^2\right) \pm 2\beta\lambda\left(\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right) + \lambda\right) \pm 8\psi\mu\beta}{2\sqrt{\sigma}\left(\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{\Omega}\right) + \lambda\right)} \quad (42)$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,8}(\xi) = \pm \frac{\beta\lambda\left(\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right) + \lambda\right) \pm 4\mu\beta\psi}{\sqrt{\delta}\left(\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right) + \lambda\right)} \quad (43)$$

where $\xi = x + \frac{2}{\delta}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,9}(\xi) = \pm \frac{\beta\left(\sqrt{-\Omega}\cot\left(\frac{\xi}{2}\sqrt{-\Omega}\right) + \lambda\right) \pm \beta\lambda}{\sqrt{\rho}} \quad (44)$$

where $\xi = x + \frac{2}{\rho}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

From equation (16) we get

$$u_{2,10}(\xi) = \pm \frac{2\mu\beta\psi\tan\left(\xi\sqrt{-\Delta}\right)}{\sqrt{\sigma}\sqrt{-\Delta}} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{\sqrt{-\Delta}}{\sqrt{\sigma}\tan\left(\xi\sqrt{-\Delta}\right)} \quad (45)$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,11}(\xi) = \pm \frac{2\mu\beta\psi\tan\left(\xi\sqrt{-\Delta}\right)}{\sqrt{\delta}\sqrt{-\Delta}} \pm \frac{\beta\lambda}{\sqrt{\delta}} \quad (46)$$

where $\xi = x + \frac{2}{\delta}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

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$$u_{2,12}(\xi) = \pm \frac{2\beta\sqrt{-\Delta}\cot\left(\xi\sqrt{-\Delta}\right) \pm \beta\lambda}{\sqrt{\rho}\sqrt{-\Delta}} \tag{47}$$

where $\xi = x + \frac{2}{\rho}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Case 3. $(U \neq 0, V \neq 0)$

From (15) we get

$$u_{2,13}(\xi) = \frac{\pm \frac{2\mu\beta}{\sqrt{\sigma}}}{-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{\psi}{\sqrt{\sigma}}\left(-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}\right) \tag{48}$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,14}(\xi) = \frac{\pm \frac{2\mu\beta}{\sqrt{\delta}}}{-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}} \pm \frac{\beta\lambda}{\sqrt{\delta}} \tag{49}$$

where $\xi = x + \frac{2}{\delta}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,15}(\xi) = \pm\frac{\beta\lambda}{\sqrt{\rho}} \pm \frac{2\beta\psi}{\sqrt{\rho}}\left(-\frac{1}{2}\frac{\lambda}{\psi} - \frac{1}{2}\frac{-V\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}}{\psi\left(V\cos\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega} + U\sin\left(\frac{1}{2}\xi\sqrt{-\Omega}\right)\sqrt{-\Omega}\right)}\right) \tag{50}$$

where $\xi = x + \frac{2}{\rho}t,\ \Omega = \lambda^2 - 4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

From (16) we get

$$u_{2,16}(\xi) = \pm\frac{2\mu\beta\psi\left(V\cos\left(\xi\sqrt{-\Delta}\right) + U\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\sqrt{\sigma}\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right) - V\sin\left(\xi\sqrt{-\Delta}\right)\right)} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{2\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right) - V\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\sqrt{\sigma}\left(V\cos\left(\xi\sqrt{-\Delta}\right) + U\sin\left(\xi\sqrt{-\Delta}\right)\right)} \tag{51}$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,17}(\xi) = \pm \frac{2\mu\beta\psi\left(V\cos\left(\xi\sqrt{-\Delta}\right)+U\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\sqrt{\delta}\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right)-V\sin\left(\xi\sqrt{-\Delta}\right)\right)} \pm \frac{\beta\lambda}{\sqrt{\delta}} \quad (52)$$

