An Intense Urge Micturition Model for a slowly varying Urethral Diameter

Ugochukwu Obinna Ugwu*

Abstract— A study of involuntary micturition (urine flow) in a slowly varying urethral diameter precipitated by an intense urge to urinate have been considered. The urethra have been assumed to be cylindrical in shape; and elastic since its made up of muscles. Even though the urethra is elastic, the pressure causing voiding is not enough to stretch the urethral walls. Thus no movement of the urethral walls during voiding implies the velocity profile is in axial direction with distance along the urethra called the axial distance. Consequently, the Navier-Stokes equation which describes the axial motion was solved to obtain an expression for velocity and pressure along the urethra assuming there is no abnormality in the Lower Urinary Tract (LUT) and that the urine is already stored in the bladder. Asymptotic limit to determine the expression for urethral axial velocity close to the end of micturition process was also considered.

Keywords: Asymptotics, Bladder pressure, Intense urge, Micturition, Urethra, Urethral axial velocity.

1. Introduction

The Lower Urinary Tract (LUT) is the only part of the body that is influenced by both voluntary and involuntary nervous system. It is made up of urinary bladder, the external sphincter and the urethra. In fact, It is a non-linear multi variable dynamic system variant in time and subject to internal alterations (convolutions, dysfunctions and infections) and external alterations (coughing, cold, fear, etc) [1].
The bladder could be seen as a spherical hollow sac made up of muscles. It collects and stores urine from the ureters. When simulated, it contracts and evacuate the urine stored in the bladder through the urethra. [2] observed that the bladder walls consist mostly of one smooth muscle, the detrusor and that the mechanical properties of the whole bladder are assumed to be those of the detrusor. Further reading on modelling bladder and its properties, see [3], [4].
Micturition is the process by which urine is expelled from the body. [5] described investigations into micturition as both obstructive and inconvenient due to the need for simultaneous knowledge of detrusor pressure and flow rate. They further observed that the detrusor pressure are normally taken to be the difference between the pressure inside the bladder and the abdominal pressure.
Results from [6], [7], [8] pointed out that the sphincter is not modelled separately but subsumed as part of the urethra. The reason could be attributed to the fact that the sphincter which is responsible for relaxing and constricting the urethra is also made up of contractile sheath of muscles around the urethra. Moreover, in the so-called VBN model proposed by [9], the changing cross sectional area and elastic properties of the urethra were considered and a simple sphincter control region included.
The urethra is a tube-like structure through which urine is expelled out of the body from the bladder during micturition. [10] proposed a model where the urethra was consider as a generally tubular lining enclosed in the so-called sheath of muscles known as pelvic floor.
The LUT nervous system cannot be left out since it controls storage/filling of the bladder and emptying/voiding processes. [8] gave a detailed treatise on LUT nervous control system.
An interesting observation from [3] revealed that in idealised voiding, urine is expelled by detrusor contraction while the contribution from gravity and abdominal straining ignored. Furthermore, as the effective abdominal pressure is only a weak contribution to the flow rate for the identification of the detrusor contractile strength, the contribution of abdominal straining has to be eliminated.
For our purpose, we consider a situation where there is no abnormality in LUT, assuming that the urine is already stored in the bladder so that neglecting the contribution from gravity and abdominal straining, we study the mechanism of involuntary micturiction precipitated by an intense urge to urinate with the behaviour of the urethra considered as a cylindrical tube during voiding so that we determine the bladder pressure and velocity along the urethra during the micturition process.

