Earthquake Versus Electric Field
(A Resistant Design with Piezo Ceramic Materials)

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Abstract This paper relates to   vibration control using piezo electric materials. The vibration control efficiency relies on the   optimization of the vibration energy transfer between a structure   and   Lead-Zirconate-Titanate (PZT) piezoelectric ceramic material. In this   paper, an analytical study of   electric field produced by   applied stresses of earthquake vibrations to control and damp them is presented .The influence of electric field formed around ceramic material forms   a shield to further counter   effect   the upcoming vibrations is discussed.

Index Terms Coupling ElectriC field , Piezo Lead-Zirconate-titanate (PZT), Shield, Strain, Stress, Voltage

1. INTRODUCTION

Earthquake resistant design structures has been a subject of engineering research for   the past few decades till now. Many   Researches in this field have undergone to   control the vibratory effect of earthquakes. Earthquakes occur   due to sudden release of   stored   energy . Most earthquakes take place along faults in the upper 25 miles of the earth's   surface   when one side rapidly moves relative to   the otherside   of the fault.   This sudden   motion causes shock waves (seismic waves) to radiate   from their point of   origin called the **focus** and travel   through   the earth. The   stresses developed due to these vibrations are firstly   affect   the foundation of a structure. Hagood and   von Flotow   (1991) have demonstrated   vibrations   of   a structure   can be controlled   with   piezo materials . The piezoelectric   materials   convert vibration   energy to electrical energy by polarizing themselves. Due to polarization voltage is induced which can be used to   create a 3d   network   or cover of electrical fields around . The cover of field formed   act as a   shield between   the foundation   and earthquake vibrations.

2. WORKING OF PIEZO MATERIALS

2.1 Direct Piezoelectric Effect

Deformation   of the piezo material causes an electrical charge on certain opposite faces of   the piezo   electric   material. The separation between   negative and positive charge sites (the electric dipole) leads   to   a   net polarization. The polarization   generates   a

voltage. The produced voltage can be used to create electric field lines. These field lines generated

are in form of concentric circles around   the crystal.

Figure 1. shows   a schematic diagram of piezo material with applied force . The force leads to polarization of the crystal with positive side on   upper side   and negative down side.
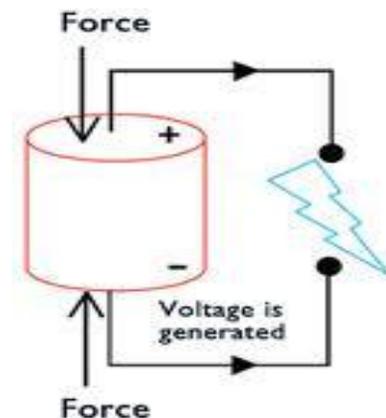

Figure 1. Generation of Voltage

This generates   a   high   power voltage.   Due to high power voltage current is set   up as shown. This set up current has tendency   to create Shield of electric field around them.

a) When a poled ceramic material is maintained below its curie temperature and is subjected to a macroscopic expansion along the poling axis and contraction perpendicular to it dipoles respond collectively to produce an effective electric field.
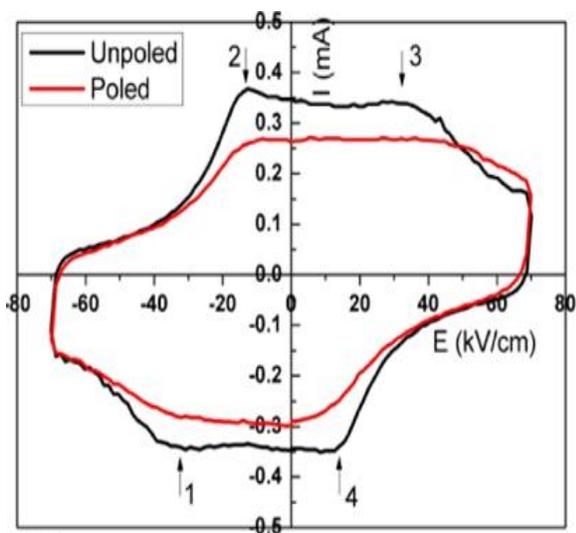

Figure 2. A plot for voltage produced current and electric field in Piezo material

b) Electric field due to geometry and deformation of PZT poled in the 3 direction of a simple cube.