where $\xi = x + \frac{2}{\delta}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{2,18}(\xi) = \pm \frac{\beta\lambda}{\sqrt{\rho}} \pm \frac{2\sqrt{-\Delta}\left(U\cos\left(\xi\sqrt{-\Delta}\right)-V\sin\left(\xi\sqrt{-\Delta}\right)\right)}{\sqrt{\rho}\left(V\cos\left(\xi\sqrt{-\Delta}\right)+U\sin\left(\xi\sqrt{-\Delta}\right)\right)} \quad (53)$$

where $\xi = x + \frac{2}{\rho}t,\ \Delta = -4\mu\nu + 4\mu < 0,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Family3. (Rational function solutions)

Case 1. $(U = 0, V \neq 0)$

From equation (17) we get

$$u_{3,1}(\xi) = \pm \frac{4\mu\beta\psi\xi}{\sqrt{\sigma}(\lambda\xi + 2)} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{\lambda\xi + 2}{\xi\sqrt{\sigma}} \quad (54)$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{3,2}(\xi) = \pm \frac{4\mu\beta\psi\xi}{\sqrt{\delta}(\lambda\xi + 2)} \pm \frac{\beta\lambda}{\sqrt{\delta}} \quad (55)$$

where $\xi = x + \frac{2}{\delta}t,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{3,3}(\xi) = \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{\lambda\xi + 2}{\xi\sqrt{\sigma}} \quad (56)$$

where $\xi = x + \frac{2}{\rho}t,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

Case 2. $(U \neq 0, V \neq 0)$

$$u_{3,4}(\xi) = \frac{\pm 2\mu\beta}{\sqrt{\sigma}\left(\frac{-\lambda}{2\psi} + \frac{V}{\psi(-V\xi + U)}\right)} \pm \frac{\beta\lambda}{\sqrt{\sigma}} \pm \frac{1}{\sqrt{\sigma}}\left(\frac{-\lambda}{2} + \frac{V}{(-V\xi + U)}\right) \quad (57)$$

where $\xi = x \pm \frac{2}{\sqrt{\sigma}}t,\ \psi = \nu - 1$ and $\sigma = 8\mu\beta - 8\mu\nu\beta - \beta\lambda^2 - 2 > 0$

$$u_{3,5}(\xi) = \frac{\pm 2\mu\beta}{\sqrt{\delta}\left(\frac{-\lambda}{2\psi} + \frac{V}{\psi(-V\xi + U)}\right)} \pm \frac{\beta\lambda}{\sqrt{\delta}} \tag{58}$$

where $\xi = x + \frac{2}{\delta}t,\ \psi = \nu - 1$ and $\delta = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

$$u_{3,4}(\xi) = \pm\frac{\beta\lambda}{\sqrt{\rho}} \pm \frac{2\beta}{\sqrt{\rho}}\left(\frac{-\lambda}{2} + \frac{V}{(-V\xi + U)}\right) \tag{59}$$

where $\xi = x + \frac{2}{\rho}t,\ \psi = \nu - 1$ and $\rho = 4\mu\nu\beta - 4\mu\beta - \beta\lambda^2 - 2 > 0$

The above equations (18 to 59) are the general solutions of the MRLW equation regarding several conditions. By choosing particular values of the parameters we avail different exact travelling wave solutions.

4 Results and Discussion

The above general solutions are investigated to check their exactness by putting them back into the equation (8) with the help of the computational software Maple-17. It is worth mentioning that majority of the solutions are found to satisfy the equation (8) and some, satisfy with particular choice of arbitrary parameters, are repeated version of previous solutions. Moreover, choosing the fixed values of λ, μ, ν and β the dynamics of some obtained exact traveling waves are presented in the following figures using Maple-17 software.

