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An Extension of the Degree of Approximation by Jackson type Operators

Huzoor H. Khan[1], Vishnu Narayan Mishra[2 *], Idrees A. Khan[3]

ABSTRACT

In this paper, we obtain the degree of approximation of functions belonging to class $\mathrm{Lip}\left(\psi\left(u_1,u_2,\cdots,u_n\right);p\right), p>1$ using the Jackson type operators of the n-Fourier series $f\left(x_1,x_2,\cdots,x_n\right)$. Here we extend the results of Huzoor H. Khan [1], Schurer, P. [4], Siddiqi, Mohammadzadeh [5], and Yoshimitsu H [7] et al.

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KEYWORDS AND PHRASES $\mathrm{Lip}\left(\psi\left(u_1,u_2,\cdots,u_n\right);p\right), p>1$ class, Jackson type operator, degree of approximation, Fourier series, Holder's inequality.

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1. INTRODUCTION AND RESULTS

Let the function $f\left(x_1,x_2,\cdots,x_n\right)$ be integrable and periodic with period 2 π. We define the n-Fourier series of $f\left(x_1,x_2,\cdots,x_n\right)$ be

$$\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\cdots\sum_{m_n=0}^{\infty} A_{m_1,m_2,\cdots,m_n}\left(x_1,x_2,\cdots,x_n\right) \qquad (1.1)$$

Where

$$A_{0,0,0,\cdots,0_n}\left(x_1,x_2,\cdots,x_n\right)=\frac{1}{2^n}a_{0,0,0,\cdots,0_n}$$

$$A_{m_1,0,0,\cdots,0_n}\left(x_1,x_2,\cdots,x_n\right)=\frac{1}{2^{n-1}}\left(a_{m_1,0,0,\cdots,0_n}\cos m_1x+b_{m_1,0,0,\cdots,0_n}\sin m_1x\right)$$

$$A_{0,m_2,0,\cdots,0_n}\left(x_1,x_2,\cdots,x_n\right)=\frac{1}{2^{n-1}}\left(a_{0,m_2,0,\cdots,0_n}\cos m_2x+b_{0,m_2,0,\cdots,0_n}\sin m_2x\right)$$

$$A_{0,0,m_3,\cdots,0_n}\left(x_1,x_2,\cdots,x_n\right)=\frac{1}{2^{n-1}}\left(a_{0,0,m_3,\cdots,0_n}\cos m_3x+b_{0,0,m_3,\cdots,0_n}\sin m_3x\right)$$

$$A_{m_1,m_2,\cdots,m_n}\left(x_1,x_2,\cdots,x_n\right)=a_{m_1,m_2,\cdots,m_n}\cos m_1x_1,\cdots,\cos m_nx_n+a_{m_1,m_2,\cdots,m_n}\cos m_1x_1,$$

$$\cdots,\cos m_{(n-1)}x_{(n-1)}\sin m_nx+a_{m_1,m_2,\cdots,m_n}\cos m_1x_1,\cdots,$$

$$\cdots,\cos m_{(n-2)}x_{(n-2)}\sin m_{(n-1)}x_{(n-1)}\sin m_nx$$

$$+\cdots a_{m_1,m_2,\cdots,m_n}\sin m_1x_1,\sin m_2x_2,\cdots,\sin m_nx_n+\cdots$$

$$+\,a_{m_1,m_2,\cdots,m_n}\sin m_1x_1,\sin m_2x_2,\cdots,\sin m_{(n-1)}x_{(n-1)}\cos m_nx_n$$

where we collect the coefficients $a_{m_1,m_2,\cdots,m_n}\to 2^n$ times.

We define the operators $L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}\left(f;x_1,x_2,\cdots,x_n\right)$,

(where, $m_1,m_2,\cdots,m_n$ are positive integers) as

$$L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}\left(f;x_1,x_2,\cdots,x_n\right)=\frac{1}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}$$

$$f\left(x_1+u_1,x_2+u_2,\cdots,x_n+u_n\right)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\left(\frac{\sin\frac{1}{2}m_n}{\sin\frac{1}{2}u_n}\right)^{2s}$$

$$du_1du_2,\cdots,du_n \qquad (1.2)$$

where

$$A_{m_ns-s}=\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s}$$

$$du_1du_2,\cdots,du_n \qquad (1.3)$$

In case s = 1, 2, 3, …, = n we have the Fejer and Jackson operators of n-Fourier series of $f\left(x_1,x_2,\cdots,x_n\right)$ respectively, i.e.

