A new class of Graceful Tree

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Abstract— A tree is graceful if it has a graceful labeling. A graceful labeling of a tree T with m edges is a function f: V(T)→ {0,…….,m} such that distinct vertices receive distinct numbers and {|f(u)-f(v)|: uv ϵ E(T)} ={1,……..,m}.The problem of whether or not all trees are graceful is still open. Many classes of trees have already been proved graceful such as paths, caterpillars, symmetrical trees, spider trees, lobster trees, star trees, firecrackers, banana trees etc. Trees of diameter at most five and trees with up to 35 vertices have been shown to be also graceful. In this paper we prove that trees consists of several stars all connected to a single star is graceful.

Keywords— Caterpillar tree, Banana Trees , Firecrackers ,Graceful labeling, Star trees, Superstar tree, Symmetrical tree, Spider Trees

1 INTRODUCTION

Let, G be a graph with m number of edges and n number of vertices. If its n vertices can be distinctly labeled using integers 0, 1,..., m so that the difference between any two distinct vertices are also all distinct , then the labeling is termed graceful and the graph is said to be a gracefully labeled graph.

A tree T with n vertices has n-1 edges. If a labeling on vertices, in which every number from 0 to n-1 appears once and only once, can induce a labeling on edges where every number from 1 to n-1 appears once and only once, this labeling on vertices is called a graceful labeling. If a tree has at least one graceful labeling, it is called a graceful tree.

Gracefully labeled graphs are finding applications in coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and database management [1].

There is a well known conjecture that says "every tree is graceful". Many classes of trees have been shown to be graceful [2]. However, it has not yet been possible to prove the conjecture for all trees. A lot of work has been done by many researchers toward proving this conjecture. So far some special classes of trees have been shown to be graceful. For example paths, caterpillars, symmetrical trees, spider trees, lobster trees, star trees, firecrackers, banana trees etc. Trees of diameter at most five and trees with up to 35 vertices have also been shown graceful. In this paper we have proved a new general class of gracefully labeled tree.

2 PRELIMINARIES

In this section we define and discuss classes of trees that have been proved graceful.

2.1 Star Trees[5] :

In graph theory, a tree with one internal node and k leaves is said to be a star S1,k that happen to be a complete bipartite graph K1,k.

Theorem 1. All Star trees are graceful.

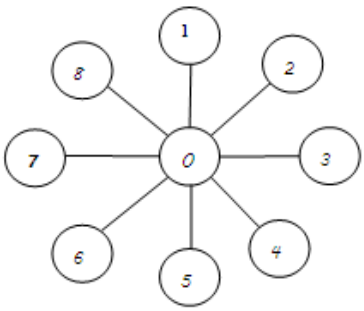

Fig. 1. Star tree (S_8)

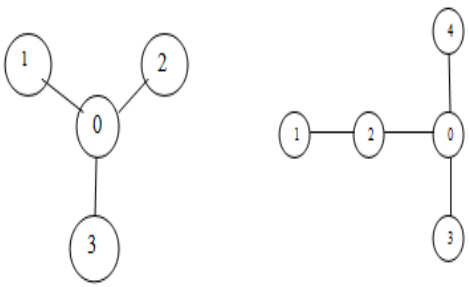

Fig. 2. Spider Tree

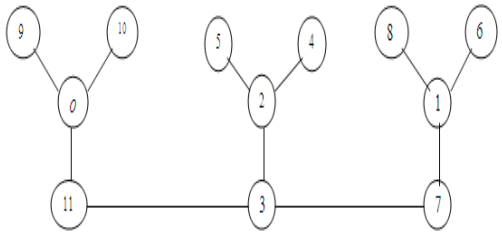

Fig. 3. Firecracker tree

2.2 Spider Trees [3]:

A spider tree is a tree with at most one vertex of degree greater than 2. If such a vertex exists, it is called the branch point of the tree. A leg of a spider tree is any one of the paths from the branch points to a leaf of the tree.[Fig 2.]

Theorem 2. All Spider trees are gracefull.

2.3 Firecrackers[2] :

An (n, k) -firecracker is a graph obtained by the concatenation of n, k-stars by linking one leaf from each. [Fig 3.]

Theorem 3. All firecrackers are graceful.

2.4 Banana Trees[2]:

A banana tree consists of a vertex v joined to one leaf of any number of stars. An n, k banana tree, as defined by Chen et al. (1997), is a graph obtained by connecting one leaf of each of n copies of an K- star graph with a single root vertex that is distinct from all the stars. [Fig 4.]

Theorem 4. All banana trees are graceful.

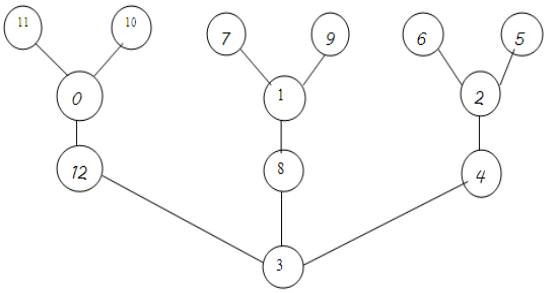

Fig. 4. Banana tree

2.5 Lobstar tree[5] :

Lobstar are defined as trees that result in caterpillar when all pendant vertices are deleted. [Fig 5.] Wang, working with Jin ,Lu, Zhang ,[6] showed gracefulness of a class of Lobsters tree.