Family1. (Hyperbolic function solutions)

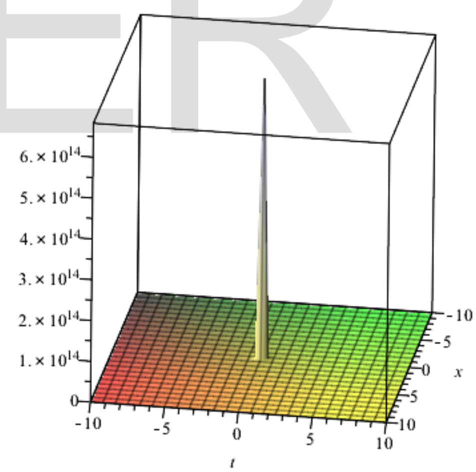

Fig1. $\lambda = 2, \mu = 10, \nu = 0.5, \beta = 5$

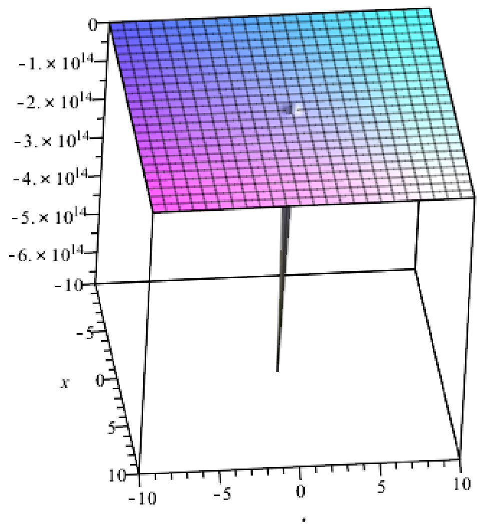

Fig2. $\lambda = 2, \mu = 10, \nu = 0.5, \beta = 5$

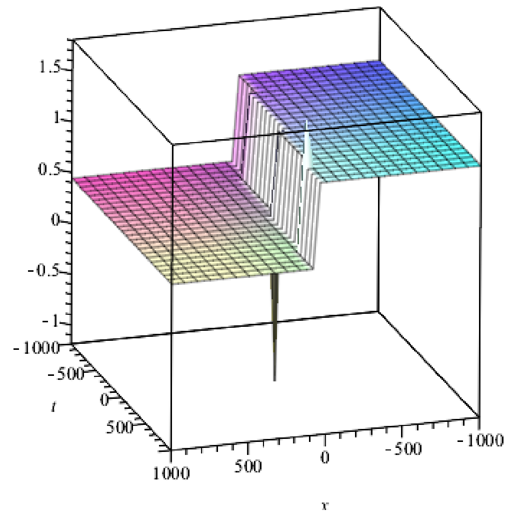

Fig4. $\lambda = 2, \mu = 5, \nu = 0.01, \beta = 1$

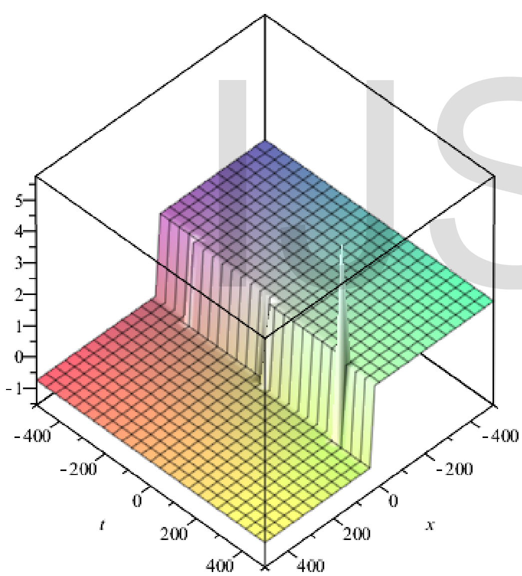

