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A New Investigation about the Artificial Intelligence in Reproducing Kernel Hilbert Spaces: An Analytical Study

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Abstract—The current paper aims to compute the artificial intelligence for regression in bounded subspaces of Reproducing Kernel Hilbert Spaces (RKHS) for the Support Vector Machine (SVM) regression. Both ε -insensitive loss function and general L_p loss functions are studied. It is shown that the artificial intelligence is finiteness. This, in turn, confirms that the probability for regression machines in RKHS subspaces using the L_ε or general L_p loss functions is uniformly converged. Further, the results are verified in a new fashion in the case of introducing a bias to the functions in the RKHS.

Index Terms— Artificial Intelligence, Applied Mathematics, Reproducing Kernel Hilbert Spaces, Regression, Support Vector Machines, Regularization Networks

1 Introduction

The artificial intelligence of real-valued functions $L(y,f(x))=|y-f(x)|^p$ and $L(y,f(x))=|y-f(x)|_\varepsilon$ with f in a bounded sphere in a Reproducing Kernel Hilbert Space (RKHS), is computed in the current paper. Considering these loss functions, it is shown that the artificial intelligence is finite and then, an upper bound is computed for the dimension. The problem is solved by two solutions. A discussion on a simple argument, leading to a loose upper bound on the artificial intelligence, is introduced, at first, and then, the results from the case of infinite dimensional RKHS, which is frequently considered in the literature as the type of hypothesis spaces [1-16], is refined. The results are applied to some standard regression learning machines including Regularization Networks (RN) and Support Vector Machines (SVM) [17, 18]. Moreover, the artificial intelligence is innovatively computed in the case of introducing a bias to the functions; i.e. when $f=f_0+b$, where $b\in R$ and f_0 is in a sphere in an infinite dimensional RKHS [19].

It is previously confirmed that when L is used as loss function in a regression learning problem, a necessary and sufficient condition for uniform convergence in probability is finiteness of the artificial intelligence for all $\gamma>0$ [20]. Accordingly, the results of the current paper confirm the uniform convergence of both RN and for SVM regressions [21, 23, 24, and 27].

The problem of patter recognition, with L works as an indicator function, was considered in

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previous related works [21-39]. However, the fat-shattering dimension [40] was introduced as the substitute of the artificial intelligence [41-45]. By presenting entropy numbers of operators as cover of number arguments, a different approach is followed to confirm uniform convergence for RN and SVM [46-62]. However, regression as well as the case of non-zero bias b was not comprehensively considered in both of them [63-73].

According to the framework of statistical learning theory, the problem of learning from examples is taken into account in this study [74]. By randomly sampling from a space $X \times Y$ with $X \subset R^d$, $Y \subset R$ a set of ℓ examples $\{(x_1, y_1), ..., (x_\ell, y_\ell)\}$ is generated [75]. It is based on an unknown probability distribution $P(x, y)$ [76]. It is assumed that X and Y are bounded [77]. The problem of learning is defined as finding a function $f : X \to Y$, according to a set of examples, through it, the value y, corresponded to new point $x \in X$, can be predicted [78].

It is well-known that the problem of learning from examples is ill-posed [79, 80]. Performing Empirical Risk Minimization (ERM), using a specified loss functions, and with limiting the solution to the problem to be in a "small" hypothesis space [80] is the tradition solution of the problem. The goal of the solution is minimizing the empirical risk

$$I_{emp}[f] = \frac{1}{\ell}\sum_{i=1}^{\ell} L(y_i, f(x_i)) \quad \text{with } f \in H,$$

where L is the loss function measuring the error as the difference of predicted, $f(x)$ and actual, y values and H is a given hypothesis space [81].

The hypothesis spaces of functions considered in the current study are hyperplanes in some feature space:

$$f(x) = \sum_{n=1}^{\infty} \omega_n \phi_n(x) \tag{1}$$

with:

$$\sum_{n=1}^{\infty} \frac{\omega_n^2}{\lambda_n} < \infty \tag{2}$$

where $\phi_n(x)$ is a set of given, linearly independent basis functions, λ_n are given non-negative constants such that $\sum_{n=1}^{\infty} \lambda_n^2 < \infty$. The form of functions' spaces represented in Eq. (1) is similar to which is used in Reproducing Kernel Hilbert Spaces (RKHS) [81, 82] with kernel K given by:

$$K(x, y) = \sum_{n=1}^{\infty} \lambda_n \phi_n(x) \phi_n(y) \tag{3}$$

Eq. (2) gives the RKHS norm of f, $\|f\|_K^2$ where f is defined according to Eq. (1). However, the number D of features ϕ_n (if D is finite, all sums above are also finite) is the dimensionality of the RKHS [83].