2. Model Formulation

In formulating this model, we put the following into consideration:
The urethra is assumed cylindrical in shape though not a perfect cylinder because it is made up of several natural constrictions. Interestingly, it is elastic (stretchable) since it is made up of several muscles.
Even though the urethra is elastic, the pressure causing voiding is not enough to stretch the urethral walls implies no slip condition assumed. In other words, there is no movement of urethral walls during voiding. The consequence is that at any given instant, the pressure and velocity profile only vary along the direction of flow and not across the urethral walls.
With the foregoing in mind, we make use of the Navier-Stokes equations to describe the axial motion of urine along the urethra.
Now, consider the Navier-Stokes equation in polar cylindrical

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coordinates as follows:

continuity equation:

$$\frac{\partial \rho}{\partial \bar{t}}+\frac{1}{\bar{r}}\frac{\partial(\rho \bar{r}\bar{V}_{\bar{r}})}{\partial \bar{r}}+\frac{1}{\bar{r}}\frac{\partial(\rho \bar{V}_{\theta})}{\partial \theta}+\frac{\partial(\rho \bar{V}_{\bar{z}})}{\partial \bar{z}}=0 \quad (2.1)$$

$\bar{r}$ *-component*:

$$\begin{aligned}&\rho(\frac{\partial \bar{V}_{\bar{r}}}{\partial \bar{t}}+\bar{V}_{\bar{r}}\frac{\partial \bar{V}_{\bar{r}}}{\partial \bar{r}}+\frac{\bar{V}_{\bar{\theta}}}{\bar{r}}\frac{\partial \bar{V}_{\bar{\theta}}}{\partial \bar{\theta}}+\bar{V}_{\bar{z}}\frac{\partial \bar{V}_{\bar{r}}}{\partial \bar{z}}-\frac{\bar{V}_{\bar{\theta}}^{2}}{\bar{r}})\\&=-\frac{\partial \bar{P}}{\partial \bar{r}}+\mu(\frac{\partial}{\partial \bar{r}}(\frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{V}_{\bar{r}})}{\partial \bar{r}})\\&+\frac{1}{\bar{r}^{2}}\frac{\partial^{2}\bar{V}_{\bar{r}}}{\partial \theta^{2}}+\frac{\partial^{2}\bar{V}_{\bar{r}}}{\partial \bar{z}^{2}}-\frac{2}{\bar{r}^{2}}\frac{\partial \bar{V}_{\theta}}{\partial \theta})+\rho g_{\bar{r}}\end{aligned} \quad (2.2)$$

θ *-component*:

$$\begin{aligned}&\rho(\frac{\partial \bar{V}_{\theta}}{\partial \bar{t}}+\bar{V}_{\bar{r}}\frac{\partial \bar{V}_{\theta}}{\partial \bar{r}}+\frac{\bar{V}_{\theta}}{\bar{r}}\frac{\partial \bar{V}_{\theta}}{\partial \bar{\theta}}+\bar{V}_{\bar{r}}\frac{\partial \bar{V}_{\theta}}{\partial \bar{r}}+\bar{V}_{\bar{z}}\frac{\partial \bar{V}_{\theta}}{\partial \bar{z}})\\&=-\frac{1}{r}\frac{\partial \bar{P}}{\partial \theta}+\mu(\frac{\partial}{\partial \bar{r}}(\frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{V}_{\theta})}{\partial \bar{r}})\\&+\frac{1}{\bar{r}^{2}}\frac{\partial^{2}\bar{V}_{\theta}}{\partial \theta^{2}}+\frac{2}{\bar{r}^{2}}\frac{\partial \bar{V}_{\bar{r}}}{\partial \theta}+\frac{\partial^{2}\bar{V}_{\theta}}{\partial \theta^{2}})+\rho g_{\theta}\end{aligned} \quad (2.3)$$

$\bar{z}$ *-component*:

$$\begin{aligned}&\rho(\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{t}}+\bar{V}_{\bar{r}}\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{r}}+\frac{\bar{V}_{\theta}}{\bar{r}}\frac{\partial \bar{V}_{\bar{z}}}{\partial \theta}+\bar{V}_{\bar{z}}\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{z}})\\&=-\frac{\partial \bar{P}}{\partial \bar{z}}+\mu(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\frac{\partial(\bar{r}\bar{V}_{\bar{z}})}{\partial \bar{r}})\\&+\frac{1}{\bar{r}^{2}}\frac{\partial^{2}\bar{V}_{\bar{z}}}{\partial \theta^{2}}+\frac{\partial^{2}\bar{V}_{\bar{z}}}{\partial \bar{z}^{2}})+\rho g_{\bar{z}}\end{aligned} \quad (2.4)$$