Fig3. $\lambda = 2, \mu = 5, \nu = 0.01, \beta = 1$

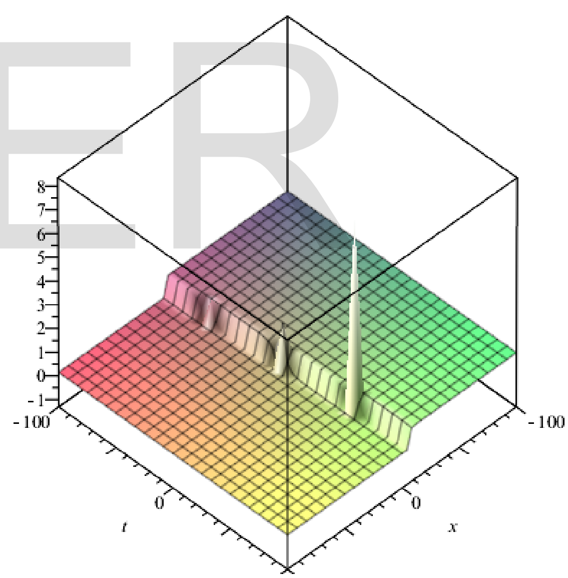

Fig5. $\lambda = 3, \mu = 5, \nu = 0.01, \beta = 1, U = 5, V = 7$

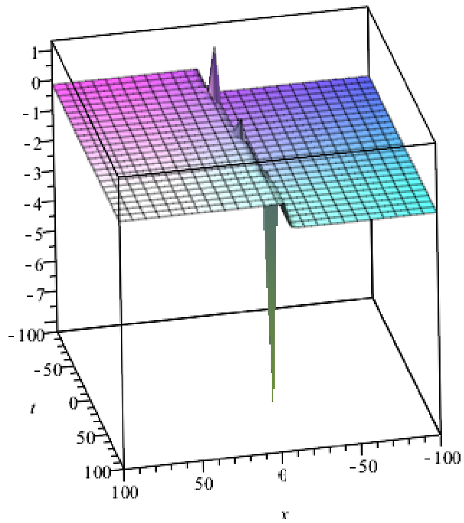

Fig6. $\lambda = 3, \mu = 5, \nu = 0.01, \beta = 1, U = 5, V = 7$

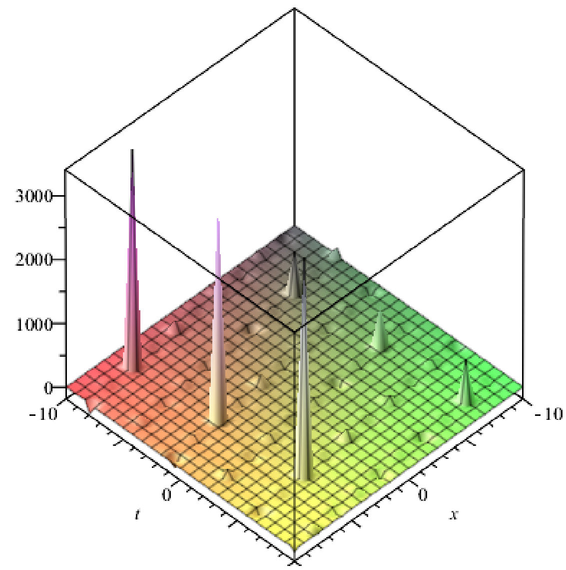

Fig8. $\lambda = 2, \mu = 5, \nu = 3, \beta = 1$

Family2. (Trigonometric function solutions)

