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# Asymptotic analysis on unsteady gravity flow of a power-law fluid flow through a porous medium

A. W.Ogunsola, B. A. Peter

Abstract— We present a paper on the asymptotic analysis of unsteady gravity flows of a power-law fluid through a porous medium. The fluids is Pseudo-plastic for n<1, Dilatant fluids for n>1 and Newtonian fluids for n=1.We assumed that porosity is large and analytical solutions are sought asymptotically. For the case of radial axisymmetric flow, the governing partial differential equations were transformed into an ordinary differential equations through similarity variable. It is further assumed that the viscosity is temperature dependent. We investigate the effects of velocity on the temperature field and the results obtained are discussed.

Index Terms— Asymptotic technique, non – Newtonian power-law fluid and porous media.

## 1 INTRODUCTION

The non-linear rheological effects of non-Newtonian fluids on the asymptotic analysis of unsteady gravity flows of a power-law fluid through a porous medium were addressed. Over the years, many research works had been carried out on power-law fluid, steady and unsteady flows of non-Newtonian fluids in a porous medium. A familiar example is an emulsion which is the dispersion of one fluid within another fluid, e.g.oil dispersed within water.

2 There have been several studies on the gravity flow of a power-law fluid through a porous medium. Peter and Ayeni[1] studied a note on unsteady temperature equation for gravity flow of a power-law fluid through a porous medium. Cortell [2] studied the unsteady gravity flows of a power-law fluid through a porous medium. Olajuwon and Ayeni [3] treated the flow of a power-law fluid with memory past an infinite plate. Pascal and Pascal [4] studied the similarity solution to some gravity flows of non-Newtonian fluids through porous media. Peter and Ayeni [5] investigated the analytical solution of unsteady gravity flow of a power-law fluid through a porous medium. Zueco [6] also investigated the numerical solutions for unsteady rotating high-porosity medium. Geetanjali Alle et al [7] studied the gravity flow of a visco-elastic fluid through an inclined channel with the effect of magnetic field. Geetanjali Alle et al [8] considered unsteady flow of a dusty visco-elastic fluid through an inclined channel .

Keeping in view the above analysis, the aim of the present paper is to analyze the influence of asymptotic techniques on unsteady gravity flow of a power-law fluid through a porous medium. In addition ,the investigation regarding the fluid problem due to the flow behavior when porosity is large are not available in the literature by means of asymptotic techniques method.

## 2.0. MATHEMATICAL FORMULATION

The governing equations for the Mathematical formulation are continuity and momentum equation. Considering a two dimensional flow in the x – z plane where the free surface is a streamline at a point on the surface, we expressed the flow by a modified Darcy's law.

$$V=\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\frac{\partial h}{\partial s}\left|\frac{\partial h}{\partial s}\right|^{\frac{1-n}{n}} \tag{2.1}$$

Where S is measured along the streamline, since z = h on the free surface. The rheological parameter n is the power-law exponent which represents shear-thinning (i.e.n<1) and shear-thickening (i.e. n>1) fluids. k is the permeability, $\rho$ is the density and $\mu_{ef}$ is the effective viscosity.

The unsteady gravity flow relevant to the problem is governed by the set of equations proposed by Cortel [2].

$$V_R=-\left(\frac{K\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\frac{\partial h}{\partial R}\left|\frac{\partial h}{\partial R}\right|^{\frac{1-n}{n}} \tag{2.2}$$

$$\frac{\partial(hv_R)}{\partial R}=-\Phi\frac{\partial h}{\partial t} \tag{2.3}$$

Where $\Phi$ being the porosity

By cylindrical coordinate we obtain

$$\frac{1}{R}\frac{\partial}{\partial R}\frac{\partial h}{\partial R}\left(\left|\frac{\partial h}{\partial R}\right|^{\frac{1-n}{n}}\right)=\Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}\frac{\partial h}{\partial t} \tag{2.4}$$

Defining new variables

Let $h(R,t)=t^{\alpha}f(\eta);\eta=Rt^{\beta}$ (2.5)

Substituting Eq.(2.5) into (2.4) and differentiating to get

$$\frac{d}{d\eta}\left(\eta f f^{1}\left|f^{1}\right|^{\frac{1-n}{n}}\right)=a^2\eta\left(\alpha f-\frac{n+\alpha}{n+1}\eta\frac{df}{d\eta}\right) \tag{2.6}$$

$$Where\quad a^2=\Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}} \tag{2.7}$$

$$f(\eta_1)=1 \tag{2.8}$$

$$\left(\frac{df}{d\eta}\right)\eta_1=1 \tag{2.9}$$

## 3.0. Methods of Solution

To solve the problem as posed in equation (2.6), we seek asymptotic techniques about $f(\eta)$.) The problem have been solved analytically by Picard iteration method.