Theorem 5. All lobstar trees are graceful.

2.6 Symmetrical tree[4] :

A rooted tree in which every level contains vertices of the same degree is called symmetrical trees. The following tree is a gracefully labeled symmetrical tree on 15 vertices. [Fig 6.]

Theorem 6. All symmetrical trees are graceful.

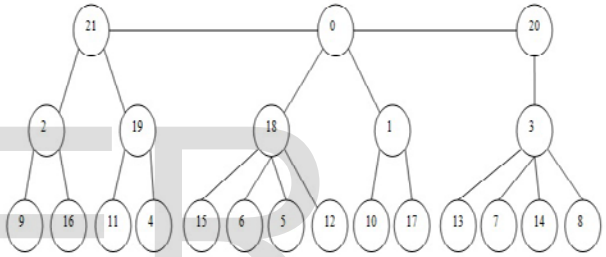

Fig. 5 Lobstar Tree

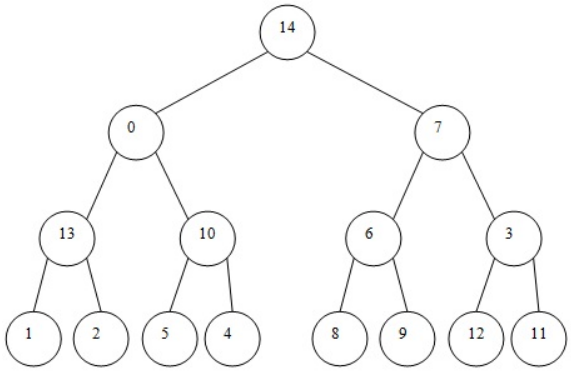

Fig. 6. Symmetric Tree

2.7 Caterpillar tree[1] :

A caterpillar is a tree such that if all leaves are removed, the remaining graph is a path. This path

can be termed as backbone of the caterpillar.Rosa proved in his paper [7] that all caterpillars are graceful. In the following we show a caterpillar and its graceful labeling. [Fig 7.]

Theorem 7. All Caterpillars are graceful.

2.8 Super Caterpillar tree[1] :

Let T0 be any arbitrary caterpillar and Ti, i = 1, ..., k be caterpillars with |Ti| = m number of vertices and sum total of vertices is the same in odd levels of all pairs T2i+1 and T2i+2. In case k being an odd number, one caterpillar will be without a pair. Let one end of each backbone be joined to the vertex v by an edge. Then the resulting tree is called a super-caterpillar. [Fig 8.]

Theorem 8. All Super Caterpillars are graceful.

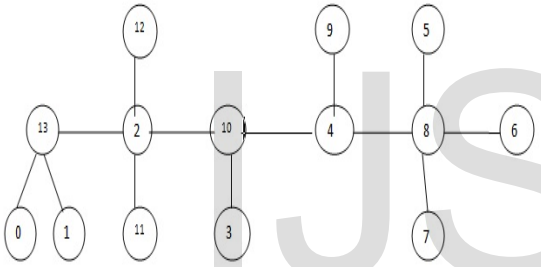

Fig. 7. Caterpillar tree

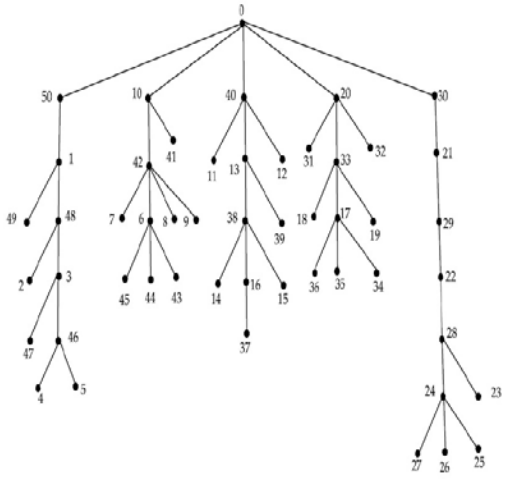

Fig. 8. Super caterpillar tree[1]

3 OUR RESULTS

Superstar tree is a tree that consists of several stars all connected to a single star is graceful. [Fig 9.]

Theorem 9. All Superstar trees are graceful.

Let Si, i=0,...,k be a set of k+1 stars with stars Si=0,1,...,k-1 connected to Sk. Let total number of vertices of all the stars be n+1. If there are at least i stars of size at least k-i then $T=\cup_{i=0}^{k}(Si)$ is graceful.

Proof . Let S_i' for i=0 to k-1 be a star of at least k-i vertices. Let N_i be the number of leaves of all stars with index ≤ i. Label centre of star S_i' by i. Start labeling leaves of S_i' by n-N_i downward in such a way that leaf connected to the star S_k will have a label equal to k- i+ n-N_i .By doing so edges will be labeled by consecutive labels up to k + n-N_i.So all labels must be generated if we follow this process.