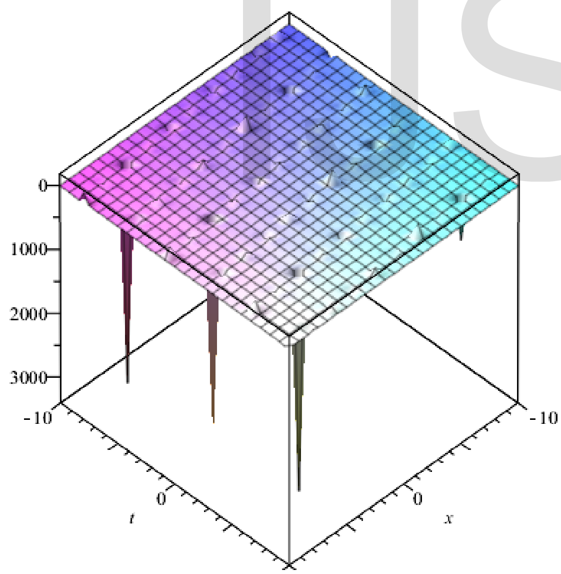

Fig7. $\lambda = 2, \mu = 5, \nu = 3, \beta = 1$

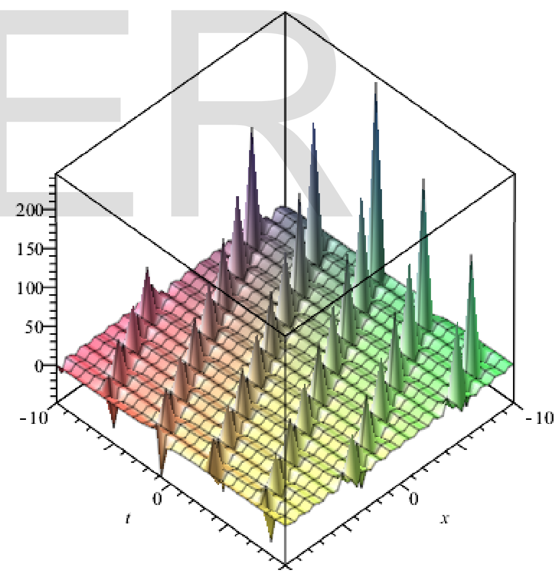

Fig9. $\lambda = 1, \mu = 6, \nu = 2, \beta = 1$

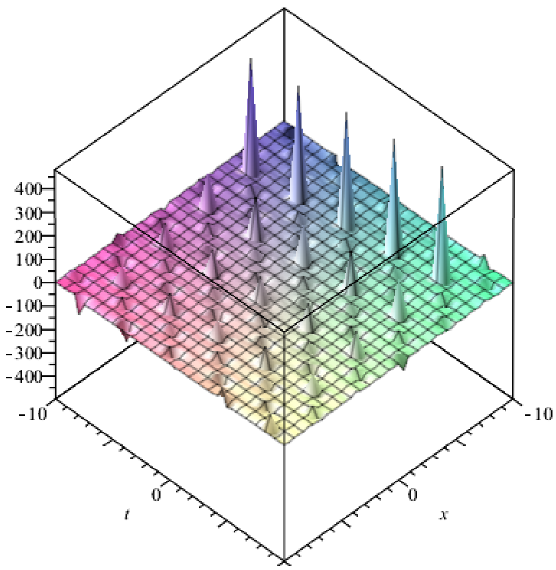

Fig10. $\lambda = 2, \mu = 6, \nu = 2, \beta = 1, U = 4, V = 2$