Figure 3. Electric field lines around cubic Crystal

2.2 Electric Field As Shield

The key concept of this paper lies in the fact that laying of such materials within the foundations of structures such that in conditions of earthquakes these material experiences the net effective stress resulting in effective strain which Creates electric field pattern This pattern acts as protecting shield or umbrella for the upcoming vibrations. The shield will continue until the effect of vibrations last longs and keep on acting against the vibrations of earthquakes by strong opposite head on collisions against them.

2.3 Actuation Of Structural Components By Piezoelectric Crystals
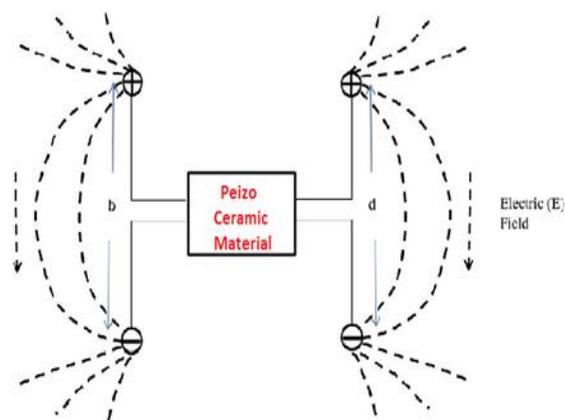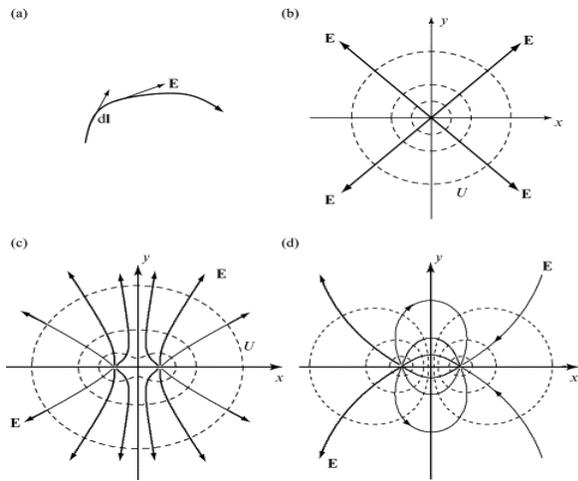
3. Basic Key Concept Model

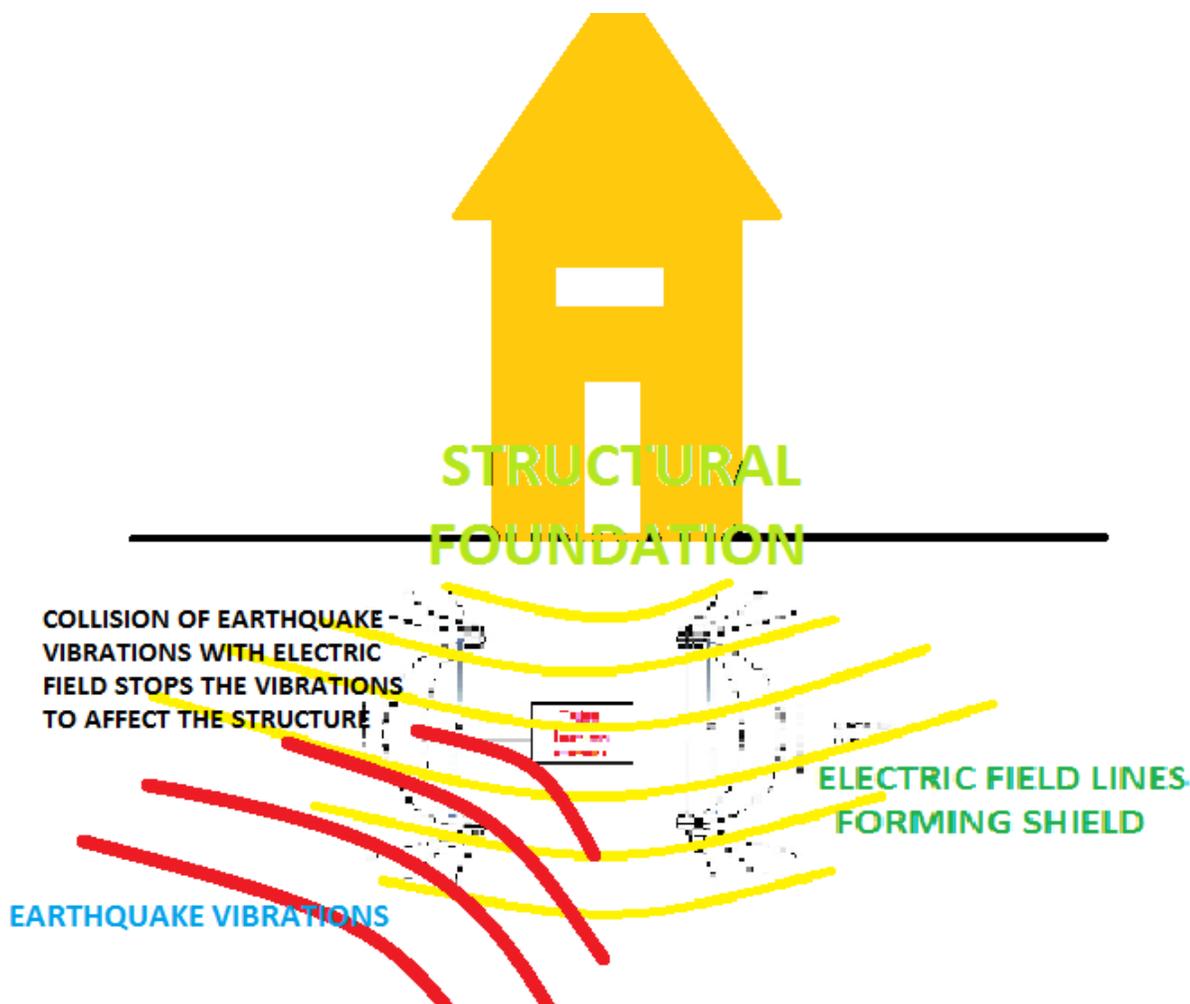Figure 4.   Electric Field Patterns forming Field
line Shield

