ISSN 2229-5518

A Super Edge Anti-Magic Total Labeling of the Cycle C_n with P_3 Chords

L.Girija[a] , A.Elumalair[b]
[a] Research Scholar, *Department of Mathematics*, Bharathiar University, Coimbatore-641046
A.Elumalai [b]
Department of Mathematics,
Valliammai Engineering College, Chennai-603203

Abstract— In this paper, we have proved super edge anti-magic total labeling of cycles with P_3 chords.

Index Terms—. Graph, Graph Labeling, edge anti-magic vertex labeling, super edge anti –magic vertex labeling, edge anti-magic total labeling, super edge anti –magic total labeling, cycle with P_3 chords.

---

1 INTRODUCTION

1.1 OVERVIEW:

All graphs are finite, simple and undirected. The *graph G* has vertex-set V (G) and edge-set E(G). Unless otherwise noted, V (G) = v and E(G) = e.

A *labeling* of a graph is any map that carries some set of graph elements to numbers (usually to the positive or non-negative integers). Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of "constant-sum" property.

Simanjuntak, Miller and Bertault [7] defined *an (a, d)-edge-antimagic vertex* ((a, d)-EAV) labeling for a graph G(V,E) as an injective mapping *f* from *V* onto the set {1, 2, . . . , n} with the property that the edge-weights $\{w(xy) : w(xy) = f(x) + f(y), xy \in E\}$, form an arithmetic sequence with the first term *a* and difference *d*, where $a > 0$ and $d \geq 0$ are two fixed integers. An (a, d)-EAT labeling is called *super (a, d)-edge antimagic total* ((a, d)-SEAT) labeling if f(V) = {1, 2, . . . , n}.

Acharya and Hegde [1] (see also [5]) introduced the concept of a strongly (a, d)-indexable labeling which is equivalent to (a, d)-EAV labeling. The relationship between the sequential graphs and the graphs having an (a, d)-EAV labeling is shown in [3].

An *(a, d)-edge antimagic total* ((a, d)-EAT) labeling is a bijection f from $V \cup E$ onto {1, 2, . . . , v + e} with the property that the sums of the label on the edges and the labels of their end points form an arithmetic sequence starting from *a* and having a common difference *d*. This labeling is a natural extension of the notion of edge magic labeling which was originally introduced by Kotzig and Rosa in [6], where edge-magic labeling is called magic valuation. Relationships between (a, d)-EAT labeling and other labelings, namely, (a, d)-EAV labeling are presented in [2]. An (a, d)-EAT labeling is called *super (a, d)-edge antimagic total* ((a, d)-SEAT) labeling if

f(V) = {1, 2, . . . , n}. This labeling is a natural extension of the notion of a super edge-magic labeling defined by Enomoto *et al.* in [4]. A graph that has an (a, d)-EAV ((a, d)-EAT or (a, d)-SEAT) labeling is called an (*a, d*)-EAV ((*a, d*)-EAT or (a, d)-SEAT) graph.

A cycle with P_3 chords is a graph obtained from a cycle C_n ($n \geq 5$, $n \neq 6$) by adding path P_3 joining two non-consecutive vertices of the cycle.

In this section we proved the super edge anti-magic labeling of the cycle C_n with P_3 chords.

Theorem: 1

A cycle C_n with $1P_3$ chord has a super edge anti-magic total labeling.

Proof:

Let C_n be a cycle on n vertices. We denote the vertices of C_n as $v_1, v_2, v_3, \ldots, v_n$ in the clockwise direction and denote the edges of C_n with P_3 chords as $e_1, e_2, e_3, \ldots, e_{n+2}$ such that $e_i = v_i v_{i+1}$ for $1 \leq i \leq n-1$, $e_n = v_n v_1$.

Case:1 C_n , $n \geq 5$ (n is odd)

A vertex which divide the chord is named as v_{n+1} and the edges of the chord are named as $e_{n+1} = v_{n+1} v_n$ and $e_{n+2} = v_2 v_{n+1}$

The labeling for the vertices of C_n with P_3 chord is given as follows. Define

$f(v_i) = (i+1)/2$, $1 \leq i \leq n$, i odd

$f(v_i) = (n+1+i)/2$, $2 \leq i \leq n-1$, i even

$f(v_{n+1}) = n+1$.

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as

$f(e_i) = f(v_i) + f(v_{i+1}) + ((n+1)/2)$ for $1 \leq i \leq n-1$,

$f(e_n) = f(v_n) + f(v_1) + ((n+1)/2)$

$f(e_{n+1}) = f(v_{n+1}) + f(v_n) + ((n+1)/2)$ and

$f(e_{n+2}) = f(v_2) + f(v_{n+1}) + ((n+1)/2)$

Hence the cycle C_n with P_3 chord has a ((3n+7/2),2)-super edge anti-magic total labelling.