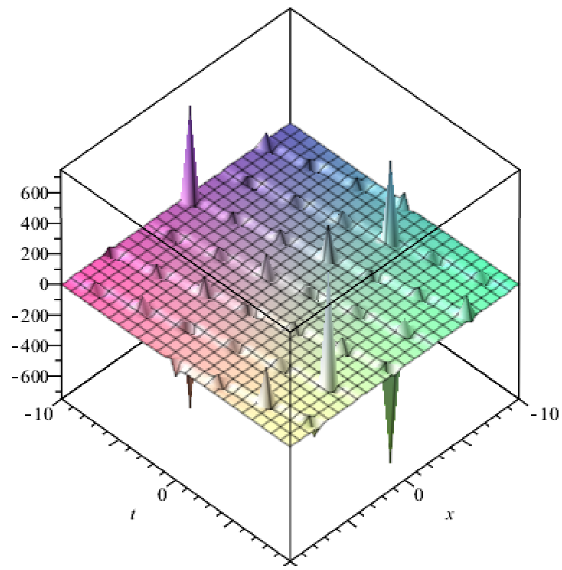

Fig12. $\lambda = 1, \mu = 6, \nu = 4, \beta = 1$

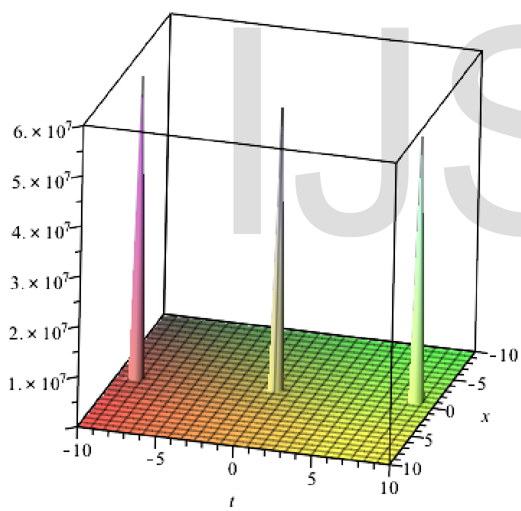

Fig11. $\lambda = 1, \mu = 3, \nu = 3, \beta = 1$

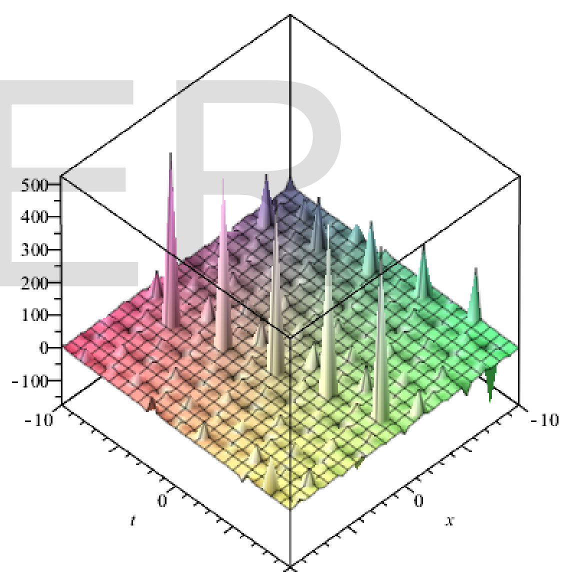

Fig13. $\lambda = 1, \mu = 3, \nu = 2, \beta = 1, U = 1, V = 1$

Family3. (Rational function solutions)