Figure 5. Research Concept Model

Figure 5 illustrates the basic concept of the research work. The model represents the structure installed with a piezo ceramic material . This material when acted upon by earthquake vibrations generates strong yellow colour electric field around the foundation of structure. Collision between electric field and earthquake vibrations occurs which results in resisting and damping of destructive vibrations and safeguarding the structure from hazardous earthquake.

4. ELECTRIC FIELD AND STRAIN RELATIONSHIP

4.1 "g" Constant:

The piezoelectric constants relating the electric field produced by a mechanical stress are termed the voltage constants, or the "g" coefficients. The units may then be

expressed as **volts/meter per newtons/square meter**.

$$g = \frac{\text{open circuit electric field}}{\text{applied mechanical stress}}$$

Output electric field is obtained by dividing the calculated voltage by the thickness of ceramic between electrodes . A

"33" subscript indicates that the electric field   and the mechanical stress are both along the   polarization   axis . A "31" subscript signifies that the pressure is applied at right angles   to   the   polarization   axis, but the voltage appears on the same electrodes as in the "33" case. A   "15" subscript   implies that the applied   stress is   shear and that the resulting electric field is perpendicular to the polarization axis. High g_{ij} constants   favor large voltage output.   Although the g coefficient are called voltage coefficients, it is   also correct to say the g_{ij} is the ratio of strain   developed over the applied   charge density with units of meters per meter over coulombs per square meter.

4.2 "d_{ij}" Constant

Strain refers   to effective   deformation produced due to   application of   stresses. Relationship   between field strength and strain is   quantified   by the piezoelectric moduli d_{ij}.

Where
i : direction of the electric field
j : direction of the resulting normal strain
For example:

$$\varepsilon_{33} = d_{33} \cdot \frac{V}{t},$$

$$\varepsilon_{11} = d_{31} \cdot \frac{V}{t},$$

Where
V : voltage applied in the 3-direction
t : thickness of the specimen

Typical values of the piezoelectric of piezoelectric   moduli are given   in following table:

Table. Typical Piezoelectric Moduli of PZT materials (V/m)

	d_{33}	d_{31}
Hard PZT	225×10^{-12}	-100×10^{-12}
Soft PZT	600×10^{-12}	-275×10^{-12}

 For the same produced strain due toequal applied stresses, soft

PZT will generate strong electric shield.