Case:2 $C_{n=4m}$ for m≥2 (n is even)

A vertex which divide the chord is named as v_{n+1} and the edges of the chord are named as $e_{n+1}=v_{n+1}v_1$ and $e_{n+2}=v_{n-2}v_{n+1}$.

The labeling for the vertices of C_n with P_3 chord is given as follows. Define

$f(v_i) = (i+1)/2, \quad 1\le i\le \frac{n-2}{2}$, i odd

$f(v_i) = (n+i+1)/2, \quad \frac{n+2}{2}\le i\le n-3$, i odd

$f(v_i) = (i/2)+1, \quad n/2\le i\le n$, i even

$f(v_i) = (n+i+2)/2, \quad 2\le i\le \frac{n-4}{2}$, i even

$f(v_{n-1}) = n+1$,

$f(v_{n+1})= n$,

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,

$f(e_i)= f(v_i)+f(v_{i+1}) +((n+2)/2)$ for $1\le i\le n-1$,

$f(e_n)=f(v_n)+f(v_1) +((n+2)/2)$

$f(e_{n+1})= f(v_{n+1})+f(v_1) +((n+2)/2)$ and

$f(e_{n+2})=f(v_{n-2})+f(v_{n+1}) +((n+2)/2)$

Hence the cycle C_n with P_3 chord has a ((3n+6/2,2)-super edge anti-magic total labellingedge anti-magic total labelling.

Case:3 $C_{n=4m+2}$ for m≥2 (n is even)

A vertex which divide the chord is named as v_{n+1} and the edges of the chord are named as $e_{n+1}=v_{n+1}v_2$ and $e_{n+2}=v_{n-3}v_{n+1}$.

The labeling for the vertices of C_n with P_3 chord is given as follows. Define

$f(v_i) = (i+1)/2, \quad 1\le i\le \frac{n}{2}$, i odd

$f(v_i) = (n+i+3)/2, \quad \frac{n+4}{2}\le i\le n-1$, i odd

$f(v_i) = (n+i+4/2), \quad 2\le i\le \frac{n-2}{2}$, i even

$f(v_i) = (i+2)/2, \quad \frac{n+2}{2}\le i\le n-4$, i even

$f(v_i) = (i+4)/2, \quad n-2\le i\le n$

$f(v_{n+1}) = n/2$,

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,

$f(e_i)=f(v_i)+f(v_{i+1}) +(n/2)$ for $1\le i\le n-1$,

$f(e_n)=f(v_n)+f(v_1) +(n/2)$

$f(e_{n+1})= f(v_{n+1})+f(v_2) +(n/2)$ and

$f(e_{n+2})=f(v_{n-3})+f(v_{n+1}) +(n/2)$

Hence the cycle C_n with P_3 chord has a ((3n+8), 2)-super edge anti-magic total labelling.

Theorem: 2

A cycle C_n with $2P_3$ chords has a super edge anti-magic total labeling

Proof:

Let C_n be a cycle on n vertices. We denote the vertices of C_n as $v_1,v_2,v_3,\ldots,v_n$ in the clockwise direction and denote the edges of C_n with $2P_3$ chords as $e_1,e_2,e_3,\ldots,e_{n+4}$ such that $e_i= v_iv_{i+1}$ for $1\le i\le n-1$, $e_n=v_nv_1$.

Case:1 C_n , n≥5 (n is odd)

The vertices which divide the chords are named as v_{n+1} and v_{n+2} and the edges of the chords are named as $e_{n+1}=v_{n+1}v_2$, $e_{n+2}=v_{n+2}v_2$ and $e_{n+3}=v_4v_{n+2}$, $e_{n+4}=v_nv_{n+1}$.

The labeling for the vertices of C_n with $2P_3$ chords are given as follows. Define,

$f(v_i) = (i+1)/2, \quad 1\le i\le n$, i odd

$f(v_i) = (n+1+i)/2, \quad 2\le i\le n-1$, i even

$f(v_{n+1}) = n+1$

$f(v_{n+2}) = n+2$

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,

$f(e_i)= f(v_i)+f(v_{i+1}) +((n+3)/2)$ for $1\le i\le n-1$,

$f(e_n)= f(v_n)+f(v_1) +((n+3)/2)$

$f(e_{n+1})= f(v_{n+1})+f(v_2) +((n+3)/2)$

$f(e_{n+2})= f(v_{n+2})+f(v_2) +((n+3)/2)$

$f(e_{n+3})= f(v_4)+f(v_{n+2}) +((n+3)/2)$ and

$f(e_{n+4})= f(v_n)+f(v_{n+1}) +((n+3)/2)$.

Hence the cycle C_n with $2P_3$ chord has a ((3n+9/2),2)-super edge anti-magic total labelling.

Case:2 $C_{n=4m}$ for m≥2 (n is even)

The vertices which divide the chords are named as v_{n+1} and v_{n+2} and the edges of the chords are named as $e_{n+1}=v_{n+1}v_1$, $e_{n+2}=v_{n+2}v_1$ and $e_{n+3}=v_{n-2}v_{n+2}$, $e_{n+4}=v_{n-1}v_{n+1}$.