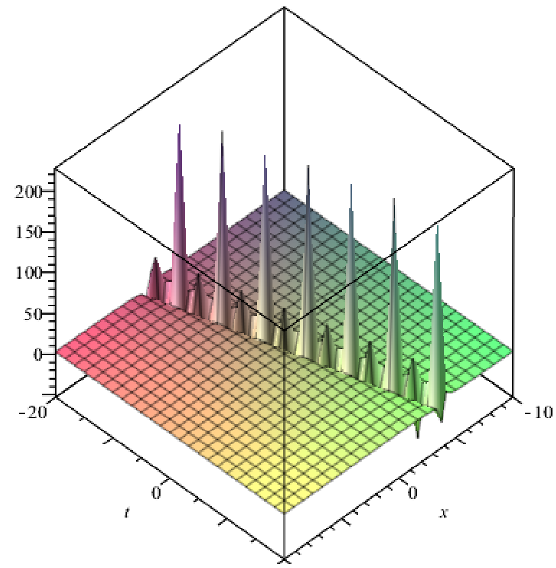

Fig14. $\lambda = 2, \mu = 5, \nu = 2, \beta = 1$

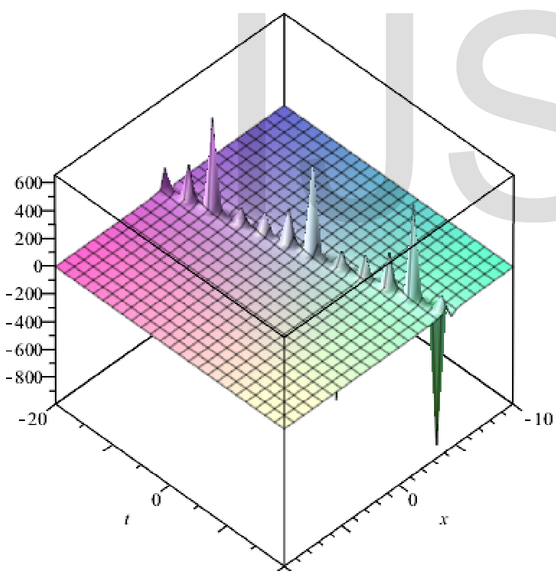

Fig15. $\lambda = 2, \mu = 5, \nu = 2, \beta = 1$

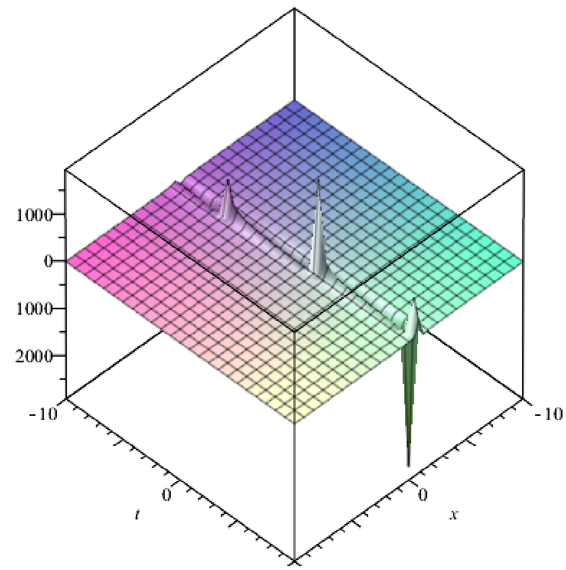

Fig16. $\lambda = 1, \mu = 5, \nu = 3, \beta = 1, U = 5, V = 3$

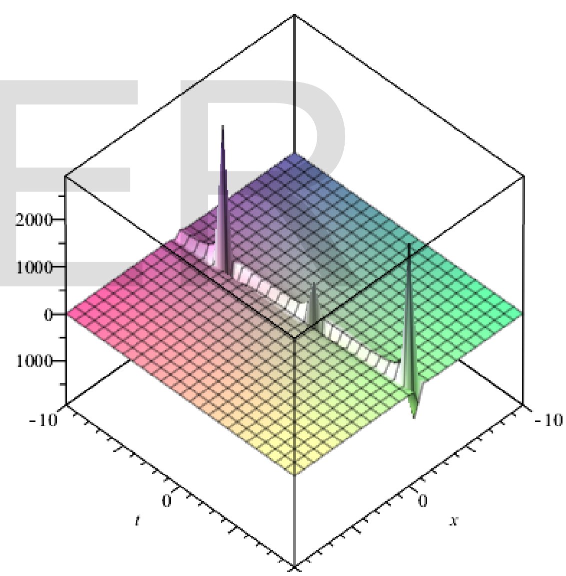

Fig17. $\lambda = 1, \mu = 5, \nu = 3, \beta = 1, U = 5, V = 3$

5 Conclusions

In this study, the generalized $\frac{G'}{G}$ expansion method is employed to explore some new exact traveling wave solutions of the MRLW equation. More traveling wave solutions have found with hyperbolic function solutions, trigonometric function solutions and rational function solutions including arbitrary parameters. The method provides copious freedom of choice of arbitrary parameters to construct exact traveling wave solutions which can be used to inquire the real structures of some physical phenomena of the considered NLEE. Interactions of different solitary waves are clearly understandable from the figures. The dynamics of the physical phenomena of the considered NLEE can be explained by analyzing the graphs.

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