5. MATHEMATICAL SOLUTIONS WITH MODEL REFERENCES   FOR GENERATED VOLTAGE AND FIELD

5.1 Electrical Equivalent Circuit

The   study   of   the   transient   dynamic characteristics   of   a   PZT   bender utilizing electrical   equivalent   models has been performed   in previous studies and   the   model   has   shown   fair accuracy in various conditions of   mechanical   stress The electrical equivalent model has been studied   and implemented   in this research compare the accuracy and validity of the and   the   analytical   results   from models   based   on   beam   theory   and   energy conservation for   PZT   beam.
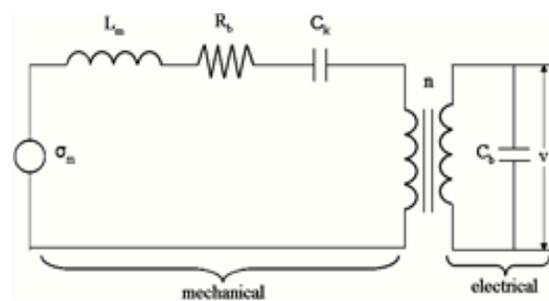

Figure 6. An electric equivalent circuit model for a PZT beam

where a voltage source is connected with an inductor, a resistor and a capacitor that build a resonant circuit. The transformer represents the voltage adaptation while the capacitor indicates the inherent capacitance of the device.

The circuit can be described by using Kirchhoff's voltage law:

$$\sigma_{in} = L_m \varepsilon + R_b \dot{\varepsilon} + \frac{\ddot{\varepsilon}}{C_k} + nV \tag{1}$$

$$i = C_k \dot{V}. \tag{2}$$

The equivalent circuits leads to the correlation between the strain e and voltage V.

$$\ddot{\varepsilon} = \frac{-Y}{k_1 k_2 m}\varepsilon - \frac{b_m}{k_1 m}\dot{\varepsilon} + \frac{Y}{k_1 k_2 m}\frac{d_{31}}{2t_c}V + \frac{\ddot{y}}{k_2},$$

and

$$V = \frac{n_p t_c d_{31} Y}{\varepsilon}\dot{\varepsilon}$$

where

$\ddot{\varepsilon}, \dot{\varepsilon} \rightarrow$ second and first time derivative of strain.

5.2 Beam Theory (Timoshenko and Euler–Bernoulli)

The static analysis of a piezoelectric cantilever sensor is typically performed by the use of calculations employed for de ection of a thermal bimorph proposed by Timoshenko The principle is based on the strain compatibility between three heterogeneous bimorph where two piezoelectric layers are bonded on cantilever beams joined along the along the bending axis . Due to forces applied by one or all of the layers, the de ection of the three layer structure is derived from a static equilibrium state. The structure considered is a piezoelectric is assumed to be approximately the same to those of the structure, simply because of the assumption both sides of a purely elastic layer, i.e. brass with a pure elasticity is sandwiched between the upper and lower layers of the PZT material. The beam thickness is much less than the piezoelectric – induced curvature. The radius of curvature for all the layers that the thickness is much less than the overall beam curvature. The total strain at the surface of each layer is the sum of the strains caused by the piezoelectric effect the axial force, and the bending.

$$\varepsilon_i = \varepsilon_{\text{piezo}} + \varepsilon_{\text{axial}} + \varepsilon_{\text{bend}} = d_{31}E_i + \frac{F_i}{A_i Y_i} \pm \frac{t_i}{2r} \quad (5)$$

where εpiezo in the linear constitutive equation above considers the transverse piezoelectric coupling coefficient d31 and the electric field across the thickness of the layer Ei· for a piezoelectric material, t1 and t3 are the thickness of the PZT layer and t2 is the thickness of the center shim, Ai is the area of the corresponding layer and Yi is the young's modulus of elasticity.