The labeling for the vertices of C_n with $2P_3$ chords are given as follows. Define

$f(v_i) = (i+1)/2, \quad 1\le i\le \frac{n-2}{2}$, i odd

$f(v_i) = (n+i+5)/2, \quad \frac{n+2}{2}\le i\le n-1$, i odd

$f(v_i) = (i+2/2), \quad n/2\le i\le n$, i even

$f(v_i) = (n+i+6)/2, \quad 2\le i\le \frac{n-4}{2}$, i even

$f(v_{n+1}) = (n+4)/2$

$f(v_{n+2})= (n+6)/2$

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,

$f(e_i)=f(v_i)+f(v_{i+1})+((n+4)/2)$ for $1\le i\le n-1$

ISSN 2229-5518

$f(e_n)=f(v_n)+f(v_1)+((n+4)/2)$
$f(e_{n+1})=f(v_{n+1})+f(v_1)+((n+4)/2)$
$f(e_{n+2})=f(v_{n+2})+f(v_1)+((n+4)/2)$
$f(e_{n+3})=f(v_{n-2})+f(v_{n+2})+((n+4)/2)$ and
$f(e_{n+4})=f(v_{n-1})+f(v_{n+1})+((n+4)/2)$.

Hence the cycle C_n with $2P_3$ chord has a ((3n+8/2,2)-super edge anti-magic total labelling.

Case:3 $C_{n=4m+2}$ for $m\geq 2$ (n is even)

The vertices which divide the chords are named as v_{n+1} and v_{n+2} and the edges of the chords are named as $e_{n+1}=v_{n+1}v_{n-3}$, $e_{n+2}=v_{n+2}v_{n-3}$ and $e_{n+3}=v_3v_{n+2}$, $e_{n+4}=v_2v_{n+1}$.

The labeling for the vertices of C_n with $2P_3$ chords are given as follows. Define

$f(v_i) = (i+1)/2, \quad 1\leq i\leq \frac{n}{2}$, i odd

$f(v_i) = (n+i+5)/2, \quad \frac{n+4}{2}\leq i\leq n-1$, i odd

$f(v_i) = (n+i+6)/2, \quad 2\leq i\leq \frac{n-2}{2}$, i even

$f(v_i) = (i+2/2), \quad \frac{n+2}{2}\leq i\leq n-4$, i even

$f(v_i) = (i+6)/2, \quad n-2\leq i\leq n$, i even
$f(v_{n+1}) = n/2$
$f(v_{n+2}) = (n+2)/2$

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,
$f(e_i)= f(v_i)+f(v_{i+1})+((n+2)/2)$ for $1\leq i\leq n-1$,
$f(e_n)= f(v_n)+f(v_1)+((n+2)/2)$
$f(e_{n+1})= f(v_{n+1})+f(v_{n-3})+((n+2)/2)$
$f(e_{n+2})= f(v_{n+2})+f(v_{n-3})+((n+2)/2)$
$f(e_{n+3})= f(v_3)-+f(v_{n+2})+((n+2)/2)$, and
$f(e_{n+4})= f(v_2)+f(v_{n+1})+((n+2)/2)$.

Hence the cycle C_n with $2P_3$ chord has a ((3n+10),2)-super edge anti-magic total labelling.

References

[1] B.D. Acharya and S.M. Hegde, Strongly indexable graphs, *Discrete Math.* **93** (1991), 275–299.

[2] M. Baˇca, Y. Lin, M. Miller and R. Simanjuntak, New constructions of magic and antimagic graph labelings, *Utilitas Math.* **60** (2001), 229–239.

[3] M. Baˇca andM.Z. Youssef, Further results on antimagic graph labelings, *Austral.J. Combin.* **38** (2007), 163–172. 272 RAHMAWATI, SUGENG, SILABAN, MILLER AND BAˇCA

[4] H. Enomoto, A.S. Llad´o, T. Nakamigawa and G. Ringel, Super edge-magic graphs, *SUT J. Math.* **34** (1998), 105–109.

[5] S.M. Hegde, On indexable graphs, *J. Combinatorics, Information and System Sciences* **17** (1992), 316–331.

[6] A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.***13** (1970), 451–461.

[7] R. Simanjuntak, M. Miller and F. Bertault, Two new (*a, d*)-antimagic graph labelings, *Proceedings of the Eleventh Australasian Workshop on Combinatorial Algorithms* (2000), 179–189.

[8] J. A. MacDougall and W.D. Wallis ,Strong edge-magic labelling of a cycle with a chord, Australasian journal of combinatorics Volume **28** (2003), Pages 245–255,

[9] Gallian. J. A. (2011), 'A Dynamic survey of graph labeling', Electronic J. Combinatirics