$$L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}\left(f;x_1,x_2,\cdots,x_n\right)=\frac{1}{2^n\pi^3\left(m_n,m_{(n-1)},\cdots,m_1\right)}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}$$

$$f\left(x_1+u_1,x_2+u_2,\cdots,x_n+u_n\right)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2},\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2}$$

$$du_1du_2,\cdots,du_n \qquad (1.4)$$

and

$$L_{m_n-2,m_{(n-1)}-2,\cdots,m_1-2}\left(f;x_1,\cdots,x_n\right)$$
$$=\frac{1}{2^n\pi^3\left(m_1^3,m_2^3,\cdots,m_n^3\right)+2\left(m_1,m_2,\cdots,m_n\right)\left(m_1^2,m_2^2,\cdots,m_n^2\right)+\left(m_1,m_2,\cdots,m_n\right)}$$

$$\times\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}f\left(x_1+u_1,x_2+u_2,\cdots,x_n+u_n\right)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^4$$
$$\times\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^4,\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^4du_1du_2,\cdots,du_n \qquad (1.5)$$

The form of the operators $L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}$ can be made more explicit by evaluation of the integral

$$A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}=\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots$$
$$\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s}du_1du_2,\cdots,du_n$$

$$>\left(2^{2n}\right)\int_0^{\frac{\pi}{2k}}\int_0^{\frac{\pi}{2n}},\cdots,\int_0^{\frac{\pi}{2m}}\left(\frac{\sin m_nu_1}{\sin u_1}\right)^{2s}\left(\frac{\sin m_{(n-1)}u_2}{\sin u_2}\right)^{2s},\cdots,$$
$$\left(\frac{\sin m_1u_n}{\sin u_n}\right)^{2s}du_1du_2,\cdots,du_n$$

$$\geq B\left(m_n2s-1,m_{(n-1)}2s-1,\cdots,m_12s-1\right) \qquad (1.6)$$

Where, B is an absolute constant and

$$\sin nt\geq nt-\frac{(nt)^3}{6}\Big\};\quad t\in\left[0,\frac{\pi}{2n}\right]$$

$$\sin t<t$$

and one can get,

$$\left(\frac{\sin m_nu}{\sin u}\right)^{2s}=\frac{1}{6}\rho_0^{m_ns-s}+\frac{1}{3}\sum_{r=1}^{m_ns-1}\rho_r^{m_ns-s}\cos ru\geq 0,\left(-\pi<u\leq\pi\right) \qquad (1.7)$$

The estimate for the integral A_{m_ns-s} (n^{th} variable, s fixed and is completely determined by the coefficients $\rho_0^{m_ns-s}$ in (1.7) by Schurer P.[4].

A function $f\left(x_1,x_2,\cdots,x_n\right)$ is said to belong to $Lip\left(\psi\left(u_1,u_2,\cdots,u_n\right);p\right),p>1$ if

$$\left|f\left(x_1+u_1,x_2+u_2,\cdots,x_n+u_n\right)-f\left(x_1,x_2,\cdots,x_n\right)\right|=O\left(\frac{\psi\left(u_1,u_2,\cdots,u_n\right)}{\left(u_1,u_2,\cdots,u_n\right)^{\frac{1}{p}}}\right),u_1,u_2,\cdots,u_n\to 0$$

where $\psi\left(u_1,u_2,\cdots,u_n\right)$ is a positive increasing function of variables u_1, u_2,…, and u_n. Yoshimitsu [7] proved a theorem for obtaining the degree of approximation of class of function Lip (α, β), 0 < α < 1, 0 < β < 1 by means of first arithmetic means of the double Fourier series. Later Siddiqi and Mohammadzadeh [5] extended the result in the two directions in terms of a positive increasing function of two variables.

Now the object of this paper is to find the degree of approximation for the functions belonging to the class $Lip\left(\psi\left(u_1,u_2,\cdots,u_n\right);p\right),p>1$ by means of operators $L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}$ (n = 1, 2, 3, …; s ≥ n + 1), and various investigators such as Huzoor H. Khan and G. Ram [2], Picugov [3] and Xhevat Z. Krasniqi [6], have obtained the degree of approximation of functions in various spaces. Our main result states as follows:

2. THEOREM

Let $f\left(x_1,x_2,\cdots,x_n\right)$ be a periodic function of period 2π with respect to each variables $\left(x_1,x_2,\cdots,x_n\right)$ belonging to $Lip\left(\psi\left(u_1,u_2,\cdots,u_n\right);p\right),p>1$, then

$$E_n^*(f)=\min_{L_n}\left|L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}\left(f;x_1,x_2,\cdots,x_n\right)-f\left(x_1,x_2,\cdots,x_n\right)\right|$$
$$=O\left\{\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)^{-\frac{1}{p}}\right\},\quad n=1,2,3\cdots;s\geq(n+1)$$

provided

$$\left\{\int_0^{\frac{\pi}{2m_n}}\int_0^{\frac{\pi}{2m_{(n-1)}}},\cdots,\int_0^{\frac{\pi}{2m_1}}\left(\frac{\left(2u_1,2u_2,\cdots,2u_n\right)\left|\phi\left(2u_1,2u_2,\cdots,2u_n\right)\right|}{\psi\left(2u_1,2u_2,\cdots,2u_n\right)}\right)^p du_1du_2,\cdots,du_n\right\}^{\frac{1}{p}}$$
$$=O\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right) \qquad (2.1)$$

and

$$\left\{\int_{\frac{\pi}{2m_n}}^{\pi}\int_{\frac{\pi}{2m_{(n-1)}}}^{\pi},\cdots,\int_{\frac{\pi}{2m_1}}^{\pi}\left(\frac{\left(2u_1,2u_2,\cdots,2u_n\right)^{-\delta}\left|\phi\left(2u_1,2u_2,\cdots,2u_n\right)\right|}{\psi\left(2u_1,2u_2,\cdots,2u_n\right)}\right)^p du_1du_2,\cdots,du_n\right\}^{\frac{1}{p}}$$
$$=O\left(m_n^\delta,m_{(n-1)}^\delta,\cdots,m_1^\delta\right) \qquad (2.2)$$

where δ is an arbitrary number such that $q(1-\delta)-1>0$ and $\frac{1}{p}+\frac{1}{q}=1$

PROOF OF THE THEOREM

We know that

$$L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}\left(f;x_1,x_2,\cdots,x_n\right)-f\left(x_1,x_2,\cdots,x_n\right)=\frac{1}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}$$

$$\times\int_{-\pi}^{\pi}\int_{-\pi}^{\pi},\cdots,\int_{-\pi}^{\pi}\phi\left(u_1,u_2,\cdots,u_n\right)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots$$

$$\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s} du_1du_2,\cdots,du_n$$

by Xhevat [6] for n-variables, one can get as

$$\phi(u_1,u_2,\cdots,u_n)=f(x_1+u_1,x_2+u_2,\cdots,x_n+u_n)+f(x_1+u_1,x_2+u_2,\cdots,x_n+u_n)+\cdots,$$

$$+f(x_1+u_1,x_2+u_2,\cdots,x_n+u_n)-2^n+\cdots,f(x_1,x_2,x_3,\cdots,x_n)$$

$$=\frac{2^n}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}\int_0^\pi\int_0^\pi,\cdots,\int_0^\pi \phi(u_1,u_2,\cdots,u_n)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}$$

$$\left(\frac{\sin\frac{1}{2}(n-1)u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s} du_1du_2,\cdots,du_n$$

$$=\frac{(2)^{2n}}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}\left[\int_0^{\frac{\pi}{2m_n}}\int_0^{\frac{\pi}{2m_{(n-1)}}},\cdots,\int_0^{\frac{\pi}{2m_1}},\cdots,\int_{\frac{\pi}{2m_n}}^{\pi}\int_{\frac{\pi}{2m_{(n-1)}}}^{\pi},\cdots,\int_{\frac{\pi}{2m_1}}^{\pi}\right]$$

$$\phi(2u_1,2u_2,\cdots,2u_n)\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s}$$

$$du_1du_2,\cdots,du_n=I_{1,1,\cdots,1_n}+I_{2,2,\cdots,2_n} \text{ (say)}$$

Applying Holder's inequality and the fact that $\phi(u_1,u_2,\cdots,u_n)\in Lip(\psi(u_1,u_2,\cdots,u_n);p),\ p>1$, for $I_{1,1,\cdots,1_n}$, we get

$$\left|I_{1,1,\cdots,1_n}\right|\le\frac{(2)^{2n}}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}\left\{\int_0^{\frac{\pi}{2m_n}}\int_0^{\frac{\pi}{2m_{(n-1)}}},\cdots,\right.$$

$$\left.\times\int_0^{\frac{\pi}{2m_1}}\left(\frac{(u_1,u_2,\cdots,u_n)\left|\phi(2u_1,2u_2,\cdots,2u_n)\right|}{\psi(2u_1,2u_2,\cdots,2u_n)}\right)^p du_1du_2,\cdots,du_n\right\}^{\frac{1}{p}}$$