Hence the radius of curvature is given by the term:

$$\frac{1}{r} = \frac{2d_{31}DA^{-1}C}{2 - DA^{-1}B}$$
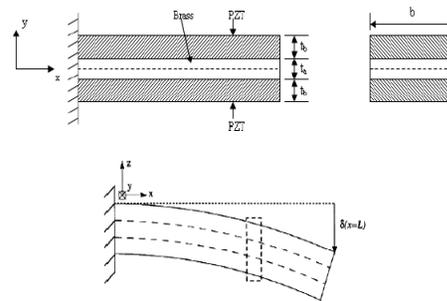

Figure7. Geometry of strain beam due to Stresses

where

$$A = \begin{bmatrix} \frac{1}{A_1 Y_1} & -\frac{1}{A_2 Y_2} & 0 \\ 0 & \frac{1}{A_2 Y_2} & -\frac{1}{A_3 Y_3} \\ 1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} t_1 + t_2 \\ t_2 + t_3 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -E \\ E \\ 0 \end{bmatrix}.$$

Euler–Bernoulli beam theory describes the relationship between the radius of curvature and the force applied, by the following equation:

$$\rho A \frac{\partial^4 w(x,t)}{\partial t^4} + YI \frac{\partial^4 w(x,t)}{\partial x^4} = F(t) \quad (7)$$

where ρ is the density, I is the moment of inertia and $F(t)$ is the applied force. A general solution for this equation is given by

$$w(x,t) = \sum q_i(t)X_i(x)$$

Equations For Vibrations Produced Due to Displacement :

$$X_i(x) = \cosh(\beta_i x) - \cos(\beta_i x) - \frac{\sinh(\beta_i L) - \sin(\beta_i L)}{\cosh(\beta_i L) + \cos(\beta_i L)}$$
$$\times (\sinh(\beta_i x) - \sin(\beta_i x)) \tag{9}$$

$$q_i(t) = \frac{1}{\omega_{di}} e^{-\zeta \omega_{ni} t} \int_0^t F_i(\tau) e^{-\zeta \omega_{ni} t} \sin(\omega_{di}(t - \tau)) \, d\tau \tag{10}$$

and

$$\beta_i^4 = \frac{\omega_{ni}^2}{C^2} \tag{11}$$

ωn, is the natural frequency obtained by solving the transcendental equation:

$$\mathrm{Cosh}(\beta_i L)\mathrm{Cos}(\beta_i L) + 1 = 0. \tag{12}$$

The radius of curvature is given by the following equations:

$$r = \frac{1}{2w(L)}L^2, \qquad \text{where } \frac{1}{r} = \ddot{w}(x) \text{ and } w(x) = \frac{1}{2r}x^2.$$

Hence by substituting the radius of curvature term in equation (6), **the voltage produced for the PZT** is given by:

$$V = \frac{2w(L)t_p}{L^2} \frac{2 - DA^{-1}B}{2d_{31}DA^{-1}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}^{-1}. \tag{13}$$

5.3 Conservation Of Energy

The principle is based on the fact that the total energy of the PZT bender stored is equal to the sum of the mechanical energy applied to the beam. When a mechanical stress is applied, the energy stored in a PZT layer is the sum of the mechanical energy and the electric-field-induced energy. Thus, the energy stored in a PZT layer is expressed as follows:

$$U_u = \frac{1}{2}(s_{11}^E \sigma_1 - d_{31} E_3)\sigma_1 = \frac{1}{2}s_{11}^E \sigma_1^2 \tag{14}$$

where σ is the stress and s is the stiffness matrix. The energy in the metal layer can be expressed by a simple equation

$$U_m = \frac{1}{2}s_m \sigma_1^2. \tag{15}$$

The total energy of the beam is given as [15]:

$$U_{\text{total}} = \int_0^L \int_0^W \left(\int_{\frac{t_2}{2}}^{\frac{t_2}{2}+t_1} dU_u \, dz + \int_{-\frac{t_2}{2}}^{\frac{t_2}{2}} dU_m \, dz \right.$$
$$\left. + \int_{-\frac{t_2}{2}-t_3}^{-\frac{t_2}{2}} dU_l \, dz \right) dy \, dx. \tag{16}$$

the electric field is given by $E = V/(2t3)$.