By limiting the hypothesis space so that it consists of functions in a RKHS with norm less than a constant A, the general setting of above mentioned learning becomes:

$$Minimize: \ \frac{1}{\ell}\sum_{i=1}^{\ell} L(y_i, f(x_i))$$
$$subject\ to: \ \|f\|_K^2 \le A^2 \tag{4}$$

The consistency of learning machines defined by Eq. (4) is a critical issue. It was found that by approaching the number of examples (x_i, y_i) to infinity, the probable error of the solution should converge in probability to the minimum expected error in the hypothesis space [80, 83]. For learning machines performing ERM in a hypothesis space Eq. (4), it was shown that consistency is related to uniform convergence in probability [84]. In addition, depending on the artificial intelligence of the considered hypothesis space, which indicates the complexity of the space, necessary and sufficient conditions for uniform convergence are defined [84, 85].

The VC-dimension is typically used in statistical learning theory as the measure of complexity. However, when RKHS is dimensionally infinite,

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the VC-dimension also is infinite both for L_p and L_ε, in the above learning setting. As a result, the VC-dimension is not applicable to investigate learning machines of the form Eq. (4). In this regard, another measure of complexity (e.g., the artificial intelligence) should be considered to demonstrate uniform convergence in infinite dimensional RKHS.

2 Results and Discussion

In the following, it is assumed that data X are within a sphere of radius R in the feature space defined by the kernel K of the RKHS and y is bounded between -1 and 1. As a result of these assumptions, a theorem can be described as follows:

Theorem. The artificial intelligence h for regression, considering L_p or L_ε as loss functions, for hypothesis spaces $H_A = \left\{ f(x) = \sum_{n=1}^{\infty} \omega_n \phi_n(x) \middle| \sum_{n=1}^{\infty} \frac{\omega_n^2}{\lambda_n} \le A^2 \right\}$ and y bounded, is finite for $\forall_\gamma > 0$. If D is the dimensionality of the RKHS, then $h \le O\left(\min\left(D, \frac{(R^2+1)(A^2+1)}{\gamma^2} \right) \right)$.

Proof. Considering L_1 as loss function and B as the upper bound of L_1, the rules for separating points can be decomposed according to the following:

$$\begin{aligned} &class\ 1 if\ \ y_i - f(x_i) \ge s + \gamma \\ &\quad or\ f(x_i) - y_i \ge s + \gamma \\ &class\ -1 if\ \ y_i - f(x_i) \le s - \gamma \\ &\quad or\ f(x_i) - y_i \le s - \gamma \end{aligned} \quad (5)$$

for some $\gamma \le s \le B - \gamma$. It should be noted that, despite the number of N points, the number of separations possible to get by rules Eq. (5) cannot more than the number of separations possible to get by the product of two indicator functions (of hyperplanes with margin):

$$\begin{aligned} &function\ (a): \text{class} 1 \quad if\ \ y_i - f_1(x_i) \ge s_1 + \gamma \\ &\qquad \text{class} -1 \quad if\ \ y_i - f_1(x_i) \le s_1 - \gamma \\ &function\ (b): \text{class} 1 \quad if\ \ f_2(x_i) - y_i \ge s_2 + \gamma \\ &\qquad \text{class} -1 \quad if\ \ f_2(x_i) - y_i \le s_2 - \gamma \end{aligned} \quad (6)$$

where f_1 and f_2 are in H_A, $\gamma \le s_1$, $s_2 \le B - \gamma$. Recovering Eq. (5), for $s_1 = s_2 = s$ and for $f_1 = f_2 = f$, gives the results same as to what will be obtained if one follows Eq. (5). For example, if $y - f(x) \ge s + \gamma$ then indicator function (a) will result -1 and indicator function (b) will result -1. Hence, their product will result $+1$, same as Eq. (5). Therefore, it can be obviously seen that the number of separations for any considered set of points can be considerably increased if more freedom give to f_1, f_2, s_1, s_2 compared to when Eq. (5) is followed.

It is previously mentioned that the number of separations, for any N points, is bounded by the growth function. However, it was shown that the growth function for products of indicator functions is enclosed by the product of the growth functions of the indicator functions. Moreover, the indicator functions in Eq. (6) are hyperplanes and its margin is in the $D+1$ dimensional space of vectors $\{\phi_n(x), y\}$ where R^2+1 is the radius of the data, the norm of the hyperplane is bounded by A^2+1, (where 1 added in both cases due to y), and the margin is at least $\frac{\gamma^2}{A^2+1}$. It was previously found that the artificial intelligence h_γ of these hyperplanes is bounded by $h_\gamma \le \min\left((D+1)+1, \frac{(R^2+1)(A^2+1)}{\gamma^2} \right)$.