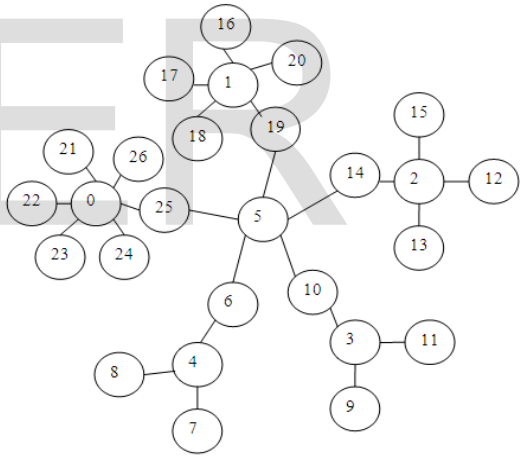

Fig. 9. Superstar tree

3.2 Algorithm

1. Let S_i, i=0,1,…,k be a set of k+1 stars with stars S_i, i=0,1,…,k-1 connected to S_k.
2. Let N_i be the number of leaves of all stars with index ≤ i and total number of vertices of all the stars be n+1.
3. Let total number of star S_i of the superstar tree is k.
4. Assign k to the root of the central star S_k.
5. i=0
6. While(i<k)
    a. if size of leaf star S_i at least k-i then

i. Assign i to the root of leaf star S_i.
ii. start labeling S_i by n-N_i downward in such a way that leaf connected to the star S_k will have a label equal to k-i+n-N_i .
iii. i++

b. end if

7. end while

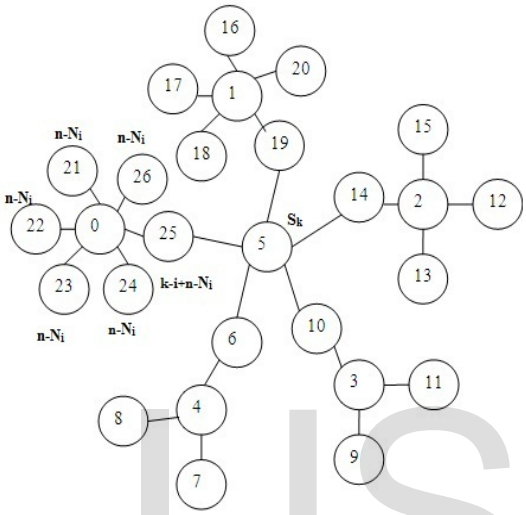

Fig. 10. Illustration of Superstar tree according to the algorithm

4 Conclusion

We have proved a general class of tree that consists of several stars all connected to a single star is graceful. In addition to caterpillar our class of tree maintains a property that every vertex has label greater than the labels of all adjacent vertices or vice-versa.

References

[1] Md. Forhad Hossain, Md. Momin Al Aziz,M Kaykobad, "New classes of graceful trees", unpublished.

[2] Elina Robeva, "An Extensive Survey of Graceful Trees", Stanford University (2011).

[3] Patrick Bahls, Sara Lake, and Andrew Wertheim, "Gracefulness of Families of Spiders".

[4] Bermond, J., Schonheim, J.: "G-decomposition of kn, where g has four vertices or less". Discrete Mathematics 19(2), 113-120 (1977).

[5] R.Lynn Watson, "A SURVEY ON THE GRACEFULL LABELING OF GRAPHS", B.S. , Roanoke College, 1972.

[6] D. J Jin and J.G. wang. "The gracefulness of the tree with a diameter tour". Acta Scientiarum Naturalium Universitaties Jilinensis, 1:17-22,1993.

[7] Rosa, A.: "On certain valuations of the vertices of a graph". Theory of Graphs p.349355 (1982).

[8] Gallian, J.A.: "A dynamic survey of graph labeling". Dynamic Survey 6, 43 (1997).

[9] Huang, C., K., Rosa, A.: "A further results on tree labellings". Utilitas Mathematica 21, 31–48 (1982).

[10] V. J. Abhyanker, "Direct methods of gracefully labelling graphs", Ph. D. thesis, University of Mumbai, 2002.

[11] R.E.L. Alfred and B.D. McKay. "Graceful and harmonious labeling of trees". Bull. Inst.Appl. 23:69-72, 1998].

[12] R.E.L Alfred, J. Siran, M. Siran, "A Note on the Number of Graceful Labelings of Paths", *Discrete Math.*, 261, 2003, 27-30.

[13] V. Bhat-Nayak and D. Deshmukh, "New families of graceful banana trees", *Proc. Indian Acad. Sci. Math. Sci.*, 106 (1996) 201-216.

[14] J. C. Bermond, "Graceful Graphs, radio antennae and French windmills", *Graph Theory and Combi-natorics*, Pitman, London (1979) 18-37.

[15] M. Burzio and G Ferrarese, "The Subdivision of a Graceful Tree is a Graceful Tree", *Discrete Math.*,181, 1998, No. 1-3, 275-281.

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