The total electrical energy is equal to a product of the charge and the voltage. Thus, the charge generated in the beam is obtained by a partial derivative of the total energy with respect to the voltage:

$$Q = \frac{\partial U_{\text{total}}}{\partial V} = -3 \frac{d_{31} s_m (t_2 + t_3) L^2}{X} F_o.$$

The capacitance of the piezoelectric material is described as the relation between the voltage and charge on the piezoelectric material.

$$C_{\text{free}} = \frac{v_{33}^T W L}{2t_3} \left(1 + \frac{(6s_m t_3(t_2 + t_3)^2 - X)}{X} K_{31}^2 \right) \tag{18}$$

where $K31$ is the coupling coefficient
Thus, the voltage generated is found as a function of the applied force:

$$V = \frac{Q}{C_{\text{free}}} = -\frac{6d_{31} s_m t_3(t_2 + t_2) L}{v_{33}^T W X (1 + (\frac{6s_m t_3(t_2+t_3)^2}{X} - 1)K_{31}^2)} F_o. \tag{19}$$

All of the models described above are solved by using Matlab/Simulink. Simulation results are compared with the experimental results in the following sections:

$$V = -\frac{6d_{31} s_m t_3(t_2 + t_3) L}{v_{33}^T W X (1 + (\frac{6s_m t_3(t_2+t_3)^2}{X} - 1)K_{31}^2)} M_{\text{end}} \times \ddot{Z}(t).$$

6. V AND E RELATIONSHIP

The scalar electric potential (voltage) is that the electric field can be calculated from it. The component of electric field in any direction is the negative of rate of change of the potential in that direction. If the differential voltage change is calculated along a direction

ds, then it is seen to be equal to the electric field component in that direction times the distance ds.

$$dV = -\vec{E} \cdot \vec{ds} = -E_s ds$$
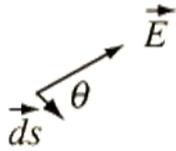
Evaluate the voltage
change dV along
the direction of ds

The electric field can then be expressed as

$$E_s = -\frac{dV}{ds} \text{ along ds, or } E_s = -\frac{\partial V}{\partial s}$$

For rectangular coordinates, the components of the electric field are

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Thus taking the partial derivative of above all
Voltage equations Eq. [3] , [13] and [19] with x, y and z gives the electric field in that direction.

7. RESULTS

The above Electric field lines obtained when encounters with the vibrations originating from focus of the earth , the collision between the two results in the slowly reducing and then simultaneously damping of the earthquake   radiations. As they approaches close to crystal   their   effect reduces   rapidly due to the continuous increase   in   intensity   of   electric field. Characteristic   functions   of   earthquake

vibrations represented as functions in the   form of their **mobility (1) or slowness (2):**

$$V(s) = H \frac{c_k s^k + c_{k-1} s^{k-2} + \ldots + c_1 s}{d_l s^l + d_{l-1} s^{l-2} + \ldots + d_0}$$

$$U(s) = H \frac{d_l s^l + d_{l-1} s^{l-2} + \ldots + d_0}{c_k s^k + c_{k-1} s^{k-2} + \ldots + c_1 s}$$

where:
k, l - natural numbers,
c, d - real numbers,
H - any positive real number.

$$V(s) = \frac{c_1}{s} + m_1 s + \cfrac{1}{\cfrac{s}{c_2} + \cfrac{1}{m_2 s + \ldots + \cfrac{1}{\cfrac{s}{c_n} + \cfrac{1}{m_n s}}}}$$

$$U(s)\frac{1}{H} = \frac{c_1}{s} + m_1 s + \cfrac{1}{\cfrac{s}{c_2} + \cfrac{1}{m_2 s}} + \ldots + \cfrac{1}{\cfrac{s}{c_n} + \cfrac{1}{m_n s}}$$

where:
c - elastic elements,
m - inertial elements.

Thus V(s) and U(s) are the slowness   functions which represents   the slowing   and damping down the effect of   consecutive vibrations of earthquake. These factors results in losing of velocity of earthquake vibrations and gradually damping of them.

8. CONCLUSIONS

This paper presents a theoretical approach in the field of civil engineering. The concept discussed above with mathematical solutions   shows that using the ability of peizo   ceramic   materials to   gets   polarized under effective   stresses resulting   strain   creates a potential

difference within the crystal. This potential can be harnessed to produce high voltages. The partial derivative of this voltage developed in three dimensions creates a network of electric field around the structure under which it is laid. This network acts a shield or cover to safeguard the structure from the consecutive effect of vibrations originating from focus of the earthquake. The voltage equations Provided in section [5.1],[5.2]and [5.3] shows voltage can be effectively produced in piezo materials whereas article [6] presents the solution to generate electric field from that voltage produced.

The slowness factor discussed under results in section [7] indicates the slowing and damping effect of vibrations under the influence of developed electric shield.

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