Let $$f=f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+\ldots\ldots \tag{3.1}$$

Where $a=\sqrt{\Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{1/n}}$

$$f'=f_0+\frac{1}{a^2}f_1^1+\frac{1}{a^4}f_2^1+\ldots\ldots. \tag{3.2}$$

Substituting Equations (3.1), (3.2) into Eq. (2.6) to get

$$\frac{d}{d\eta}[\eta(f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+......)\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+......\right)\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+........\right)^{\frac{1-n}{n}}$$

$$=a^2\eta\left|\alpha\{\ f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+......\ \}-\frac{\alpha+n}{n+1}\eta\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+.....\right)\right.$$

(3.3)

Suppose $n=\frac{1}{2}$

Substitute for n in Eq.(3.3) to get

$$\frac{d}{d\eta}[\eta(f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+......)\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+......\right)\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+........\right)$$

$$=a^2\eta\left|\alpha\{\ f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+......\ \}-\frac{2\alpha+1}{3}\eta\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+.....\right)\right.$$

(3.4)

Multiply Eq.(3.4) through by $\frac{1}{a^2}$ to get

$$\frac{d}{d\eta}[\eta(\frac{1}{a^2}f_0+\frac{1}{a^4}f_1+\frac{1}{a^6}f_2+......)\left(\frac{1}{a^2}f_0'+\frac{1}{a^4}f_1'+\frac{1}{a^6}f_2'+......\right)^2$$

$$=\eta\left|\alpha\{\ f_0+\frac{1}{a^2}f_1+\frac{1}{a^4}f_2+......\ \}-\frac{2\alpha+1}{3}\eta\left(f_0'+\frac{1}{a^2}f_1'+\frac{1}{a^4}f_2'+.....\right)\right.$$

(3.5)

Equating the values of $\frac{1}{a^2}$ to get

$$\eta\alpha f_1-\frac{2\alpha+1}{3}\eta^2 f_1'=0$$

(3.6)

$$f_0(f_0')^2=\eta\alpha f_2-\frac{2\alpha+1}{3}\eta^2 f_2'$$

(3.7)

$$f_0(\eta)=1$$

(3.8)

Integrating Eq. (3.6) together with the boundary condition (3.8) to get

$$f_0(\eta)=\eta^{\frac{3\alpha}{2\alpha+1}}$$

(3.9)

Substitute (3.8) in (3.7) to get

i. $$f_2'=\frac{3}{2\alpha+1}\left|\frac{1}{\eta}\alpha f_2-\left(\frac{3\alpha}{2\alpha+1}\right)^2\eta^{\left(\frac{3\alpha}{2\alpha+1}\right)2}\right|$$

(3.10)

Case 1

Definition: The function $f(f_2,\eta)$ is Lipschitz continuous in $f$ over $D$ if there exists a constant $k$, $0<k<\infty$, such that

$\left|\ f(f_{21},\eta)-f(f_{22},\eta)\right.\leq k\left|f_{22}-f_{21}\right|$ and $\frac{\partial f}{\partial f_2}$ exist.

Let $f_2'=f(f_2,\eta)$

$$f(f_2,\eta)=\frac{3}{2\alpha+1}\left|\frac{1}{\eta}\alpha f_2-\left(\frac{3\alpha}{2\alpha+1}\right)^2\eta^{\left(\frac{3\alpha}{2\alpha+1}\right)2}\right|$$

(3.11)

$$\frac{\partial f}{\partial f_2}=\frac{3\alpha}{2\alpha+1}\eta^{-1}$$

(3.12)

$$k=\frac{3\alpha}{2\alpha+1}\eta^{-1}$$

(3.13)

Remark: The function $f(f_2,\eta)$ is Lipschitz continuous.

Case 2

Defining a sequence of functions $\{ (f_{2n}, \eta) \}$ by Picard iterations

$$f_n(\eta) = f_0 + \int_0^\eta y(s, f_{n-1}(s))ds \tag{3.14}$$

$$f_{21}(\eta) = f_{20} + \int_0^\eta y(s, f_{20}(s))ds \tag{3.15}$$

Integrating Eq.(3.15) to get

$$f_{21}(\eta) = -\left(\frac{3}{2\alpha+1}\right)\left(\frac{3\alpha}{2\alpha+1}\right)^2\left[\left(\frac{2\alpha+1}{3\alpha}\right)^2+1\right]\eta^{\left(\frac{3\alpha}{2\alpha+1}\right)^2+1}$$

$$+\left(\frac{3}{2\alpha+1}\right)\left(\frac{3\alpha}{2\alpha+1}\right)^2\left[\left(\frac{2\alpha+1}{3\alpha}\right)^2+1\right]\eta_1^{\left(\frac{3\alpha}{2\alpha+1}\right)^2+1} \tag{3.16}$$

Let $\alpha = -1, \eta_1 = \frac{1}{2}$ in Eq.(3.16) to get

$$f_{2(1)} = 4.5\eta^6 - 4.5\eta_1^6 \tag{3.17}$$

**4.0 Results**

Analytical solutions of Equations (3.9) and (3.17) together with the boundary condition (3.8) were provided for various parameters in the flow equations.