Model Assumptions

1.0 Urine flow assumed incompressible. i.e. density of urine, ρ is assumed constant.

2.0 The effects of gravity and abdominal straining would be neglected since in conformity with [3], urine is expelled by detrusor contraction with the contribution from gravity and abdominal straining ignored in an idealised voiding. i.e. $g_{\bar{r}}=g_{\theta}=g_{\bar{z}}=0$

3.0 No slip condition assumed since no movement across the walls of urethra.

4.0 Assume the pressure causing voiding is not enough to stretch the urethral wall. In other words the urine flow through the urethra is only in the axial direction since no movement of the urethral walls. i.e. $\bar{V}_{\bar{r}}=\bar{V}_{\theta}=0$ and $\bar{V}_{\bar{z}}\neq 0$.

5.0 Urine flow is assumed unsteady since the axial velocity and pressure describing the flow at a give instant vary along the direction of flow.

6.0 Lastly, we shall assume there is no abnormality in the lower urinary tract and the urine is already stored in the bladder.

To formulate our governing equations, we impose model assumptions 1.0 - 5.0 on (2.1)-(2.4) to get:

continuity equation:

$$\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{z}}=0 \quad (2.5)$$

$\bar{r}$ *-component*:

$$\frac{\partial \bar{P}}{\partial \bar{r}}=0 \quad (2.6)$$

θ *-component*:

$$\frac{\partial \bar{P}}{\partial \theta}=0 \quad (2.7)$$

$\bar{z}$ *-component*:

$$\rho\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{t}}=-\frac{\partial \bar{P}}{\partial \bar{z}}+\mu(\frac{1}{\bar{r}}\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{r}}+\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{r}^{2}}) \quad (2.8)$$

A quick note from (2.5)-(2.7) is that bladder pressure is independent of θ and $\bar{r}$ and the axial velocity is independent of $\bar{z}$ from the continuity condition. Note that by axis symmetry, $\frac{\partial \bar{V}_{\bar{z}}}{\partial \theta}=0$ i.e. V_z is independent of θ. Hence,

$$\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{z}}=0 \quad (2.9)$$

$$\rho\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{t}}=-\frac{\partial \bar{P}}{\partial \bar{z}}+\mu(\frac{1}{\bar{r}}\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{r}}+\frac{\partial \bar{V}_{\bar{z}}}{\partial \bar{r}^{2}}) \quad (2.10)$$

(2.9)-(2.10) gives the governing equations that describe the axial motion of urine through the urethra in the axial direction. (2.9) justifies no slip condition assumed.

3. Mathematical Analysis

3.1 Nondimensionalisation

To solve (2.10), we nondimensionalise by introducing the following scaling where bars denote the dimensional variables as before. Let V_0 and R_0 be the characteristic velocity and characteristic length respectively. Then

$$\begin{aligned}&\bar{V}_{\bar{z}}=V_0V_z,\ \bar{P}=\rho PV_0^2,\ \bar{t}=\frac{R_0t}{V_0},\\&\bar{z}=R_0z(R_0\equiv Z_0),\ \bar{r}=rR_0\end{aligned} \quad (3.1)$$

Differentiating (3.1) gives

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$$\partial \overline{V}_{\overline{z}} = V_0 \partial V_z,\ \partial^2 \overline{V}_{\overline{z}} = V_0 \partial^2 V_z,\ \partial t = \frac{R_0 \partial t}{V_0} \tag{3.2}$$

$$\partial \overline{P} = \rho V_0^2 \partial P,\ \partial^2 \overline{P} = \rho V_0^2 \partial^2 P, \quad \partial \overline{r} = R_0 \partial r,\ \partial \overline{r}^2 = R_0^2 \partial r^2 \tag{3.3}$$