.

$$\left\{\int_0^{\frac{\pi}{2m_n}}\int_0^{\frac{\pi}{2m_{(n-1)}}},\cdots,\int_0^{\frac{\pi}{2m_1}}\left[\frac{\psi(2u_1,2u_2,\cdots,2u_n)}{(u_1,u_2,\cdots,u_n)}\left|\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\right.\right.\right.$$

$$\left.\left.\left.\times\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s}\right|\right]^q du_1du_2,\cdots,du_n\right\}^{\frac{1}{q}}$$

$$\le\frac{C_1}{m_n^{2s-1},m_{(n-1)}^{2s-1},\cdots,m_1^{2s-1}}\cdot O\left[\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\right].$$

$$O\left(m_n^{2s-\frac{1}{q}},m_{(n-1)}^{2s-\frac{1}{q}},\cdots,m_1^{2s-\frac{1}{q}}\right)$$

where C_1 is absolute constant, using (2.1), we have

$$=O\left\{\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)^{-\frac{1}{p}}\right\}$$

Similarly

$$\left|I_{2,2,\cdots,2_n}\right|\le\frac{(2)^{2n}}{A_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}}\left\{\int_{\frac{\pi}{2m_n}}^{\pi}\int_{\frac{\pi}{2m_{(n-1)}}}^{\pi},\cdots,\right.$$

$$\left.\times\int_{\frac{\pi}{2m_1}}^{\pi}\left(\frac{(u_1,u_2,\cdots,u_n)^{-\delta}\left|\phi(2u_1,2u_2,\cdots,2u_n)\right|}{\psi(2u_1,2u_2,\cdots,2u_n)}\right)^p du_1du_2,\cdots,du_n\right\}^{\frac{1}{p}}.$$

$$\left\{\int_{\frac{\pi}{2m_n}}^{\pi}\int_{\frac{\pi}{2m_{(n-1)}}}^{\pi},\cdots,\int_{\frac{\pi}{2m_1}}^{\pi}\left[\frac{\psi(2u_1,2u_2,\cdots,2u_n)}{(u_1,u_2,\cdots,u_n)}\left|\left(\frac{\sin\frac{1}{2}m_nu_1}{\sin\frac{1}{2}u_1}\right)^{2s}\right.\right.\right.$$

$$\left.\left.\left.\times\left(\frac{\sin\frac{1}{2}m_{(n-1)}u_2}{\sin\frac{1}{2}u_2}\right)^{2s},\cdots,\left(\frac{\sin\frac{1}{2}m_1u_n}{\sin\frac{1}{2}u_n}\right)^{2s}\right|\right]^q du_1du_2,\cdots,du_n\right\}^{\frac{1}{q}}$$

$$\le\frac{C_2}{m_n^{2s-1},m_{(n-1)}^{2s-1},\cdots,m_1^{2s-1}}\cdot O\left[\psi\left(m_n^\delta,m_{(n-1)}^\delta,\cdots,m_1^\delta\right)\right].O\left[\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\right].$$

$$O\left(m_n^{\left(-\delta+2s-\frac{1}{q}\right)},m_{(n-1)}^{\left(-\delta+2s-\frac{1}{q}\right)},\cdots,m_1^{\left(-\delta+2s-\frac{1}{q}\right)}\right)$$

where C_2 is absolute constant, using (2.2), we get

$$=O\left\{\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)^{-\frac{1}{p}}\right\}$$

$$E_n^*(f)=\min_{L_n}\left|L_{m_ns-s,m_{(n-1)}s-s,\cdots,m_1s-s}(f;x_1,x_2,\cdots,x_n)-f(x_1,x_2,\cdots,x_n)\right|$$

$$=O\left\{\psi\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)\left(\frac{1}{m_n},\frac{1}{m_{(n-1)}},\ldots,\frac{1}{m_1}\right)^{-\frac{1}{p}}\right\},$$

$$n=1,2,3\cdots;s\ge n+1$$

which completes the proof of the theorem.

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[1]Department of Mathematics,
Aligarh Muslim University, Aligarh - 202002, India.
E-mail: huzoorkhan@yahoo.com

[2*] Department of Applied Mathematics and Humanities,
Sardar Vallabhbhai National Institute of Technology, Ichchhanath
Mahadev Road, Surat – 395 007 (Gujarat), India
E-mail: vishnu_narayanmishra@yahoo.co.in

[3]Department of Applied Mathematics,
Aligarh Muslim University, Aligarh - 202002, India.
E-mail: idrees_maths@yahoo.com