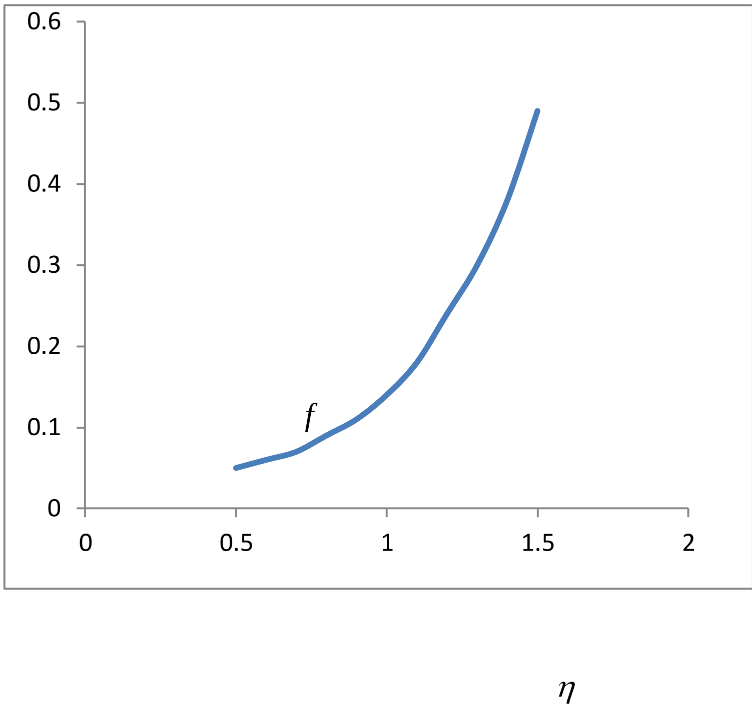


Fig4.1: Graph of the velocity function $f$ against the similarity variable $\eta$ when $\alpha = -1$ and $n = \frac{1}{2}$.

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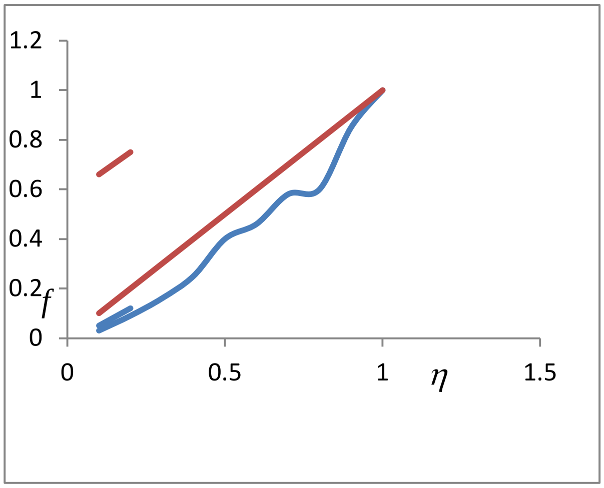


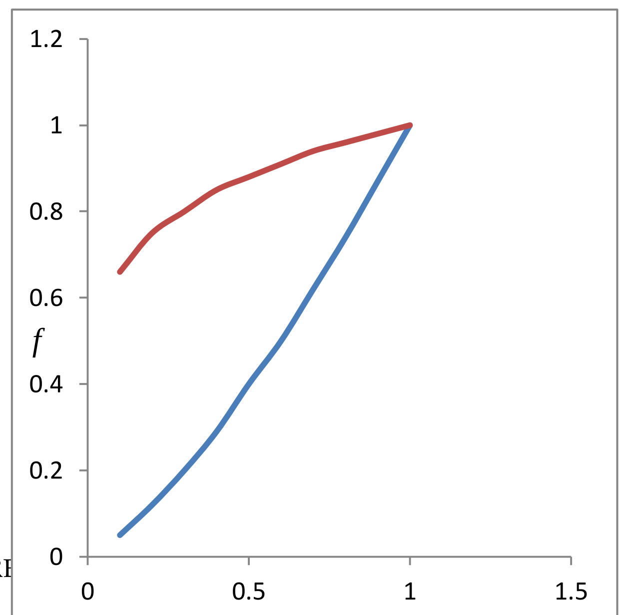

$\eta$

Fig4.3: Graph of the velocity function $f$ against the similarity $\eta$ wh

when $\alpha = \frac{1}{10}, 2$ and $n = 1$.

Fig4.2: Graph of the velocity function $f$ against the similarity $\eta$ when $\alpha \leq 2$ and $n \leq 2$.

variable $\eta$ when $\alpha \leq 2$ and $n \leq 2$.

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DEPT. OF PURE AND APPLIED MATHEMATICS,LADOKE
AKINTOLA UNIVERSITY OF TECHNOLOGY,
OGBOMOSO,NIGERIA.

amossolawale@yahoo.com

pbenjamin2008@yahoo.ca