Substitute (3.2)-(3.3) into (2.10) to get

$$\frac{\rho V_0^2}{R_0}\frac{\partial V_z}{\partial t} = -\frac{\rho V_0^2}{R_0}\frac{\partial P}{\partial z} + \mu \frac{V_0}{R_0^2}\left(\frac{1}{r}\frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial r^2}\right) \tag{3.4}$$

Multiply (3.4) by $\frac{R_0}{\rho V_0^2}$ to obtain

$$\frac{\partial V_z}{\partial t} = -\frac{\partial P}{\partial z} + \frac{\mu}{\rho V_0 R_0}\left(\frac{1}{r}\frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial r^2}\right) \tag{3.5}$$

Let $Re = \frac{\rho V_0 R_0}{\mu}$ where Re is the Reynolds number. Then (3.5) becomes

$$\frac{\partial V_z}{\partial t} = -\frac{\partial P}{\partial z} + (Re)^{-1}\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\frac{\partial V_z}{\partial r}\right) \tag{3.6}$$

(3.6) gives the nondimensionalised form of the governing equation (2.10).

3.2 Method of Solution

Consider solutions to (3.6) of the form

$$V_z(r,t) = V(r)e^{-\alpha t} \tag{3.7}$$

$$P(z,t) = P(z)e^{-\alpha t} \tag{3.8}$$

α should be chosen such that the anticipated solution does not become unbounded over time. i.e. grow infinitely since voiding at intense urge occurs over a very short period.

Let $V = V_z(r,t)$, $P = P(z,t)$, $v = V(r)$ and $p = P(z)$. Then differentiate (3.7) and (3.8) to obtain

$$\frac{\partial V}{\partial t} = -\alpha v e^{-\alpha t},\quad \frac{\partial V}{\partial r} = \frac{dv}{dr}e^{-\alpha t},\quad \frac{\partial^2 V}{\partial r^2} = \frac{d^2 v}{dr^2}e^{-\alpha t},\quad \frac{\partial P}{\partial z} = \frac{dp}{dz}e^{-\alpha t}. \tag{3.9}$$

Substituting (3.9) into (3.6) gives

$$-\alpha v e^{-\alpha t} = -\frac{dp}{dz}e^{-\alpha t} + (Re)^{-1}\left(\frac{1}{r}\frac{dv}{dr}e^{-\alpha t} + \frac{d^2 v}{dr^2}e^{-\alpha t}\right) \tag{3.10}$$

Rearrange (3.10) to get

$$(Re)^{-1}\left(\frac{1}{r}\frac{dv}{dr} + \frac{d^2 v}{dr^2}\right) + \alpha v = \frac{dp}{dz} \tag{3.11}$$

Now we separate variables since the RHS and LHS of (3.11) are respectively functions of r and z alone so that (3.11) becomes

$$(Re)^{-1}\left(\frac{1}{r}\frac{dv}{dr} + \frac{d^2 v}{dr^2}\right) + \alpha v = \frac{dp}{dz} = A \tag{3.12}$$

where A is constant. i.e.

$$(Re)^{-1}\left(\frac{1}{r}\frac{dv}{dr} + \frac{d^2 v}{dr^2}\right) + \alpha v = A \tag{3.13}$$

$$\frac{dp}{dz} = A \Rightarrow p = Az + B \tag{3.14}$$

where B is a constant of integration.
From (3.13) we have that

$$\frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} + k^2 v = C \tag{3.15}$$

where $k^2 = \alpha(Re)$ and $C = A(Re)$.