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Therefore, whenever $\ell \geq h_\gamma$, the growth function of the separating rules Eq. (5) is bounded by $g(\ell) \leq \left(\frac{e\ell}{h_\gamma}\right)^{h_\gamma} \left(\frac{e\ell}{h_\gamma}\right)^{h_\gamma}$. Considering h_γ^{reg} as the artificial intelligence, h_γ^{reg} is limited to be smaller than the larger number ℓ for which inequality $2^\ell \leq \left(\frac{e\ell}{h_\gamma}\right)^{h_\gamma} \left(\frac{e\ell}{h_\gamma}\right)^{h_\gamma}$ holds. In this regard, as $\ell \leq 5h_\gamma$, therefore

$$h_\gamma^{reg} \leq 5\min\left(D+2, \frac{(R^2+1)(A^2+1)}{\gamma^2}\right).$$

It is proved the theorem for the case of L_1 loss functions.

By rewriting Eq. (5) as follows, a same proof can be achieved for general L_p loss functions:

$$\begin{aligned} &class\ 1 if\ \ y_i - f(x_i) \geq (s+\gamma)^{\frac{1}{p}} \\ &\quad or\ f(x_i) - y_i \geq (s+\gamma)^{\frac{1}{p}} \\ &class\ -1 if\ \ y_i - f(x_i) \leq (s-\gamma)^{\frac{1}{p}} \\ &\quad or\ f(x_i) - y_i \leq (s-\gamma)^{\frac{1}{p}} \end{aligned} \tag{7}$$

Moreover, for $p > 1$, $(s+\gamma)^{\frac{1}{p}} \geq s^{\frac{1}{p}} + \frac{\gamma}{pB}$ (since $\left(\text{since}\ \gamma = \left((s+\gamma)^{\frac{1}{p}}\right)^p - \left(s^{\frac{1}{p}}\right)^p \leq \left((s+\gamma)^{\frac{1}{p}} - s^{\frac{1}{p}}\right)(pB)\right)$) and $(s-\gamma)^{\frac{1}{p}} \leq s^{\frac{1}{p}} - \frac{\gamma}{pB}$ (similarly). Similar to above argument, it can be found that the artificial intelligence is bounded by $5\min\left(D+2, \frac{(pB)^2(R^2+1)(A^2+1)}{\gamma^2}\right)$.

Finally, Eq. (5) can be rewritten as follows for the L_ε loss function:

$$\begin{aligned} &class\ 1 if\ \ y_i - f(x_i) \geq s+\gamma+\varepsilon \\ &\quad or\ f(x_i) - y_i \geq s+\gamma+\varepsilon \\ &class\ -1 if\ \ y_i - f(x_i) \leq s-\gamma+\varepsilon \\ &\quad or\ f(x_i) - y_i \leq s-\gamma+\varepsilon \end{aligned} \tag{8}$$

where calling $s' = s+\varepsilon$, the above mentioned proof can be used to find the upper bound on the artificial intelligence same as to that found for the L_1 loss function. (It should be noted that if the constraint $\gamma \leq s \leq B-\gamma$ is considered, it seems that it would have a little effect on the artificial intelligence for L_ε).

It can be concluded from these results that the artificial intelligence is still finite and is influenced only by $5\frac{(R^2+1)(A^2+1)}{\gamma^2}$ when RKHS is dimensionally infinite.

3 Conclusions

A novel approach is introduced in the current paper to compute the artificial intelligence of RKHS when L_p and L_ε are considered as loss functions. It is found that better bounds can be achieved if ε takes into ε account in the computations when L_ε considered as loss function. As an instance, it is clearly proved that the artificial intelligence is bounded by $\frac{p^2(B-\varepsilon)^2 R^2 A^2}{\gamma^2}$, when $|f(x)-y|_\varepsilon^p, p>1$ considered as the loss function. However, it is found that ε has a low influence (given that $\varepsilon \ll B$). Moreover, more

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general loss functions can be introduced to the presented computations. Appearing the eigenvalues of the matrix G in the computation of the artificial intelligence is very interested. By computing the number of separations for a given set of points, similar to that performed for the largest and smallest eigenvalues in the proofs, all the eigenvalues of G can be considered. As a result of this computation, interesting relations can be found. In addition, for obtaining the bounds on the generalization performance of regression machines of the form Eq. (4), the bounds on the artificial intelligence can be effectively used.

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