3.21 Complementary Solution to (3.15)

Consider

$$\frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} + k^2 v = 0 \tag{3.16}$$

Let $r \equiv R$, $X \equiv v$ and observe that (3.16) is a Bessel differential equation with solution of the form

$$X^c = \alpha_1 J_0(kR) + \alpha_2 J_1(kR) \tag{3.17}$$

Since we require velocity to be finite, we demand that the solution be finite so that $\alpha_2 = 0$. i.e.

$$X^c = \alpha_1 J_0(kR) \tag{3.18}$$

3.22 Particular solution to (3.15)

A critical look at (3.15) revealed that the particular solution is given by

$$X^p = \frac{C}{k^2} \tag{3.19}$$

In general, the complete solution to (3.15) is

$$X = X^c + X^p \quad \Rightarrow \quad X = J_0(kR) + \frac{C}{k^2} \tag{3.20}$$

To see that (3.20) satisfies (3.15), let $\alpha_1 = 1$. Expand and differentiate (3.18) as follows:

$$X = J_0(kR) = 1 - \frac{(kR)^2}{2^2} + \frac{(kR)^4}{2^2.4^2} - \frac{(kR)^6}{2^2.4^2.6^2} + \ldots \tag{3.21}$$

$$\frac{dX}{dR} = -\frac{2k^2 R}{2^2} + \frac{4k^4 R^3}{2^2.4^2} - \frac{6k^6 R^5}{2^2.4^2.6^2} + \ldots \tag{3.22}$$

$$\frac{1}{R}\frac{dX}{dR} = -\frac{2k^2}{2^2} + \frac{4k^4 R^2}{2^2.4^2} - \frac{6k^6 R^4}{2^2.4^2.6^2} + \ldots \tag{3.23}$$

$$\frac{d^2 X}{dR^2} = -\frac{2k^2}{2^2} + \frac{12k^4 R^2}{2^2.4^2} - \frac{30k^6 R^4}{2^2.4^2.6^2} + \ldots \tag{3.24}$$

Substituting (3.21)-(3.24) into (3.15) gives

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$$-k^2(1-\frac{(kR)^2}{2^2}+\frac{(kR)^4}{2^2.4^2}-\frac{(kR)^6}{2^2.4^2.6^2}+...)$$
$$+k^2(J_0(kR)+\frac{C}{k^2})=C \quad (3.25)$$

$$-k^2J_0(kR)+k^2J_0(KR)+C=C \quad (3.26)$$

which clearly justifies that (3.20) is a solution to (3.15).
Now, substitute (3.14) and (3.20) into (3.7)-(3.8) accordingly to obtain

$$V_z(R,t)=(J_0(kR)+\frac{C}{k^2})e^{-\alpha t}$$
$$\Rightarrow V_z(R,t)=(J_0(kR)+\frac{A}{\alpha})e^{-\alpha t} \quad (3.27)$$

$$P(z,t)=(Az+B)e^{-\alpha t} \quad (3.28)$$

(3.27)-(3.28) gives the expressions for velocity along the urethra and bladder pressure respectively. It is noteworthy that the above derivation have been based on the dominance of dynamical effects at intense urge. In other words, we would anticpate that $Re \gg 1$.

Relationship between bladder pressure and velocity along the urethra:

$$P(z,t)=\frac{V_z(R,t)}{\alpha J_0(kR)+A}\alpha(Az+B) \quad (3.29)$$

An interesting observation from (3.27) would be to investigate (3.15) as $Re \to 0$. In other words, we would anticipate the dominance of viscous effects close to the end of the micturition process since naturally, velocity along the urethra tends to decrease. i.e. becomes smaller and smaller. This we hope to see in a sequel.

3.23 Asymptotic Analysis

Here we investigate extensively the behaviour of (3.15) as $k^2 \to 0$ since we would anticipate that the Reynold number is negligible, $Re \ll 1$. Invariably, we seek the asymptotic limit as $Re \to 0$ since relative to the bladder pressure at intense urge, we would anticipate the dominance of viscous effects close to the end of micturition process as velocity gets smaller and smaller.
Pose:

$$v=v_0+(k^2)v_1+(k^2)^2v_2+... \quad (3.30)$$

Differentiating (3.30) and substituting into (3.15), we obtain a leading order coefficient $(k^2)^0$:

$$\frac{d^2v_0}{dr^2}+\frac{1}{r}\frac{dv_0}{dr}=C \quad (3.31)$$

Equivalently, we have

$$\frac{1}{r}\frac{d}{dr}(r\frac{dv_0}{dr})=C \Rightarrow \frac{d}{dr}(r\frac{dv_0}{dr})=Cr \quad (3.32)$$

Integrating (3.32) gives

$$v_0=\frac{Cr^2}{4}+D \quad (3.33)$$

Hence, for $Re \ll 1$, the expression for velocity along the urethra is given by

$$V(r,t)=(\frac{Cr^2}{4}+D)e^{-\alpha t} \quad (3.34)$$

i.e.

$$V_z(R,t)=(\frac{CR^2}{4}+D)e^{-\alpha t} \quad (3.35)$$

(3.35) gives an asymptotic approximation to the urethral axial velocity close to the end of the micturition process where viscous effect is dominant.

4. Discussion of Results

Here we present a detailed discussion of our results aimed at answering our objective of determining the profiles of axial velocity along the urethra and pressure from the bladder. Consequently, we would also determine the relationship between the axial velocity and pressure during the micturition process.
Notice from Fig. 1 that at intense urge, the velocity along the urethra decreases exponentially over time. This could be attributed to the bladder pressure which also decreases (decays) exponentially over time as can be seen in Fig. 2.

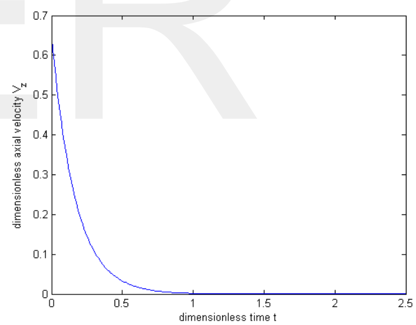

Fig. 1: urethral axial velocity versus time.

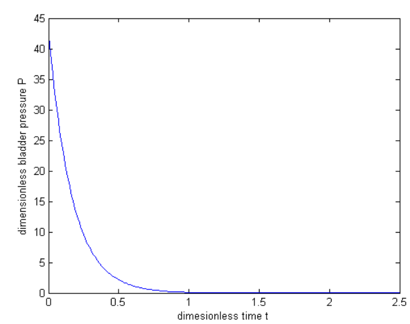

Fig. 2: shows bladder pressure versus time.

The relationship between velocity and pressure at intense urge have been found to be proportional to each other. In other words, as the bladder pressure decreases, the velocity along the urethra also decreases over time. This is immediate from

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Fig. 3.

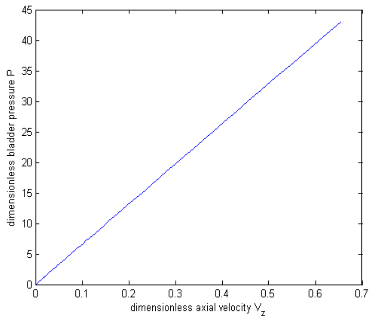

Fig. 3: relationship between bladder pressure and axial velocity.

Comparison between the radial distance and velocity along the urethra have been shown in Fig. 4. Therein, we see that the urine flow oscillates along the radial direction as urine flows down the urethra as a result of urethral constrictions. It's glaring the presence of rapid oscillations at the beginning of the micturition process which further justifies the dominance of dynamical effects. Also, we would anticipate that this could be as a result of anatomical differences between the urethral sphincter and prostatic urethra but is seen to become less erratic as the urine flows down the urethra into a region called membranous urethra.

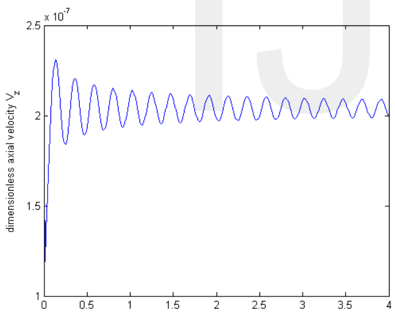

Fig. 4: radial direction versus axial velocity

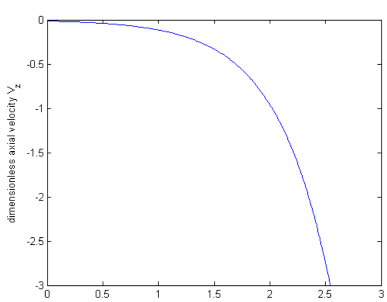

Fig. 5: Asymptotic approximation to axial velocity versus time

The asymptotic approximation to axial urethral velocity have been shown to decrease over time as in Fig. 5. This is expected since from Fig. 5, as the bladder pressure becomes zero i.e when the bladder have been emptied, the urethral sphincter closes, so that over time, the axial velocity at the upper part of the urethra becomes zero. Notice that the flow now moves down the urethra with a negative velocity, which of course gets smaller and smaller down the urethra with increasing regions of zero velocity at the upper part of the urethra. Thus further justifies the dominance of viscous effects close to the end of the micturition process.

5. Conclusions

A mathematical model describing the behaviour of bladder pressure and axial velocity at intense urge during voiding under an ideal condition have been presented with the assumption that there is no abnormality in the lower urinary tract (LUT).

In formulating the model, Navier-Stokes equation was used since the urethra is assumed to be cylindrical in shape. Urine was also assumed to have been stored in the bladder so the there is no bladder filling during the micturition process with the effects of gravity and abdominal straining neglected in conformity with [3]. Additionally, no slip condition was also assumed since there is no movement of urethral walls during voiding.

The governing equations were nondimensionalised and solved to obtaining expressions for bladder pressure, urethral axial velocity with their relationship considered. Interesting features of the method of solutions were the separation of variables, recognition of presence of Bessel's equations and the assumption that velocity is required to be finite.

The solutions to our model were analysed using MATLAB with suitable choice of values for arbitrary constants - $A=4$, $B=3$ and $\alpha=6$ and dimensionless parameters - $z=10$, $Re=100$, $R=4$ used to describe the effects of dominance of dynamic effects in the early stage of the micturition process. The dominance of viscous effects characterised by low velocity were described with values of A, R as before, $D=-0.03$, $Re=0.001$ and $\alpha=-2.112$. It's pertinent to note that the values of Re were chosen to reflect the dominance of dynamical/viscous effects at various stages of the micturition process.

Our model have been seen to have plausibly described the micturition process since in reality, at intense urge, the bladder pressure and velocity along the urethra decrease over time. Our model is not only in concordance with realism, it went further to inform that the profiles of bladder pressure and velocity decay exponentially over time at the initial stage of micturition and decreases with negative velocity when the bladder pressure becomes zero i.e. bladder becomes emptied.

Further research could be to investigate a problem for the bladder pressure enough to stretch the urethral walls. Additionally, we suspect that the behaviour of the asymptotic urethral axial velocity may not have been clearly depicted in the current model since it probably tends to smear out the possible associated discontinuities at the end of the micturition process. It's noteworthy that this phenomenon

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could vary in individuals who would patiently may not want to shake off/wipe their urinary organ close to the end of the micturition process. One may also wish to consider the effects of abdominal pressure as against [3].

A possible limitation to the model is the difficulty in determining exact values of the parameters and arbitrary constants which could vary in individuals. it is almost impossible to ascertain if our model is unisex.

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